NONLINEAR BUCK CIRCUIT IDENTIFICATION USING ORTHONORMAL FUNCTIONS WITH HEURISTIC OPTIMIZATION

Elder Oroski*, Rafael Holdorf Lopez†, Adolfo Bauchspiess‡

*Laboratory of Automation and Robotic (LARA), Department of Electrical Engineering (ENE)
Universidade de Brasília-UnB
70970-900 Brasília/DF BRAZIL

†Center for Optimization and Reliability (CORE), Departament of CIvil Engineering
Universidade Federal de Santa Catarina-UFSC
88037-000 Florianópolis/SC BRAZIL

Emails: eoroski@gmail.com, rafael.holdorf@ufsc.br, adolfobs@unb.br

Abstract— Nonlinear systems identification is a research topic of great interest in several fields, such as Economics, Electrical and Control Engineering. Therefore, this paper pursues the identification of a buck converter through the orthonormal basis functions of Laguerre and Kautz. In order to evaluate the coefficients of the functions, a Genetic Algorithm (GA) is proposed. This GA uses the Nelder Mead algorithm and the Least Mean Square method as local optimizers during the search process. A comparison between the models obtained from each of the orthonormal basis is presented. Moreover, the functionality of the proposal is proved through the comparison between the proposed GA and a classic GA.

Keywords— System Identification, Orthonormal Basis Function, Genetic Algorithms, Simplex Optimization.

Resumo— Os problemas de identificação de sistemas não lineares vem se tornando cada vez mais presentes nas áreas de Economia, Engenharia Elétrica e Engenharia de Controle. Neste contexto, este artigo aborda a identificação de um conversor CC-CC do tipo Buck. Para tanto são utilizadas funções de base ortonormal, como as bases de Laguerre e de Kautz. Para a determinação dos parâmetros de modelo são utilizados métodos heurísticos, como os Algoritmos Genéticos e Nelder Mead. Tais métodos se fazem necessários devido a extensão do espaço de busca dos parâmetros de modelo. Ao final do artigo são apresentadas comparações entre os modelos obtidos a partir cada uma das bases ortonormais.

Palavras-chave— Identificação de sistemas, Funções Ortonormais, Algoritmos Genéticos, Nelder Mead.

1 Introduction

The use of orthonormal basis functions in dynamic systems identification is widely found in literature (CAMPELLO et al., 2007; HEUBERGER et al., 2005; MACHADO et al., 2010; ROSA et al., 2009). Among these functions, the classic functions of Laguerre and Kautz are the most used. The combination of these functions, parameterized by different poles, is known as Generalized Orthonormal Functions and it constitutes an appealing alternative for the identification of systems whose dynamic is more complex (HEUBERGER et al., 2005).

The final objective of this paper is the identification of a real Buck converter using the discrete orthonormal functions of Laguerre and Kautz, implemented as digital filters. In order to accomplish this goal, first a Genetic Algorithm (GA) is proposed. The novelty in this algorithm is the use of the Nelder Mead, NM, algorithm and the Least Mean Square, LMS, method as local optimizers in order to obtain the parameters of the model. These local optimizers are expected to accelerate the convergence of the optimization problem as well as to obtain more accurate solutions for the identification problem.

The functions of Laguerre are better suitable to model systems with poles purely real or whose imaginary component value is small (HEUBERGER et al., 2005). On the other hand, the functions of Kautz usually obtain better performance in modeling underdamped dynamical systems (MACHADO et al., 2010).

The orthonormal basis functions approach is interesting due to two key aspects: first, the knowledge of past terms of the input and output signals is not necessary and, second, there is no output feedback, therefore, there is no propagation of recursion errors (CAMPELLO et al., 2007).

Identification process aim to find a model for a system, based on empirical data (LJUNG, 1999). This could be a very complex problem, depending on how difficult is to finding the derivatives, Jacobian and Hessian matrix of the system. Thus, the use of heuristic methods is appealing.

This paper is organized as follows: Section 2 explains the basic concepts of dynamic systems identification and presents some simulations in order to illustrate the application of orthonormal basis functions. Then, in section 3, the functionality of the proposed method is shown through the identification of a real Buck converter. Section 4 presents a comparison between the results obtained and, finally, section 5 gives the main conclusions of this work.

2 Dynamic System Identification

This section presents a discussion of the identification of simulated linear systems. The identification was based on Laguerre and Kautz functions. The main goal is to show how these functions can be employed in the identification of dynamic systems.

2.1 Orthonormal Functions

The main property of the orthonormal functions is expressed by (1):

$$\langle \psi_m, \psi_n \rangle = \begin{cases} 0 & m \neq n \\ 1 & m = n \end{cases} \qquad (1)$$

where ψ_m and ψ_n are orthonormal functions and $\langle . \rangle$ is an inner product, proper to those continuous functions, defined by equation (2) as given by (HEUBERGER et al., 2005):

$$\langle \psi_m(z), \psi_n(z) \rangle = \frac{1}{2\pi i} \oint_{\mathcal{C}} \psi_m(z).\psi_n^*(1/z^*) \frac{dz}{z}, \qquad (2)$$

where $\mathcal{C}$ is the unitary circle and i is the imaginary unity. For discrete functions one can define an inner product by equation (3).

$$\langle \psi_m(k), \psi_n(k) \rangle = \sum_{k=-\infty}^{\infty} \psi_m(k).\psi_n^*(k). \qquad (3)$$

Thus, the orthonormal functions must meet the following requirements:

- $\psi_m \perp \psi_n$ for $m \neq n$;
- $|\psi_n| = 1, \quad \forall n.$

The main orthonormal functions are the Hermit, Jacobi, Laguerre, Legendre, Kautz and the generalized orthonormal functions (GOBF), (WILLEM BELT, 1997). In the next subsection, the orthonormal functions employed in this paper are presented.

2.1.1 Laguerre Functions

Laguerre functions are parameterized by a real pole (ROSA et al., 2009; LEMMA et al., 2010), and they can be expressed by:

$$L_n(z) = \frac{\sqrt{1-p^2}}{z-p} \left(\frac{1-pz}{z-p} \right)^{n-1} \qquad (4)$$

where p is the pole of the Laguerre functions and z is the complex variable associated with the Z transform. The functions of a Laguerre basis can be implemented in the form of filters, so that each function will be associated to a filter, as can be seen in figure 1.

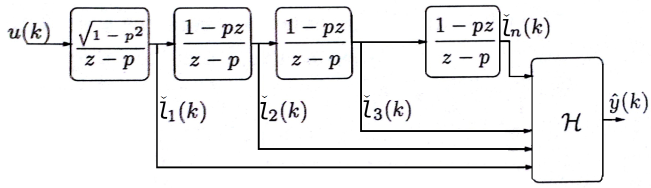

Figura 1: OBF model with Laguerre dynamics.

In the case of Linear Time-Invariant (LTI) systems, the operator $\mathcal{H}$ can be defined as the linear combination of the Laguerre filters $l_1, l_2, \cdots, l_n$.

In order to exemplify the concept of modeling a LTI system with Laguerre functions, regard the following stable LTI system sampled at a rate of 50 ms:

$$H(z) = \frac{z+2}{z^2 - 0.3z + 0.02} \qquad (5)$$

The response of the system to a step input (5) is shown in figure 2.

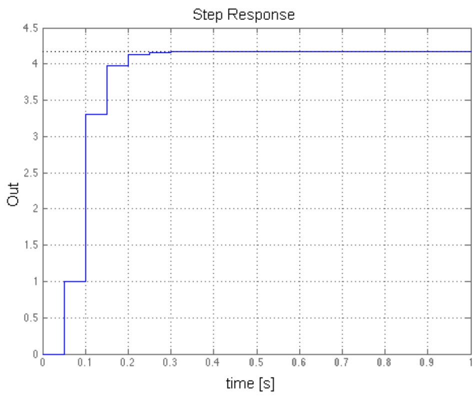

Figura 2: Step response of system with real poles

As can be seen in figure 2, the system response presents no overshoot. As already mentioned, Laguerre functions perform better in modeling underdamped systems. Thus, it can be concluded that Laguerre functions are the most suitable for representing this system.

In order to identify the model, a simulation was performed. The input to that simulation was a PRBS (*Pseudo Random binary Sequence*) with 128 samples.

As a result, a model, $M_{nL}(z)$, based on linear combinations of n Laguerre functions was obtained. This model approximates the system described in equation (6).

$$M_{nL}(z) = c_1 L_1(z) + c_2 L_2(z) + \ldots + c_n L_n(z) = \sum_{i=1}^{n} c_n L_n(z), \quad (6)$$

where $L_n(z)$ represents the Laguerre functions and c_n, the coefficients.

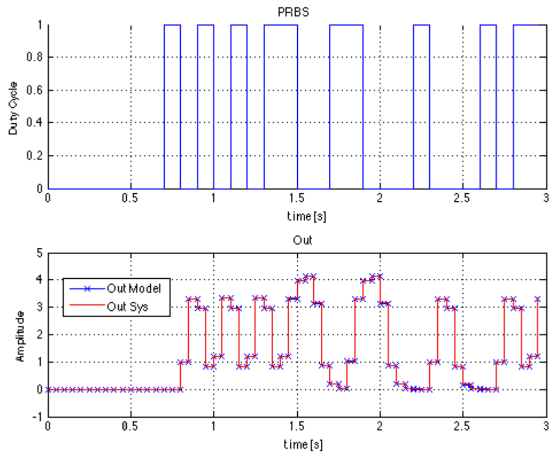

Figura 3: Comparison of the simulated system and an model approximation, with 3 Laguerre functions.

In figure 3, the simulation of the linear model obtained through the combination of 3 Laguerre functions is shown. The PRBS input is applied to the original system (5) and to the model. The output of the system is presented in continuous line and the model output is in x markers. In order to obtain this result, the GA proposed in this paper was used. Section 3.2 presents details of this algorithm. The best result, represented by a chromosome, provided by the GA is shown below.

$$[1.448 \quad 2.151 \quad -0.172 \quad 0.192]$$

where the last gene, 0.192, corresponds to the pole that parameterized the 3 Laguerre functions. The other genes express the coefficients in (7).

$$M_{3L}(z) = 1.448.L_1(z) + 2.151.L_2(z) + -0.172.L_3(z) \quad (7)$$

where M_{3L} is the model comprehending 3 Laguerre functions and $L_1(z)$, $L_2(z)$ e $L_3(z)$ are the Laguerre's functions of 1^{st}, 2^{nd} e 3^{rd} order, respectively. This model reached a fitness, Mean Square Error (MSE), of $7,01 \times 10^{-4}$.

2.1.2 Kautz Functions

The Kautz functions are parameterized by a complex pole (CAMPELLO et al., 2007; LEMMA et al., 2010). These functions can be expressed by the equations (8) and (9).

$$K_{2m}(z) = \frac{\sqrt{(1-c^2)(1-b^2)}}{z^2 + b(c-1)z - c} \times \left[\frac{-cz^2 + b(c-1)z + 1}{z^2 + b(c-1)z - c}\right]^{m-1} \quad (8)$$

$$K_{2m-1}(z) = \frac{z(z-b)\sqrt{1-c^2}}{z^2 + b(c-1)z - c} \times \left[\frac{-cz^2 + b(c-1)z + 1}{z^2 + b(c-1)z - c}\right]^{m-1} \quad (9)$$

where $K_{2m}(z)$ and $K_{2m-1}(z)$ are the even and odd Kautz functions, respectively. β and $\overline{\beta}$ are the poles that parameterize these functions. The terms b and c can be expressed as (10) and (11).

$$b = (\beta + \overline{\beta})/(1 + \beta.\overline{\beta}), \quad (10)$$

$$c = -\beta\overline{\beta}. \quad (11)$$

In this section the modeling using Kautz functions will be made considering the following stable, discrete, LTI system:

$$H(z) = \frac{2z - 1}{z^2 - 0.2z + 0.26} \quad (12)$$

this system was also sampled at a period of 50 ms.

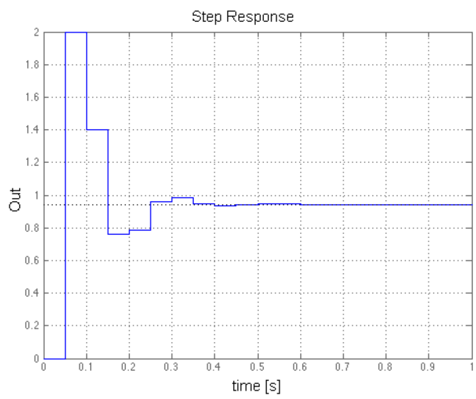

Figura 4: Step response of the system with complex poles.

The step response of the system (12), presented in figure 4, shows the underdamped nature of this system, $0 < \xi < 1$, being ξ the dumping ratio. Such systems can be well represented by combining linear Kautz functions (ROSA et al., 2009). In the same way as given for system (5), the system (12) was fed with 128 samples of a PRBS input.

Since the system is linear, we can easily obtain a model, $M_{nk}(z)$, that represents it, by a linear combination of n Kautz functions:

$$M_{nk}(z) = c_1K_1(z) + c_2K_2(z) + ... + c_nK_n(z) = \sum_{i=1}^{n} c_nK_n(z), \quad (13)$$

Through the application of the proposed GA, the parameterizing pole and the coefficients of the functions can be determined. The result provided by the GA is shown in figure 5.

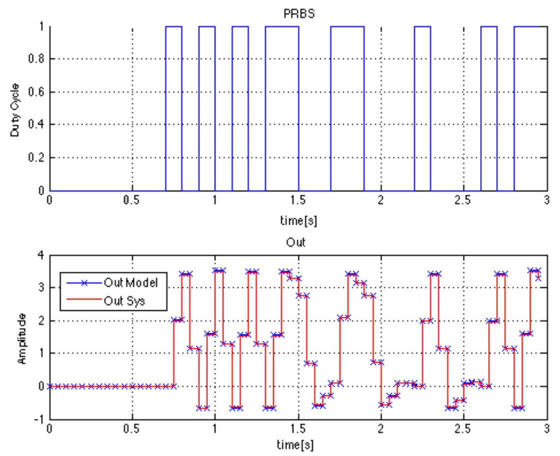

Figura 5: Model output (x markers) compared to the simulated output (continuous line).

The resulting chromosomes was:

$$[1.393 \quad 2.059 \quad 0.089 \quad -0.490]$$

where the last 2 genes, $0.089 - 0.490i$, represent the real and the imaginary parts of the pole that parameterized the 2 Kautz functions. The other terms correspond to the Kautz function coefficients, as expressed by equation (14).

$$M_{2k}(z) = 1.393.K_1(z) + 2.059.K_2(z), \quad (14)$$

where M_{2k} is the model, composed by 2 Kautz functions, and $K_1(z)$ and $K_2(z)$ are the Kautz functions of first and second order, respectively. This model was approximated with fitness, Mean Square Error (MSE), of 1.30×10^{-3}.

3 Identification of a Real Buck Converter

Real systems usually present nonlinear components and input/output signals perturbed by noise. In this section, the identification of the *Buck* circuit depicted in figure 6 is described.

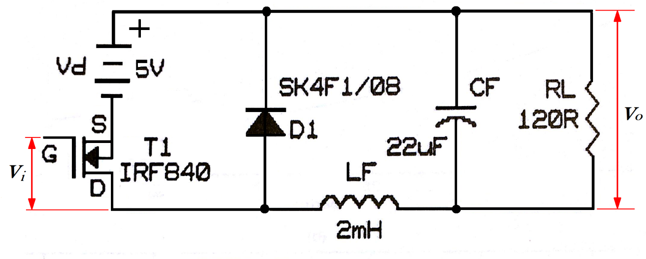

Figura 6: DC-DC Buck converter.

The input of this circuit is applied at the gate, G, of the MOSFET, Metal-Oxide Field Effect Transistor. The signal used as input is a *Duty Cycle*, D, modulated in a square wave. The output of this circuit, V_o, is defined as the voltage over the resistor R_L.

In order to obtain a continuous operation from the conversor, a period of $T = 30$ ms was employed in the input signal. Thus, the current in the inductor is never equal to zero and the continuous operation is achieved.

In the converter, when $D \to 1$ the voltage at the load, V_o, rises because the switch T_1 is closed, allowing the source V_d to energize the path given by the capacitor and the inductor. On the other hand, when $D \to 0$, the switch T_1 opens and V_o reduces according to a dynamic behavior different from the previously described, pointing out the nonlinear characteristic of this converter (AGUIRRE, 2007).

The identification of a system requires the acquisition of the input and output data of the system. In this work, the data acquisition was made with a microcontroller MBED, ARM® family Cortex M3, 32 bits architecture and Clock frequency of 96 MHz.

All the procedures necessary to the identification were executed in the software Matlab, version 2013. The communication between the software and the microcontroller was established through the RS-232 serial protocol, with a rate of 9600 kbps. An ADC, Analog-to-Digital Converter, with 16 bits, internal to the microcontroller was used in order to read the analog signals from the Buck converter.

The experimental setup used to acquire the input/output data from the Buck converter can be seen in figure 7.

In order to accomplish the identification of the proposed systems, linear and nonlinear combina-

Figura 7: Experimental setup used for the Buck converter.

tions of Laguerre and Kautz functions were used. These functions were implemented as digital filters on the frequency domain. For the purpose of treating the system' s nonlinearity a polynomial block is applied on the output of the linear model. This structure is known as Wiener model, as depicted in figure 8.

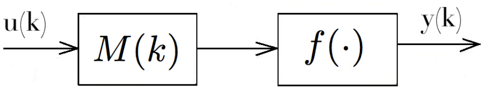

Figura 8: Wiener Model - nonlinearity at the output.

Wiener models use a cascade composition of dynamic linear models, $M(k)$, followed by a static nonlinear function, $f(.)$, (AGUIRRE, 2007). It is important to mention that the nonlinear degree of each output of the orthonormal filter is independent from each other.

3.1 Input Signals

Since the DC/DC system shown in figure 6 is nonlinear, it is necessary that the excitation signal presents variations in frequency and amplitude in order to provide a effective identification (AGUIRRE, 2007). Thus, summation of *PRBS* signals was used with different spectral composition, as it can be seen in figure 9.

The frequency band of the excitation signal determines the range in which the system characteristics could be found by the identification procedure (NEMETH and KOLLAR, 2002).

In this scenario, the input signal needs to be

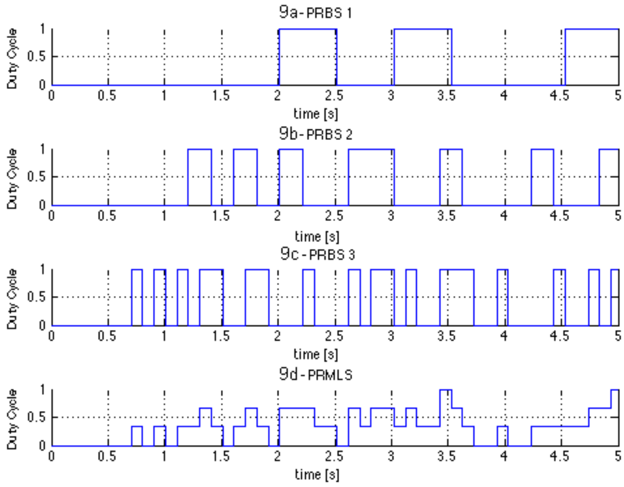

Figura 9: Composition of PRBS signals to generate a Pseudo Random Multi-Level Signal (PRMLS).

persistently excitatory. A signal is said to be persistently excitatory when its components are able to sweep all the space of the parameters (NOWAC and VAN VEEN, 1993). With this in mind, the signal given in figure 9d was generated by summing the signals of figures 9a (low frequencies), 9b (medium frequencies) and 9c (high frequencies). Hence, the resulting signal is called PRMLS, *Pseudo Random Multilevel Sequence* and it is better suited for the identification of nonlinear systems.

3.2 Genetic Algorithms

In this section, the main aspects of the proposed Genetic Algorithm, GA, employed to identify the Buck converter, of figure 6, are detailed.

3.2.1 Coding of the Genetic Algorithm

The gene codification applied in this work can be represented as follows:

c_1	c_2	$\cdots$	c_n	a	b

where $c_1, c_2, \cdots, c_n$ are the orthonormal function coefficients and $a + bi$ is the parameterizing pole of this functions.

For each of the n orthonormal functions, we have n genes representing the nonlinearity of the output filters. These genes are given by:

g_1	g_2	$\cdots$	g_n

each of these genes can assume one of the following values: 0, 1 or 2. The effect that the genes produce is described in table (1).

The outputs of the orthonormal filters, shown in table 1 and expressed by $\check{l}_1(k), \check{l}_2(k), \cdots, \check{l}_n(k)$,

Tabela 1: Gene coding for the OBF structure.

Value of g_n	Output of n-th orthonormal filter
0	$(.)^0$ - (OBF not present)
1	$(.)^1$ -(standard OBF)
2	$(.)^2$ -(OBF squared)

can be seen in figure 1. In this figure, Laguerre functions are the components of the orthonormal basis function.

It is worth to point out that the chosen model does not apply the nonlinearity on the input vector, $u(k)$ (Hammerstein Model). It applies the nonlinearity on the orthonormal filtered output signals (Wiener Model), as it can be seen in figure 8.

Assuming, for example, that it would be convenient for the data fitting to consider a 2nd degree of nonlinearity on the Laguerre filter output (see figure 1), then the vector $\check{l}_1(k)$ would have their elements squared by the mapping $\mathcal{H}$. The same procedure is applied to the other orthonormal filter outputs. For instance, when identifying a linear system, the genes g_n tend to converge to 0 or 1.

3.2.2 GA Parameters

The GA parameters employed to find the pole and the coefficients of the orthonormal functions are given in table 2.

Tabela 2: Genetic Algorithm Parameters

Parameter	Value
Population	200 Chromosomes
Selection Met.	Tournament and Recombination
Mutation Rate	Variate (20 to 60 %)
Crossover Rate	Variate (80 to 20 %)
Nelder Mead	after each 9 generations
LMQ	after each 7 generations

The fitness employed here is the Mean Squared Error (MSE) between the measured circuit output and the output estimated by the approximate model. In the GA, the mutation and crossover ratios varies with respect to the given generation. The crossover rate starts with 80% focusing on local searches, and then it is progressively reduced during the optimization process. On the other hand, the mutation starts at 20%, and it is increased as the generation number increases, focusing on global searches. It is worth to mention that the mutation rate usually found in the literature seldom are greater than 10%. In this work, much higher rates were applied due to the use of a mutation operator based on the disturbance of the gene. This disturbance consists on adding a small random constant to a given gene. Thus, the result is the reduction of the random search effect normally produced by the mutation operator (KOZA, 1998).

In the proposed GA, two local search operators have been introduced: the NM algorithm and the LMS method. A detailed description of NM can be found in (NELDER and MEAD, 1965). The LMS is described in the next subsection.

3.2.3 Least Squares Operator

The Least Mean Squares (LMS) is one of the pioneers and most used methods in systems identification, (LJUNG, 1999). In this paper, this method is applied as an operator of a Genetic Algorithm. In other words, this operator applies the LMS, given by equation (15), to randomly chosen chromosomes.

$$\theta = (\Psi^T \Psi)^{-1} \Psi^T Y, \tag{15}$$

where θ is the vector of the parameters and Ψ is the regression matrix (composed by input signals filtered by the orthonormal functions). It is important to mention that, despite the name, the matrix Ψ, in this work, has no regressive versions of the input signal, $u(k)$, it has only filtered versions of $u(k)$.

For instance, if the Laguerre basis were used, the genes related to the poles, in a chromosome, would generate the Laguerre's functions and the matrix Ψ would be composed of filtered versions of the input vector u(k), as is exposed in equation (16).

$$\check{l}_n(k) = \sum_{\tau=0}^{N-1} l_n(\tau).u(k-\tau). \tag{16}$$

where $\check{l}_n(k)$ represents the filtered version of $u(k)$ by the filter $l_n(k)$.

The structure of Ψ matrix is given by the equation (17).

$$\Psi = \begin{bmatrix} \vdots & \vdots & \vdots & \vdots \\ \check{l}_1(k) & \check{l}_2(k) & \cdots & \check{l}_n(k) \\ \vdots & \vdots & \vdots & \vdots \end{bmatrix}. \tag{17}$$

Using the equation (15) one can obtain the coefficients, θ, for the ortonormal functions. It is worth to mention that the pole that parameterize the orthonormal function is not changed by this operator.

After using the LMS operator, some genes of the chosen chromosomes will be replaced by the elements of the vector θ.

This operator will be used by the GA after each 7 generations in order to avoid the premature convergence to a local minimum.

4 Results

The models were obtained using vectors with the 128 samples of the input/output of the system. The approximated Buck converter models, using Laguerre and Kauts basis, can be seen in the tables 3 and 4, respectively.

Tabela 3: *MSE* and Laguerre components.

NF	pole	Ord.	NLD	MSE
1	-0.020	1	2	$4.52.10^{-2}$
2	-0.084	[1 2]	[2 1]	$4.36.10^{-2}$
3	-0.085	[1 2 4]	[2 1 2]	$4.36.10^{-2}$
4	-0.017	[1 3 4 5]	[2 2 2 1]	$4.32.10^{-2}$
5	-0.050	[1 2 3 4 5]	[2 1 2 1 2]	$4.32.10^{-2}$

where: NF is the number of functions used in the model, Ord. represents the order of the functions employed, NLD is the nonlinearity degree and MSE is the Mean Square Error of the model with respect to the measured signals.

Tabela 4: *MSE* and Kautz functions.

NF	pole	Ord.	NLD	MSE
1	0.073 - -0.123i	2	2	$4.42.10^{-2}$
2	-0.121- 0.230i	[1 2]	[1 2]	$4.36.10^{-2}$
3	0.058+ 0.322i	[1 2 3]	[1 2 1]	$4.35.10^{-2}$
4	-0.349+ 0.100i	[1 2 3 4]	[1 2 2 1]	$4.33.10^{-2}$
5	-0.008+ 0.299i	[1 2 3 5 6]	[1 2 1 1 2]	$4.29.10^{-2}$

These results were obtained after running 200 generations of the GA, using NM and LMS operators. The simulation of the model given in line 3 of the table 3 can be seen in figure 10.

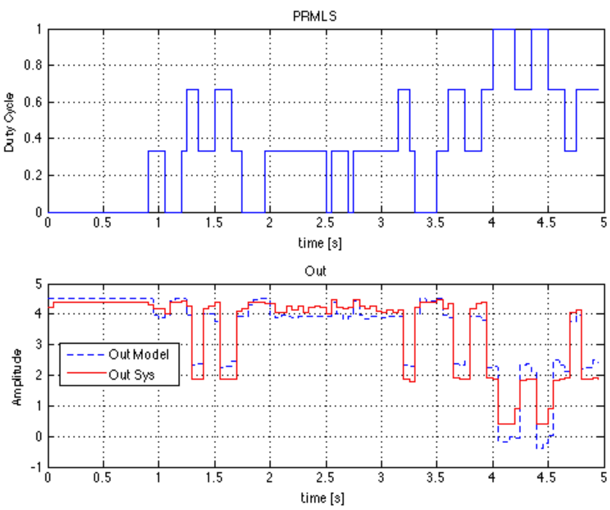

Figura 10: Example (3) of table 3 - Buck model using 3 functions of Laguerre.

This identification procedure resulted in the following chromosomes, with all genes grouped in the vectors c and g:

$$c: \quad [-5.174, -0.550, \quad 0.085, \quad 4.521, -0.103]$$
$$g: \quad [2 \quad 1 \quad 1 \quad 0]$$

where the first genes of c are associated to the Laguerre function coefficients. The last gene of c, -0.103, corresponds to the pole that parameterized the functions. The genes expressed by g correspond to the exponents of each function. The complete model, M_{3L}, can be seen in (18), and the obtained fitness was (MSE) = 0,0436.

$$M_{3L}(z) = -5.174.[L_1(z)]^2 - 0.550.L_2(z) + \\ +0.085.L_3(z) + 4.521. \quad (18)$$

For the Kautz basis, the model expressed by line 3 of table 4 can be seen in figure 11.

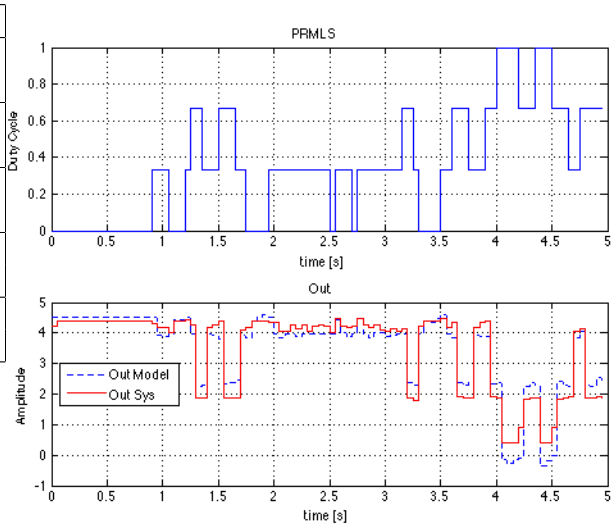

Figura 11: Buck model, example (3) of table 4, using 3 functions of Kautz.

This identification procedure resulted in the following chromosomes:

$$c: [4.522 - 5.265 - 0.326 - 0.155 \quad 0.058 \quad 0.322]$$
$$g: [1 \quad 2 \quad 1 \quad 0]$$

where the first genes of c are associated to the Kautz function coefficients. The last two genes, $0.058 + 0.322i$, correspond to the pole that parameterized the functions. Moreover, the genes expressed by g correspond to the exponents in each function. The complete model, M_{3L}, can be seen in (19), and the obtained fitness was (MSE) = 0,0435.

$$M_{3K}(z) = 4.522K_1(z) - 5.265K_2^2(z) \\ -0.326K_3(z) - 0.155. \qquad (19)$$

The convergence curves of the GA, with and without the local search operators, for the model in line 3 of table 3 can be seen in figure 12. It can be seen that the local search operator not only accelerated the search, but also made it possible to find more accurate results.

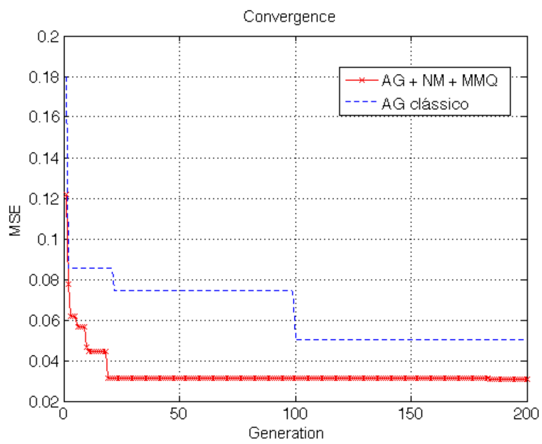

Figura 12: Convergence process of classic GA (blue dashed line) and with NM and LMS operators (continuous red lines with 'x' markers).

5 Conclusion

In this paper, the identification of nonlinear systems based on Laguerre and Kautz functions was discussed. In order to pursue this identification, the model and structure parameters were evaluated by minimizing the MSE between the measured output of the circuit and the output estimated by the approximated model. The main contribution of this paper was the proposal of a new GA with two local search operators: the NM algorithm and the LMS method. Based on the results, presented in section 5, it can be concluded that the Laguerre and Kautz functions provided very similar results for the case studied, with sensitive better performance for function of Kautz. More importantly, the GA with NM and LMS operators not only converged much faster than the classical GA, but also was able to provide a much more accurate approximated model.

As future work, the use of Genetic Programming can be seen as a good option to determine the degree of nonlinearity of the system and also to obtain the parameters of the nonlinear system.

Acknowledgement

The authors acknowledge Rafael Rodrigues Luz Benevides for the technical and theoretical support and productive discussions.

Referências

AGUIRRE, L. A. (2007). *Introdução a Identificação de Sistemas - Técnicas Lineares e não lineares Aplicadas a Sistemas Reais*, 3^a edn, Editora UFMG.

CAMPELLO, R., AMARAL, W. C. and Gustavo, O. (2007). Identificação de controle de processsos via desenvolvimento de séries ortonormais. Revista Controle e Automação, Vol. 18, pp 301-319.

HEUBERGER, P., VAN DEN HOF, P. and WAHLBERG, B. (2005). *System Identification: Theory for the User*, 1^{st} edn, Springer Press.

KOZA, J. (1998). *Genetic Programming: On the Programming of Computers by Means of Natural Selection*, 6^{th} edn, MIT Press.

LEMMA, D., RAMASSAMY, M. and SHUHAIMI, G. M. (2010). System identification using orthonormal basis filter. Journal of Applied Sciences, Vol. 10, pp 2516-2522.

LJUNG, L. (1999). *Modelling and Identification with Rational Orthogonal Basis Functions*, 2^{nd} edn, Prentice Hall.

MACHADO, J., CAMPELLO, R. and AMARAL, W. (2010). Modelos lineares gobf com funções internas. XVIII Congresso Brasileiro de Automática, pp 505-512.

NELDER, J. A. and MEAD, R. (1965). A simplex for function minimization. Computer Journal, Vol. 7, pp 308-313.

NEMETH, J. and KOLLAR, I. (2002). Identification of volterra kernels using interpolation. IEEE Transations on Instrumentation and Measurements, Vol. 51.

NOWAC, R. and VAN VEEN, B. (1993). Nonlinear system identification with pseudorandom multilevel excitation sequence. IEEE Transations, pp 456-459.

ROSA, A., CAMPELLO, R. and AMARAL, W. (2009). Exact search directions for optimizations of linear and nonlinear models based on generalized orthonormal functions. IEEE Transations on Automatic Control, Vol. 54, N^o 12.

WILLEM BELT, H. (1997). Orthonormal bases for adaptative filtering.