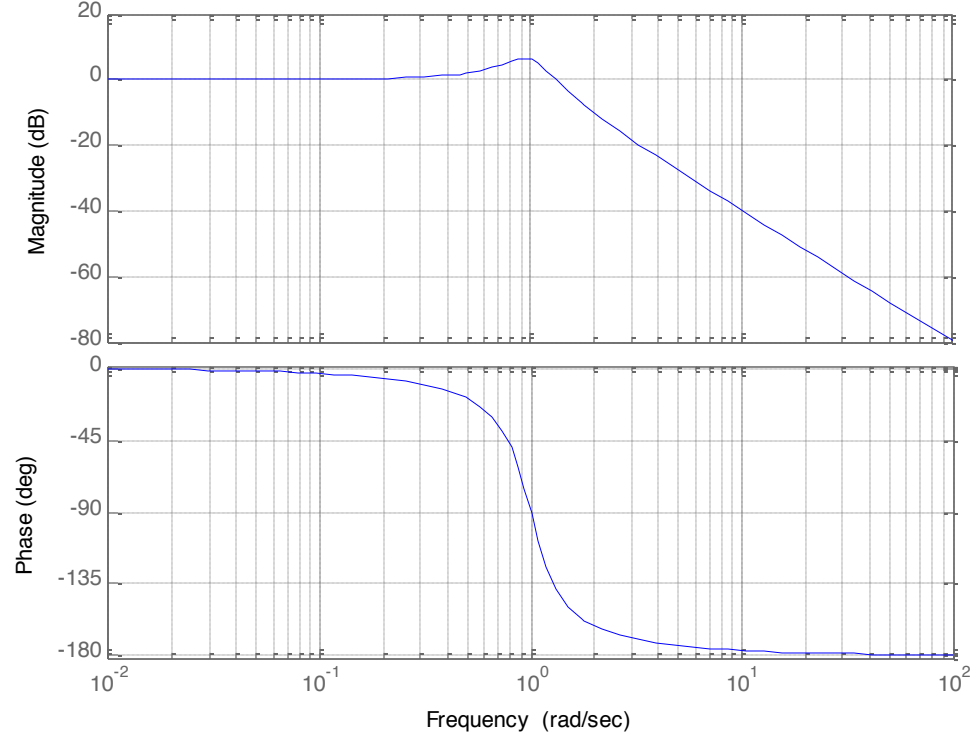


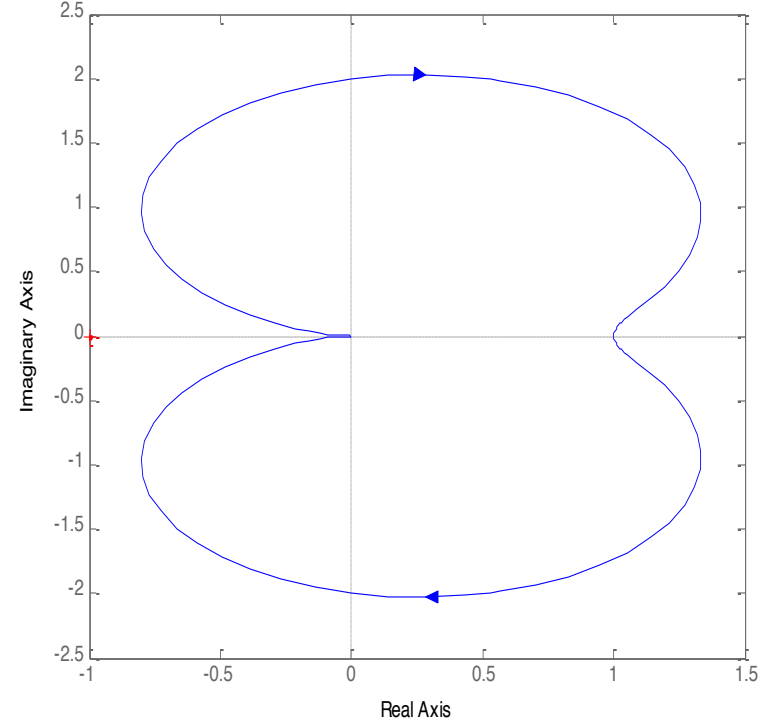
# Resposta em Frequência

$$G(s) = \frac{1}{s^2 + 0,5s + 1}$$

Bode Diagram

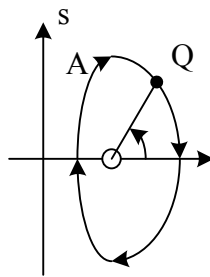


Nyquist Diagram

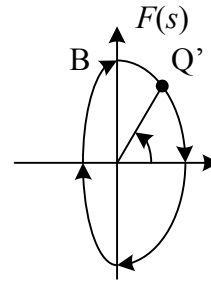


# Mapeamento de A via $F(s)$ em B

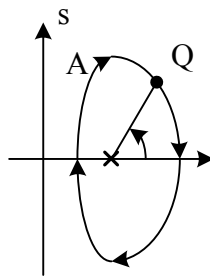
Contorno A  
 → sentido horário



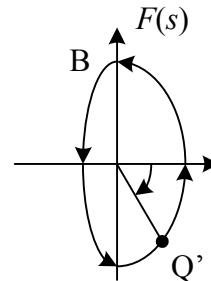
$$F(s) = s - a$$



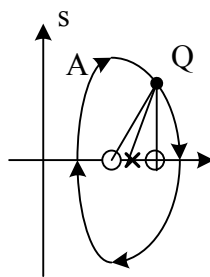
B: Envolve a origem  
 → sentido horário



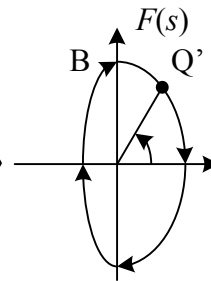
$$F(s) = 1/(s - c)$$



B: Envolve a origem  
 → **sentido anti-horário**



$$F(s) = \frac{(s - a)(s - b)}{(s - c)}$$

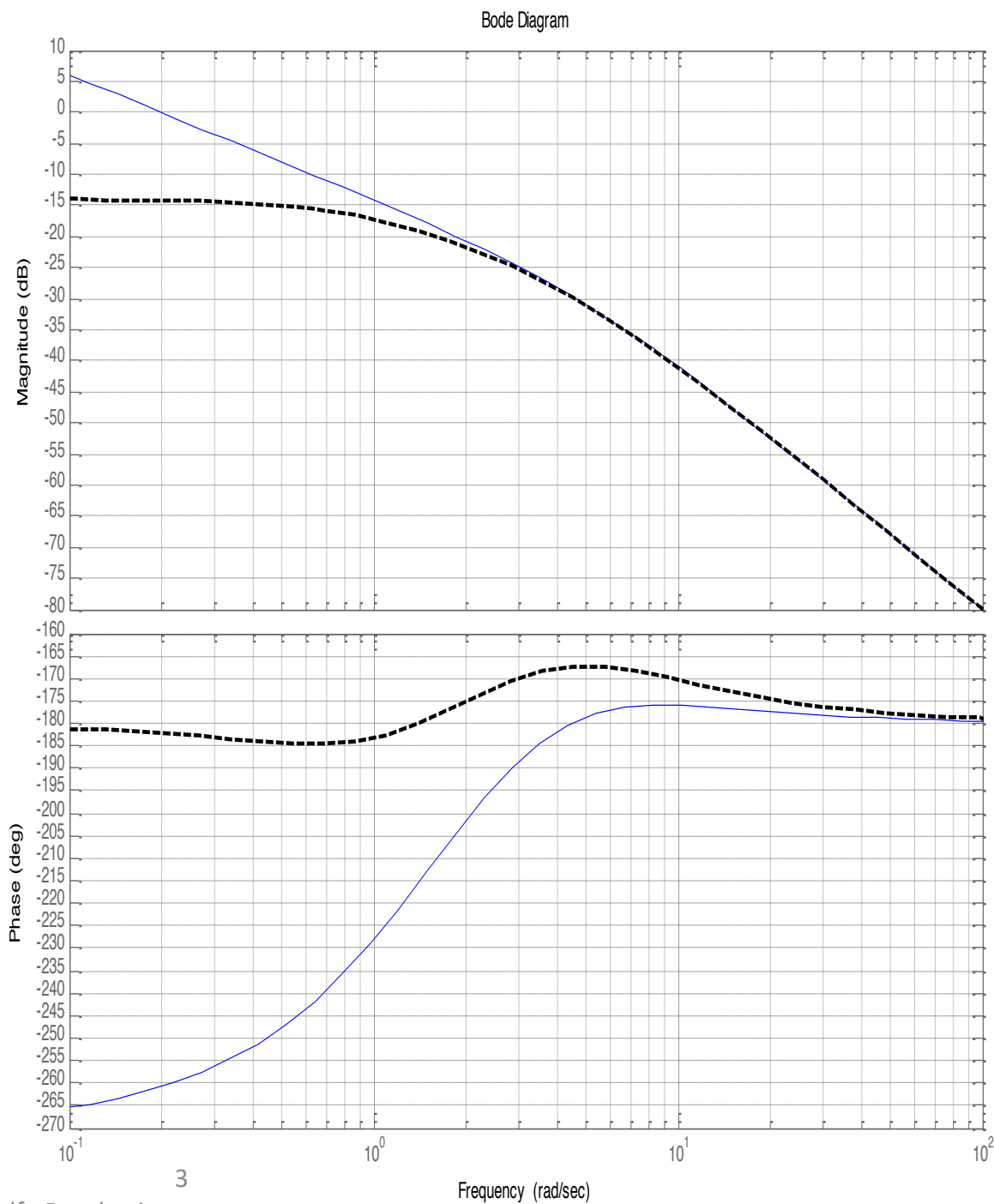


B: Envolve a origem  
 → sentido horário

# Exercício Extra 6

Considere a resposta em frequência  $G(j\omega)$  de dois sistemas que tem um polo no SPD. Para (I)--- e (II) —:

- Esboce os diagramas de Nyquist correspondentes.
- Em malha fechada, quais valores de  $K$ ,  $-\infty < K < \infty$ , produzem respostas estáveis?
- Para quais valores de  $K$  se tem a maior margem de fase possível?
- Nas condições do item c), qual o erro em regime permanente para um degrau de referência.
- Para quais valores de  $K$  o erro em regime permanente para um degrau de referência é  $e_{ss} \leq 0,01$ .

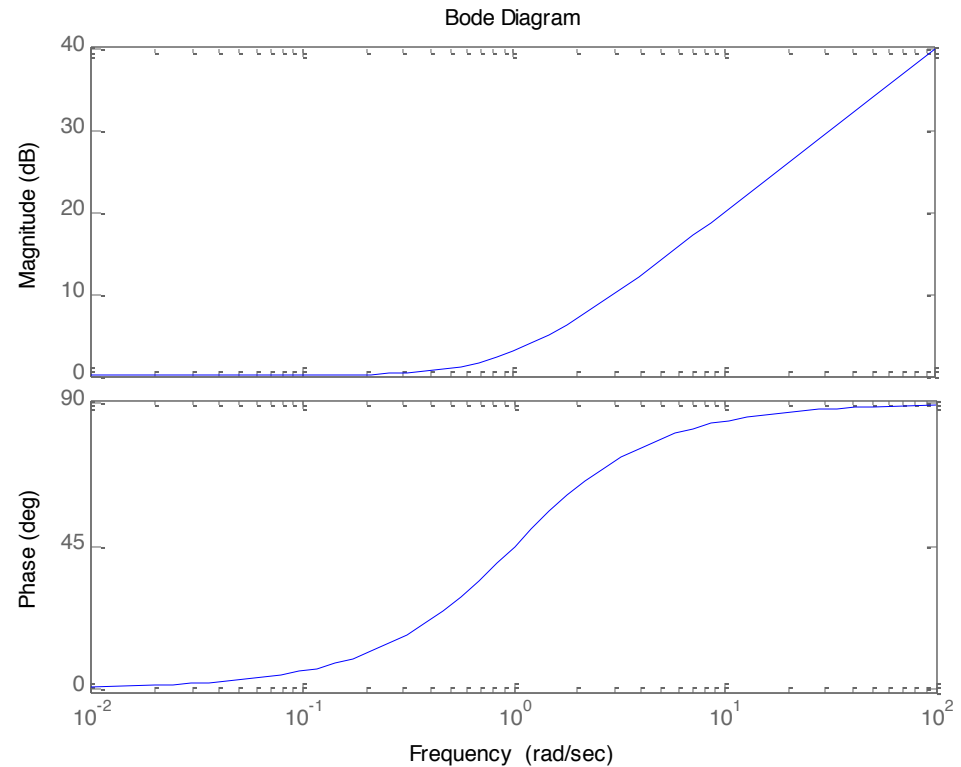


# Compensação no domínio- $\omega$

- Adição de elementos dinâmicos ao sistema de controle para melhorar a estabilidade e diminuir o erro em regime

- Compensador PD  
Pouco utilizado →  
amplifica ruído de medida

$$D(s) = K(T_d s + 1)$$



# Compensador em avanço

➤ Muito Popular

$$D(s) = K \frac{Ts + 1}{\alpha Ts + 1}$$

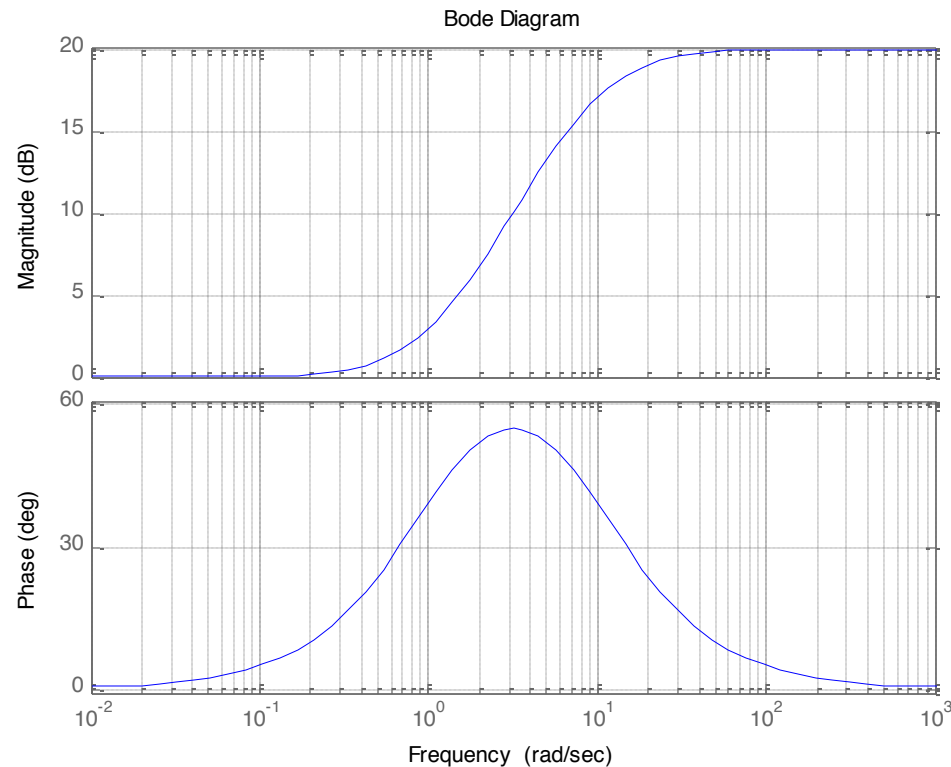
$$\alpha < 1$$

$$\phi = \tan^{-1}(\omega T) - \tan^{-1}(\alpha \omega T)$$

$\omega_m$  – média geométrica das  
freqs de canto

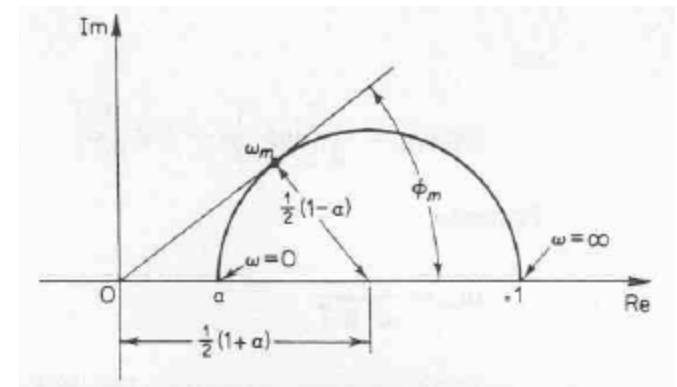
$$\log \omega_m = \frac{1}{2} \left( \log \frac{1}{T} + \log \frac{1}{\alpha T} \right)$$

$$\omega_m = \frac{1}{\sqrt{\alpha T}}$$



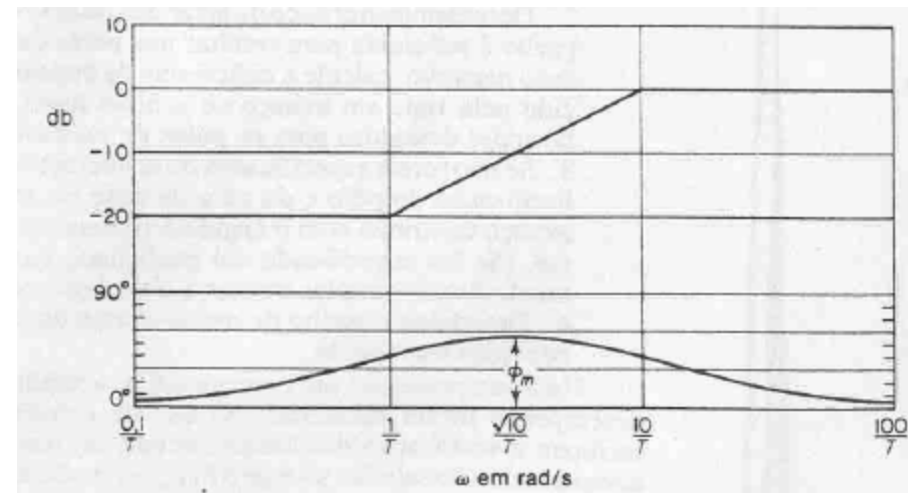
# Compensador em avanço

- Avanço máximo depende apenas de  $\alpha$



$$\text{Fator de avanço} \frac{1}{\alpha} = \frac{1 + \text{sen}\phi_m}{1 - \text{sen}\phi_m}$$

$$\text{sen}\phi_m = \frac{1 - \alpha}{1 + \alpha}$$



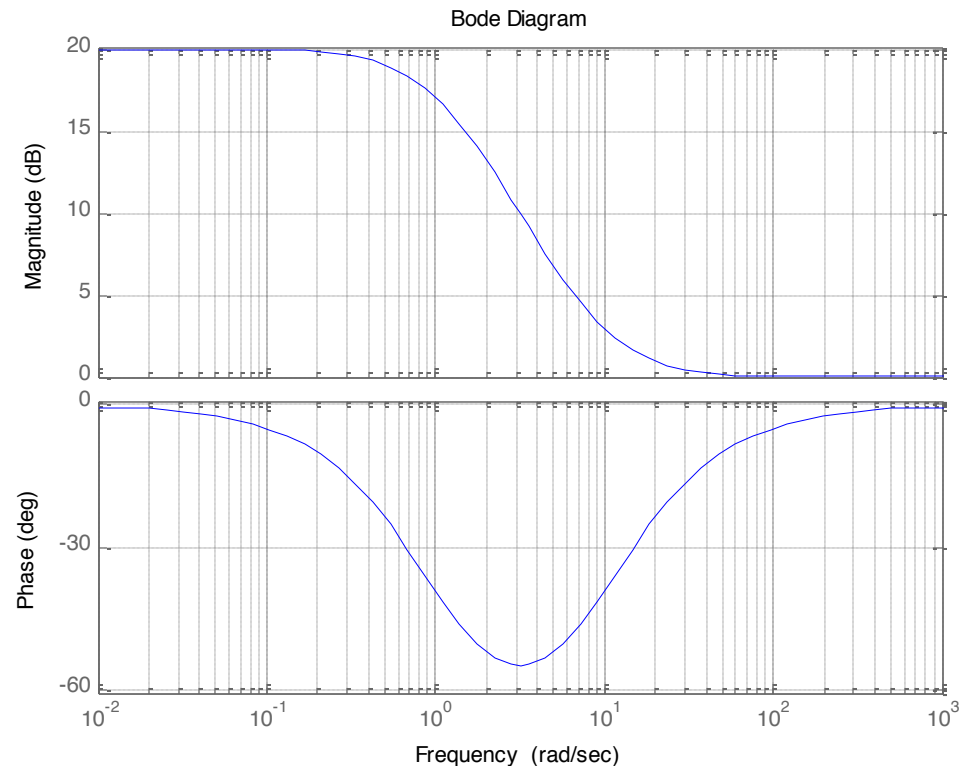
# Compensador em atraso

$$D(s) = \alpha \frac{Ts + 1}{\alpha Ts + 1}$$

$$\alpha > 1$$

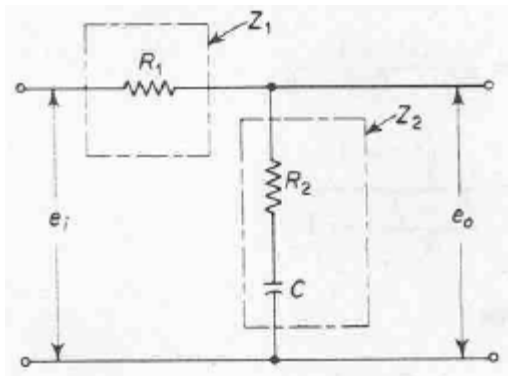
*fornece ganho adicional de  $+20\log\alpha$   
em baixas frequencias, mantendo margem  
de fase suficiente.*

*$\Rightarrow$  pólo e zero bem próximos à origem  
para não afetar a resposta transitória.*



# Compensador em atraso

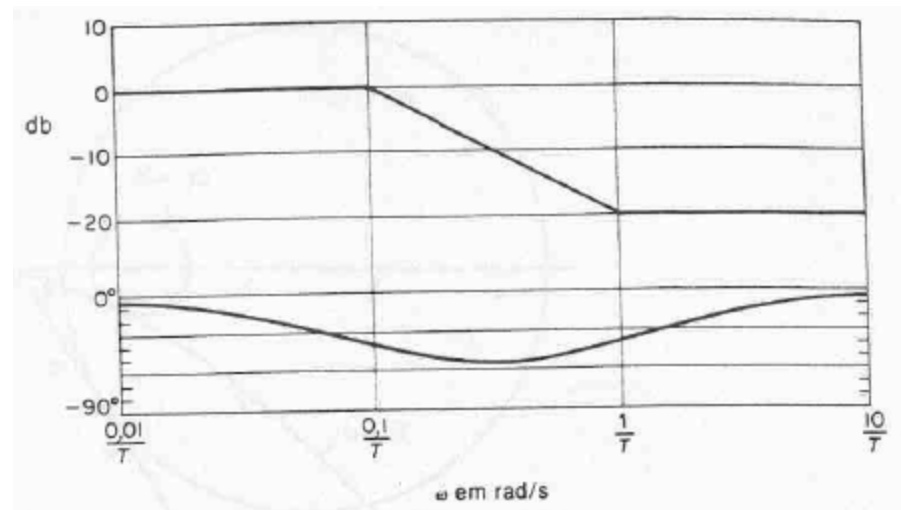
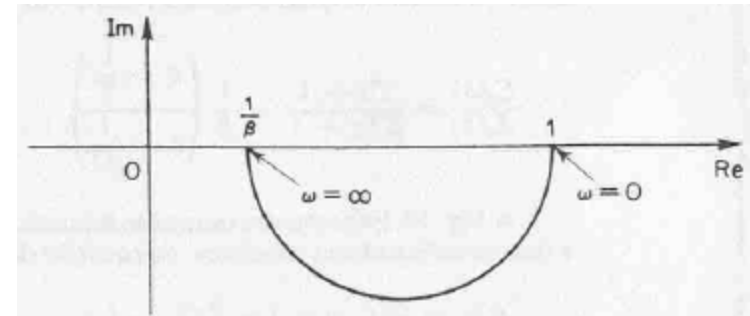
$$D(s) = \alpha \frac{Ts + 1}{\alpha Ts + 1}$$



$$\frac{E_o(s)}{E_i(s)} = \frac{Z_2}{Z_1 + Z_2} = \frac{R_2Cs + 1}{(R_1 + R_2)Cs + 1}$$

$$R_2C = T, \quad \frac{R_1 + R_2}{R_2} = \beta > 1$$

$$\frac{E_o(s)}{E_i(s)} = \frac{Ts + 1}{\beta Ts + 1} = \frac{1}{\beta} \left( \frac{s + \frac{1}{T}}{s + \frac{1}{\beta T}} \right)$$

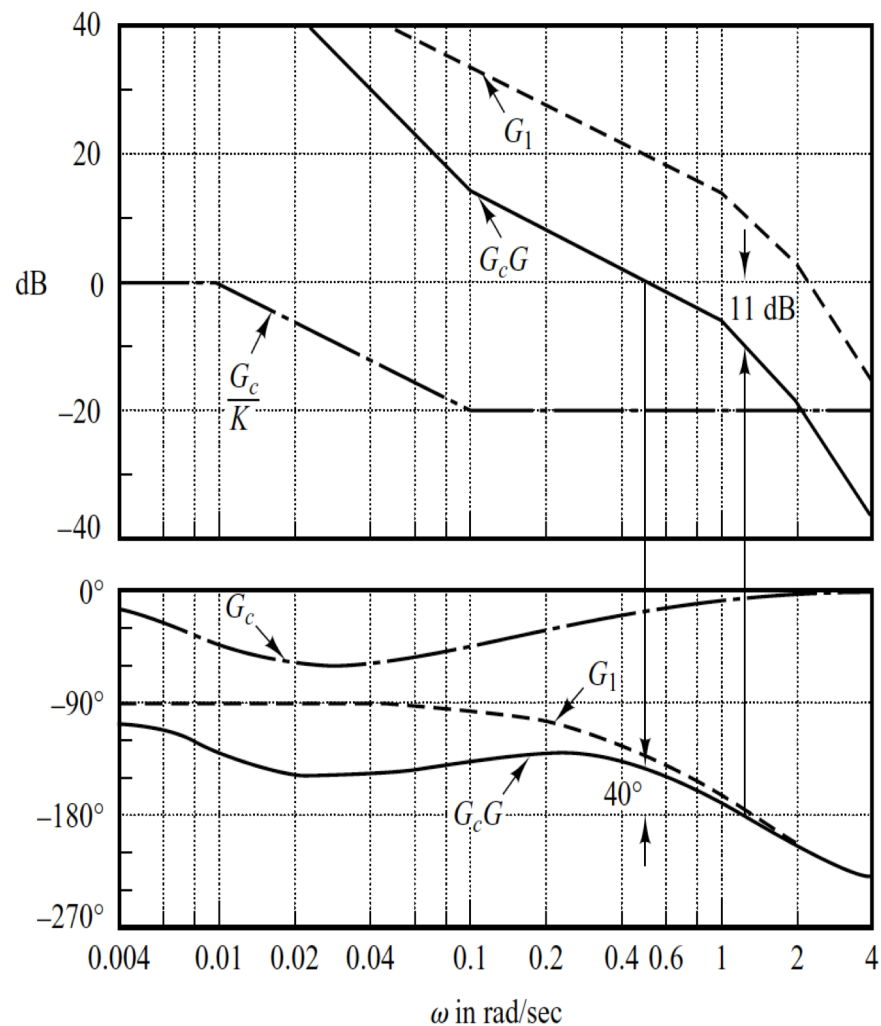




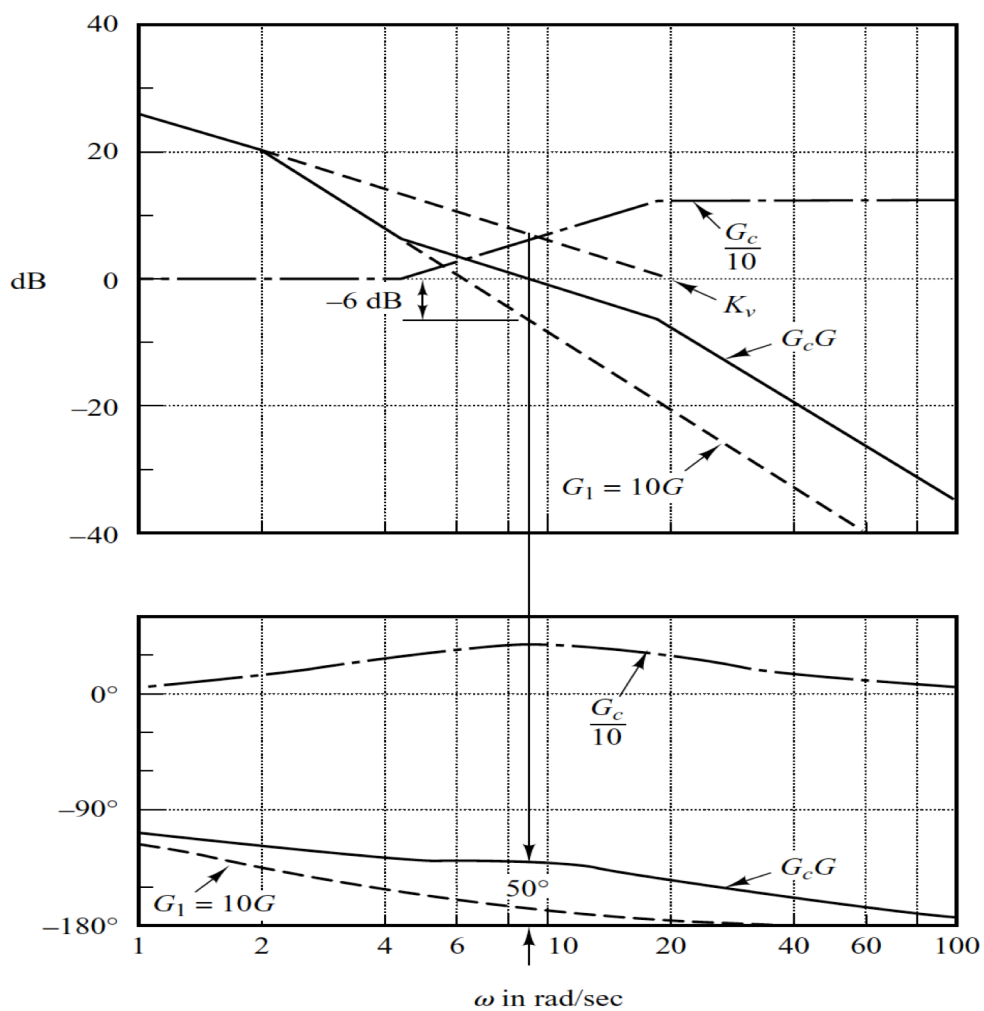
# Compensadores Dinâmicos em $\omega$

Estratégias de Projeto:

Atraso: Usar MF da curva de  $G(j\omega)$

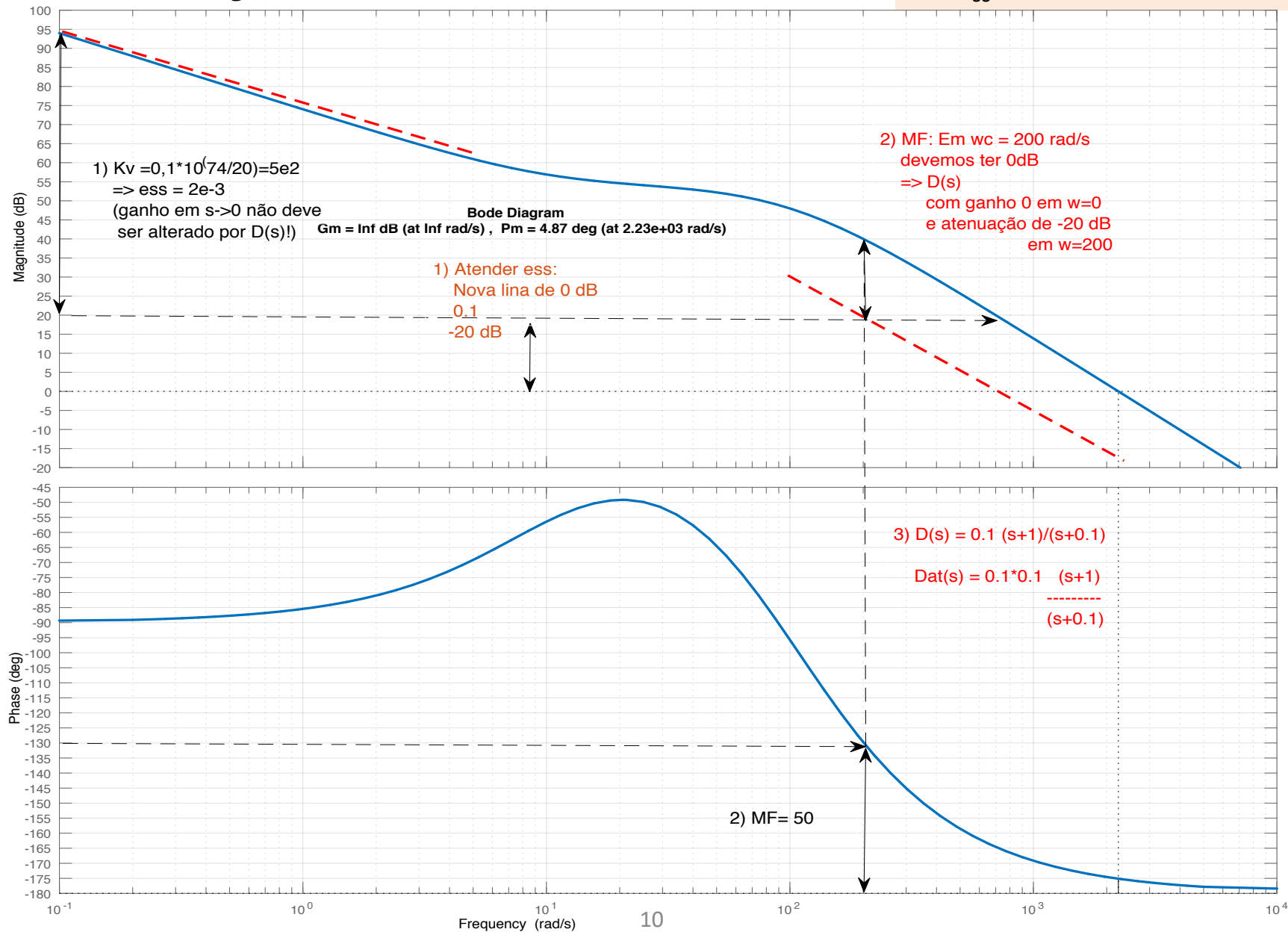


Avanço: Acrescentar Fase à  $G(j\omega)$



# Ex.: Projeto Ctrl. em Atraso

- MF = 50°
- $e_{ss} = 0,002$  (rampa unitária)



# Exemplo - Compensador em atraso

```
g1=zpk([-10],[0 -100 -100],5000000);
```

- MF = 50°
- $e_{ss} = 0,002$  (rampa unitária)
- Menor redução da banda passante possível

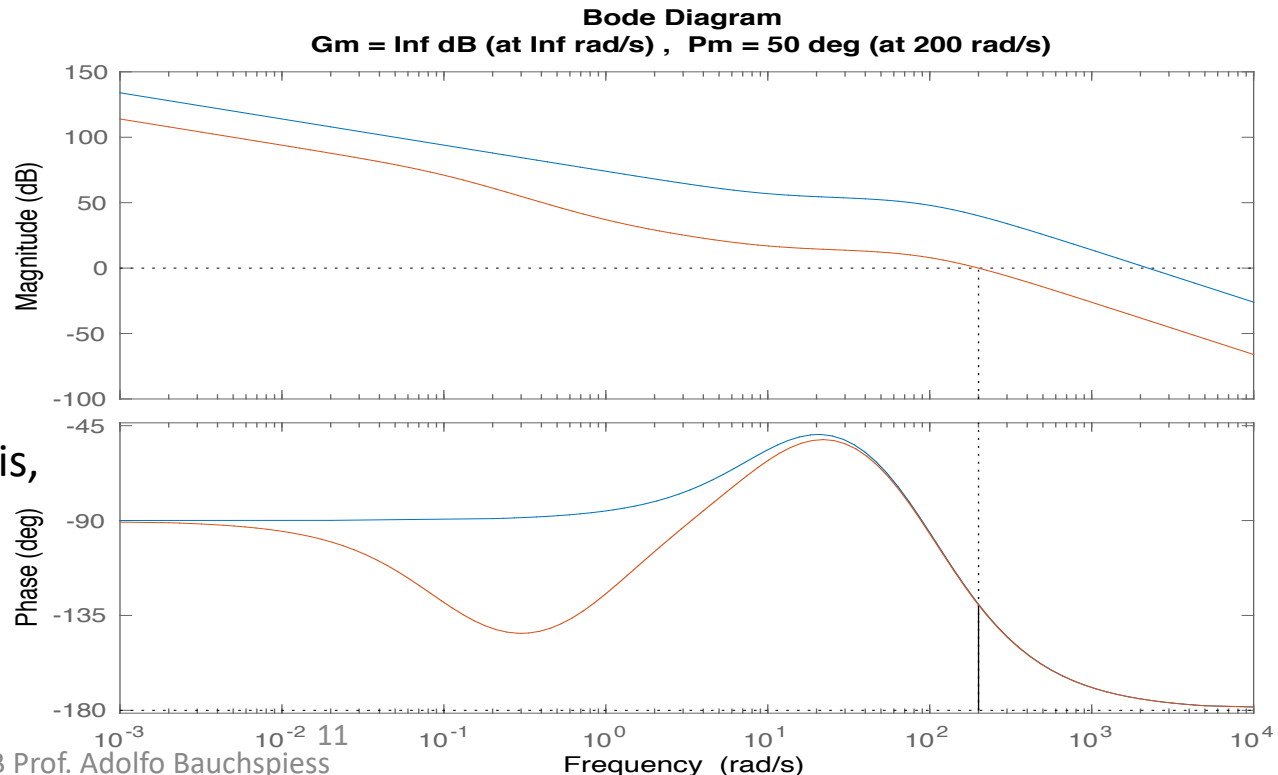
$K1 = 0,01$  (-40 dB)  $Kv' = 0,1 \times 501,18$  (54 dB) = 50,118

$1/Kv' = 0,02 \Rightarrow K2 = 10$

$$D(s) = 0,01 \frac{(s + 1)}{s + 0,1}$$

```
d=zpk(-1,-.1,0.01);  
margin(d*g1)
```

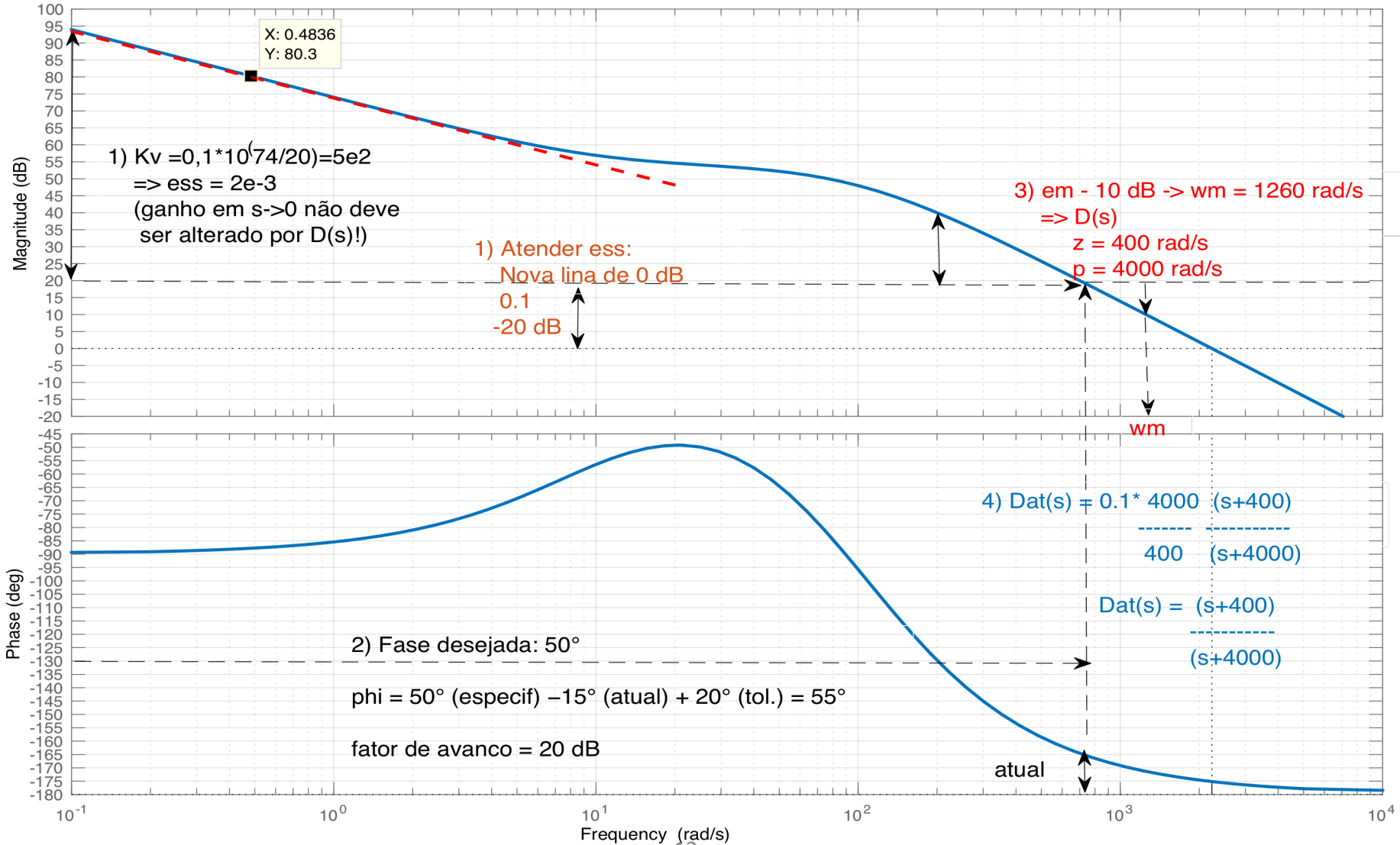
Obs: várias soluções possíveis,  
 $z/p = 10$   
e “longe” de 200 rad/s



# Ex.: Projeto Ctrl. em Avanço

- MF = 50°
- $e_{ss} = 0,002$  (rampa unitária)

Bode Diagram  
 Gm = Inf dB (at Inf rad/s) , Pm = 4.87 deg (at 2.23e+03 rad/s)



# Exemplo - Compensador em avanço

$$K_v = 10^{94/20} * 0,1 = 5e3 \rightarrow e_{ss} = 2e - 4$$

$$\Rightarrow \text{atenuar } 20 \text{ dB para obter } e_{ss} = 2e - 3$$

- MF = 50°
- $e_{ss} = 0,002$  (rampa unitária)

$$\text{Fase: } 50^\circ (\text{Espec.}) - 15^\circ (\text{Atual}) + 20^\circ (\text{Tol.}) = 55^\circ \Rightarrow \text{fator de avanço} = 10 \text{ (20 dB)}$$

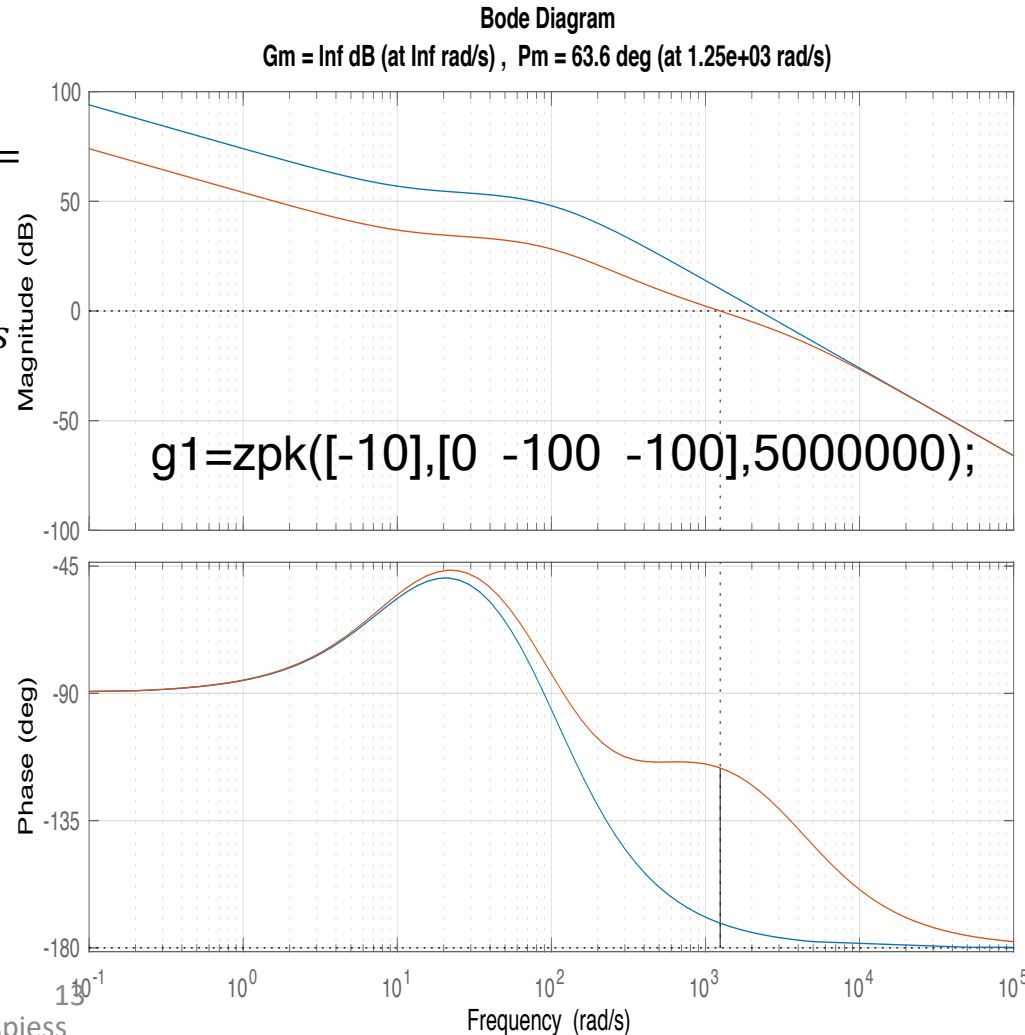
$$\text{Queda de } 10 \text{ dB} \rightarrow \text{frequência } \omega_m = 1260 \text{ rad/s}$$

$$z = \sqrt{\alpha}\omega_m = 398,7 \approx 400 \text{ rad/s}$$

$$p = \frac{\omega_m}{\sqrt{\alpha}} = 3981,6 \approx 4000 \text{ rad/s}$$

$$D(z) = 0,1 \frac{4000}{400} \frac{s + 400}{s + 4000} = \frac{s + 400}{s + 4000}$$

Obs: Margem de Fase obtida de fato: 64°

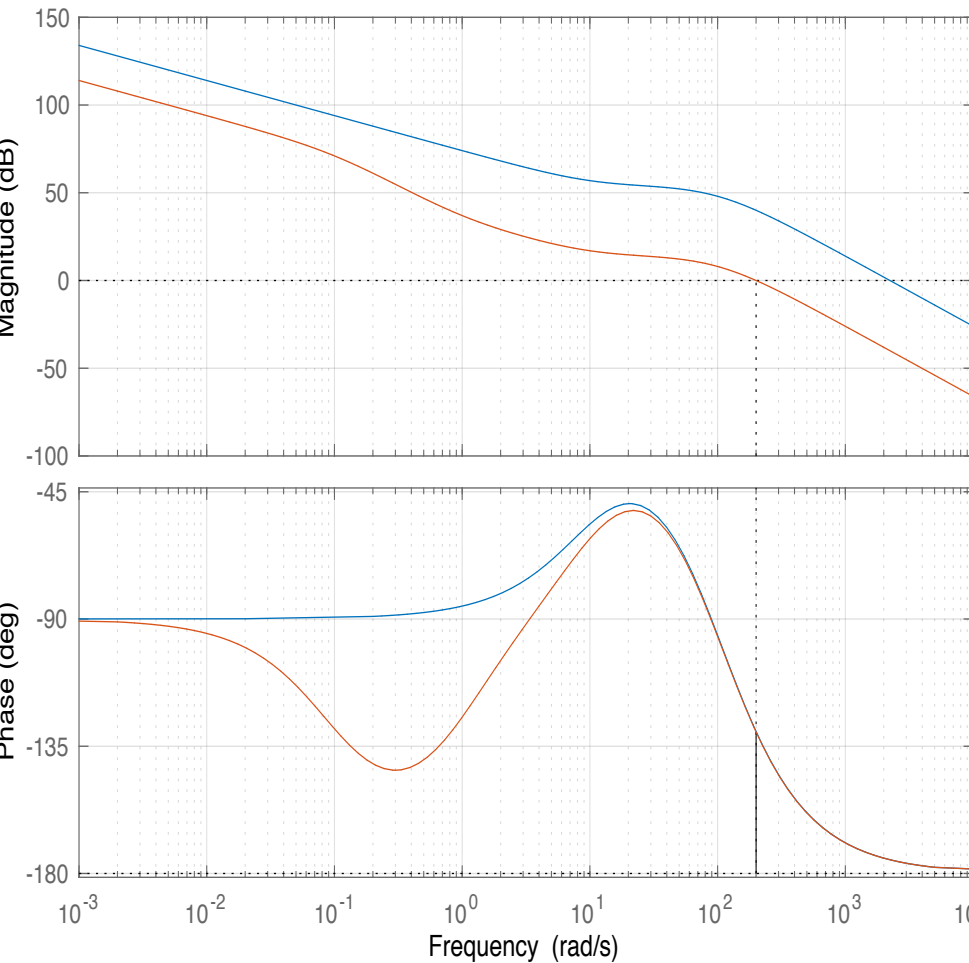


# Comparação: Atraso x Avanço

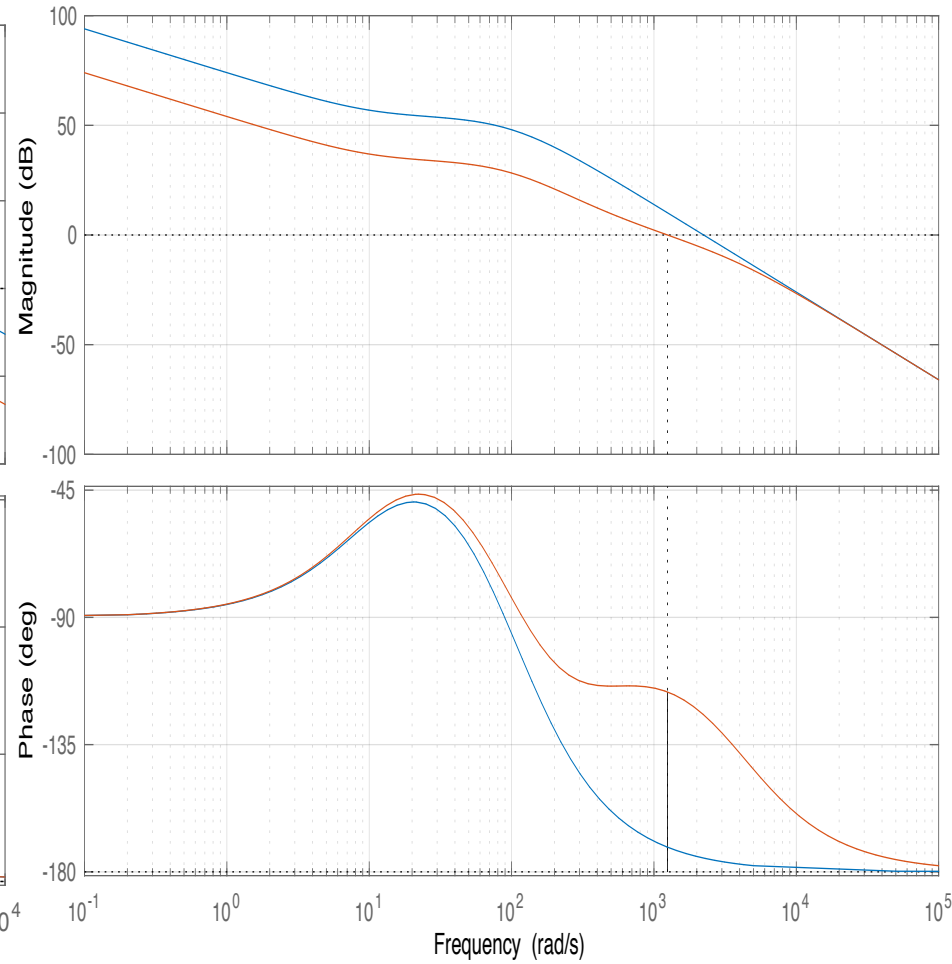
```
g1=zpk([-10],[0 -100 -100],5000000);
```

- MF = 50°
- $e_{ss} = 0,002$  (rampa unitária)

**Bode Diagram**  
Gm = Inf dB (at Inf rad/s) , Pm = 50 deg (at 200 rad/s)



**Bode Diagram**  
Gm = Inf dB (at Inf rad/s) , Pm = 63.6 deg (at 1.25e+03 rad/s)



# Comparação: $D_{at}(s)$ x $D_{av}(s)$ x $D(s)=1$

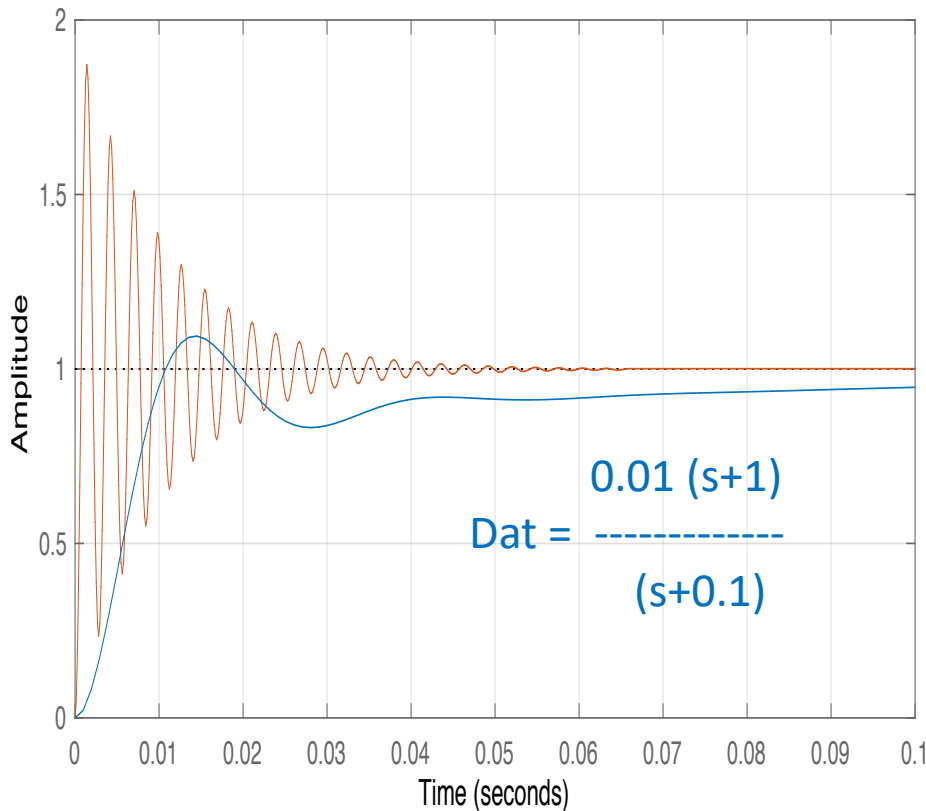
`g1=zpk([-10],[0 -100 -100],5000000);`

- MF = 50°
- $e_{ss} = 0,002$  (rampa unitária)

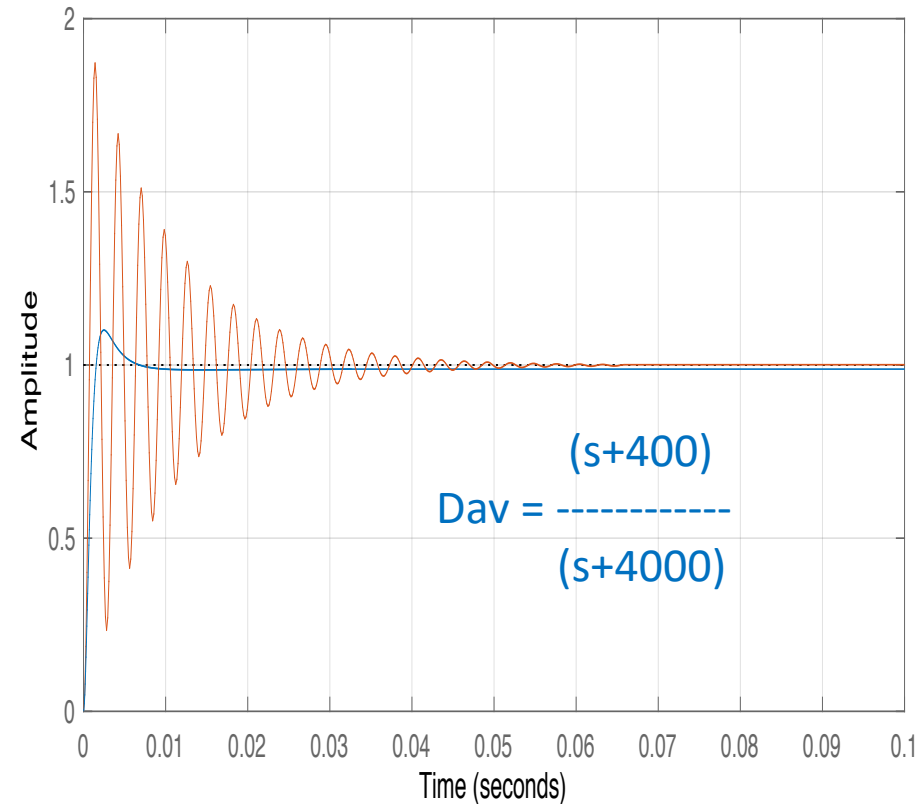
O compensador em avanço torna o sistema mais rápido!

O compensador em atraso deve ser utilizado quando há saturação do atuador!

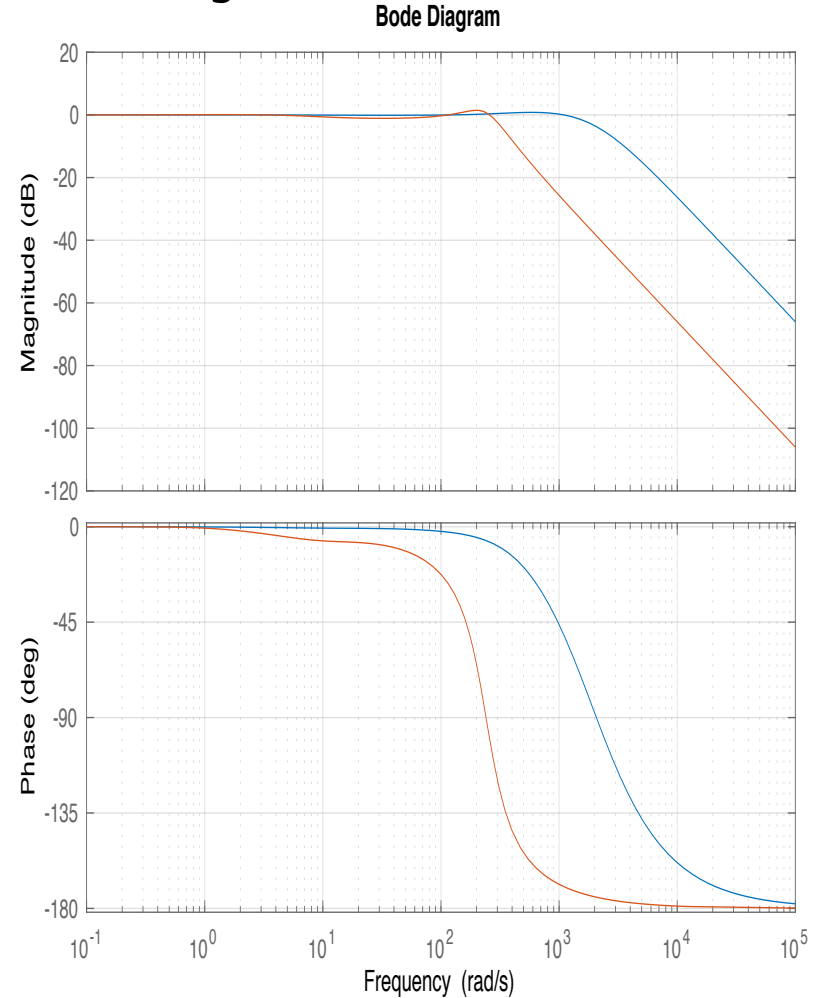
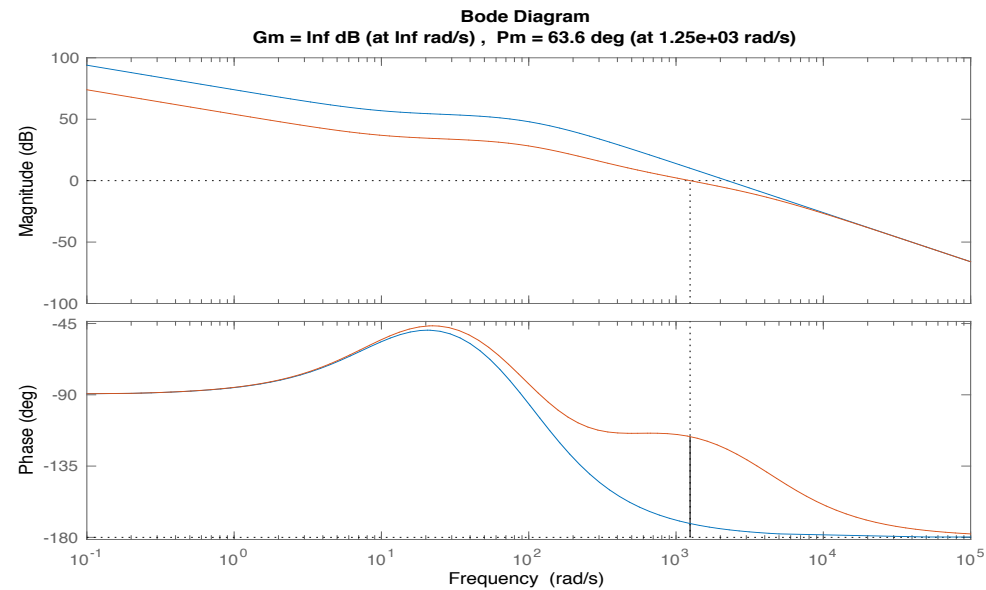
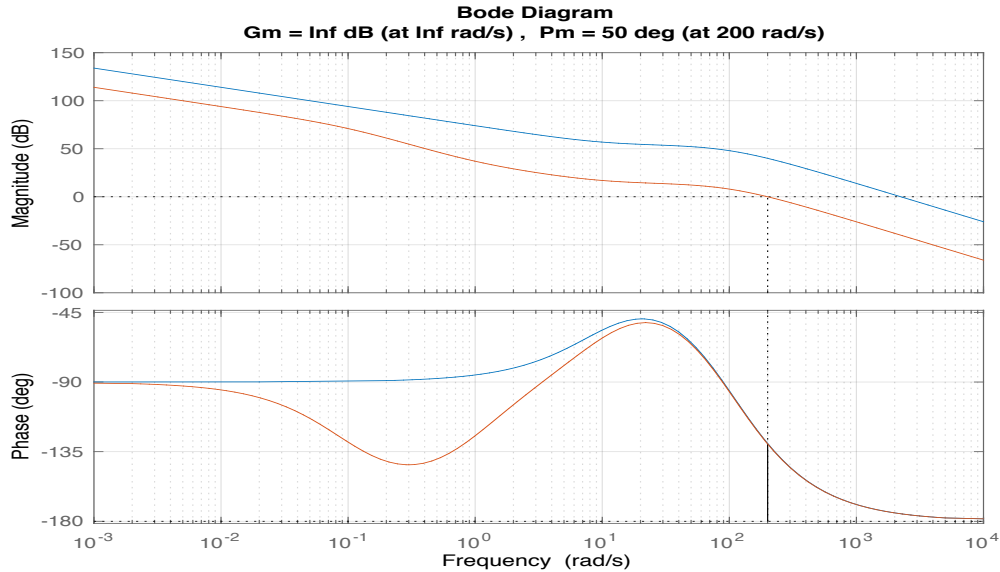
Step Response



Step Response



# Comparação: Atraso x Avanço e Bode MF



A Banda Passante do compensador em avanço é mais larga!

Verificar  $U(s)/R(s)$ !!