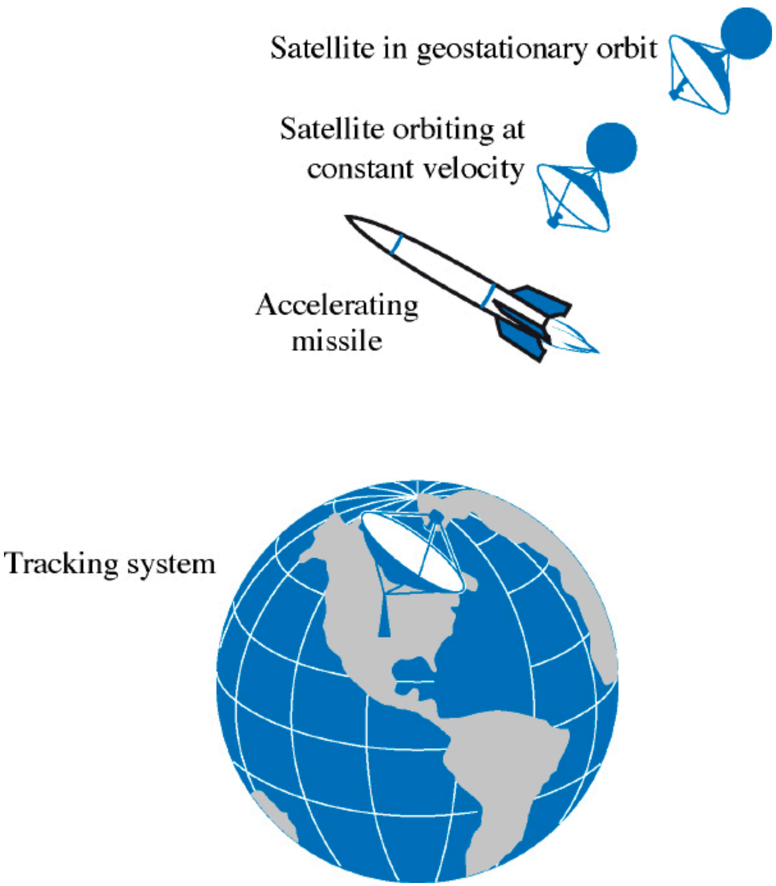
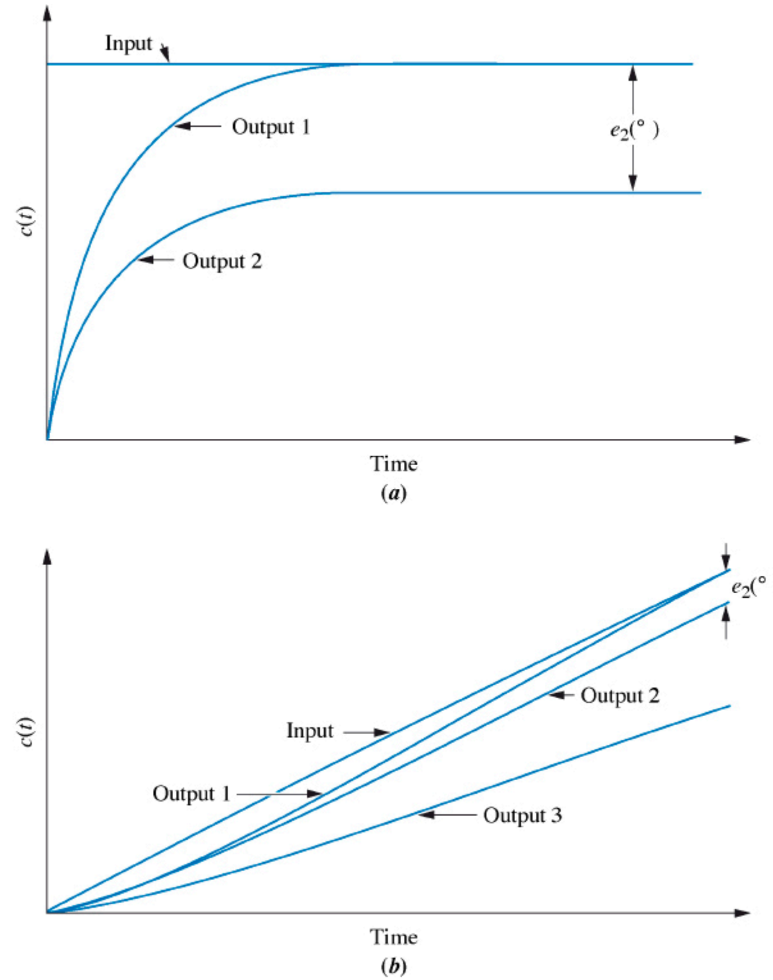
# Resposta em Regime Permanente

*Erro em Regime*: e(t→∞)

Mede a capacidade de sistemas de acompanhar sinais em regime permanente



# Resposta em Regime Permanente

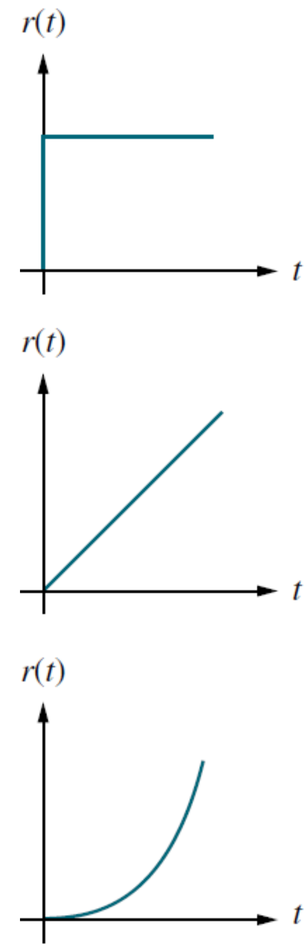
*Classificação de sistemas de controle*:
tipo 0,1,2,...

Quanto à habilidade de seguir entradas:
degrau, rampa, parábola,...

$$r(t) = \frac{t^k}{k!}1(t) \Leftrightarrow R(s) = \frac{1}{s^{k+1}}$$ Polinômios de grau k

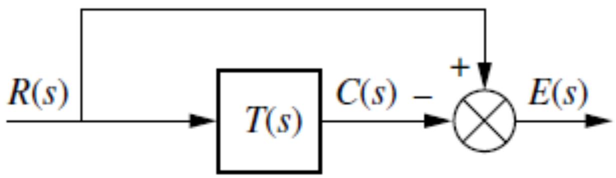
erro $\overset{\Delta}{=}$ saída desejada – saída real

e = r - y




| | | |
|---|---|---|
| (degrau) | $1$ | $\frac{1}{s}$ |
| (rampa) | $t$ | $\frac{1}{s^2}$ |
| (parábola) | $\frac{1}{2}t^2$ | $\frac{1}{s^3}$ |

# Erro em malha fechada

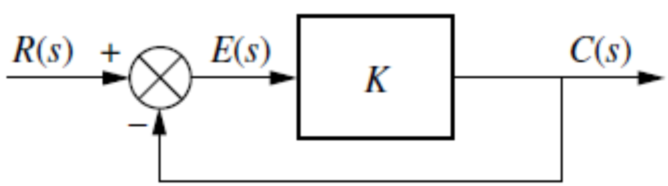



**(a)**

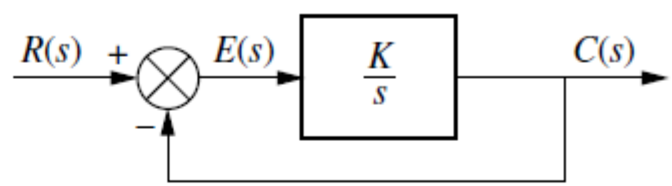

**(b)**

Erro genérico

Erro para realimentação unitária




**(a)**




**(b)**

$e_{ss}$ finito para degrau de referência

$e_{ss} = 0$ para degrau de referência

# Erro em função de G(s)

$$E(s) = R(s) - C(s)$$

$$C(s) = E(s)G(s)$$

$$E(s) = \frac{R(s)}{1 + G(s)}$$

$$\boxed{e(\infty) = \lim_{s\to 0} \frac{sR(s)}{1 + G(s)}}$$

$$com\ R(s) = \frac{1}{s}$$

$$\boxed{e(\infty) = e_{\text{step}}(\infty) = \lim_{s\to 0} \frac{s(1/s)}{1 + G(s)} = \frac{1}{1 + \lim_{s\to 0} G(s)}}$$

$$G(s) \equiv \frac{(s + z_1)(s + z_2)\cdots}{s^n(s + p_1)(s + p_2)\cdots}$$

$$e_{ss} \to 0 \quad \text{se} \quad \lim_{s\to 0} G(s) = \infty$$

# Erro em função de G(s)

$$G(s) \equiv \frac{(s+z_1)(s+z_2)\cdots}{s^n(s+p_1)(s+p_2)\cdots}$$

$$Rampa: R(s) = \frac{1}{s^2}$$

$$\boxed{e(\infty) = e_{\text{ramp}}(\infty) = \lim_{s\to 0}\frac{s(1/s^2)}{1+G(s)} = \lim_{s\to 0}\frac{1}{s+sG(s)} = \frac{1}{\lim\limits_{s\to 0} sG(s)}}$$

$e_{ss} \to 0$ se $sG(s) \to \infty$

$$Parábola: R(s) = \frac{1}{s^3}$$

$$\boxed{e(\infty) = e_{\text{parabola}}(\infty) = \lim_{s\to 0}\frac{s(1/s^3)}{1+G(s)} = \lim_{s\to 0}\frac{1}{s^2+s^2G(s)} = \frac{1}{\lim\limits_{s\to 0} s^2G(s)}}$$

$e_{ss} \to 0$ se $s^2G(s) \to \infty$

# Constantes de Erro

$$\boxed{e(\infty) = e_{\text{step}}(\infty) = \frac{1}{1 + \lim_{s \to 0} G(s)}}$$

Coeficiente de erro de posição:

$$\boxed{K_p = \lim_{s \to 0} G(s)}$$

$$\boxed{e(\infty) = e_{\text{ramp}}(\infty) = \frac{1}{\lim_{s \to 0} sG(s)}}$$

Coeficiente de erro de velocidade:

$$\boxed{K_v = \lim_{s \to 0} sG(s)}$$

$$\boxed{e(\infty) = e_{\text{parabola}}(\infty) = \frac{1}{\lim_{s \to 0} s^2 G(s)}}$$

Coeficiente de erro de aceleração:

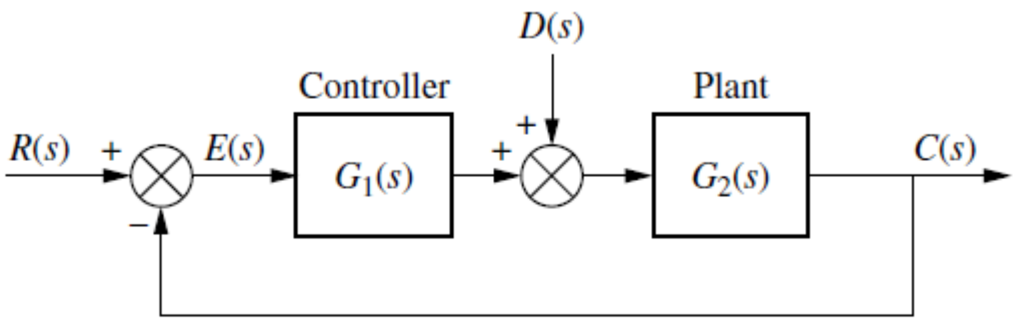
$$\boxed{K_a = \lim_{s \to 0} s^2 G(s)}$$

...Coeficiente de erro da derivada da aceleração

# Tabela
# erro x tipo sistema x coef. erro x $e_{ss}$

| | | Type 0 | | Type 1 | | Type 2 | |
|---|---|---|---|---|---|---|---|
| Input | Steady-state error formula | Static error constant | Error | Static error constant | Error | Static error constant | Error |
| Step, $u(t)$ | $\frac{1}{1+K_p}$ | $K_p = \text{Constant}$ | $\frac{1}{1+K_p}$ | $K_p = \infty$ | $0$ | $K_p = \infty$ | $0$ |
| Ramp, $tu(t)$ | $\frac{1}{K_v}$ | $K_v = 0$ | $\infty$ | $K_v = \text{Constant}$ | $\frac{1}{K_v}$ | $K_v = \infty$ | $0$ |
| Parabola, $\frac{1}{2}t^2u(t)$ | $\frac{1}{K_a}$ | $K_a = 0$ | $\infty$ | $K_a = 0$ | $\infty$ | $K_a = \text{Constant}$ | $\frac{1}{K_a}$ |

# Erro em regime para perturbações



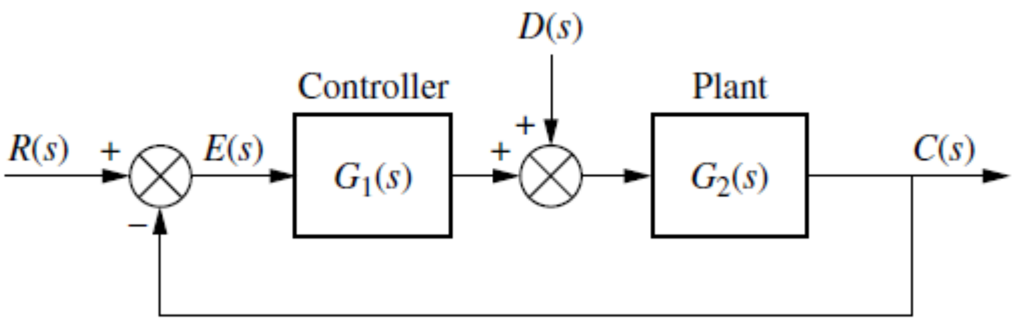

$$E(s) = \frac{1}{1+G_1(s)G_2(s)}R(s) - \frac{G_2(s)}{1+G_1(s)G_2(s)}D(s)$$

$$\begin{aligned} e(\infty) = \lim_{s\to 0} sE(s) &= \lim_{s\to 0}\frac{s}{1+G_1(s)G_2(s)}R(s) - \lim_{s\to 0}\frac{sG_2(s)}{1+G_1(s)G_2(s)}D(s) \\ &= e_R(\infty) + e_D(\infty) \end{aligned}$$

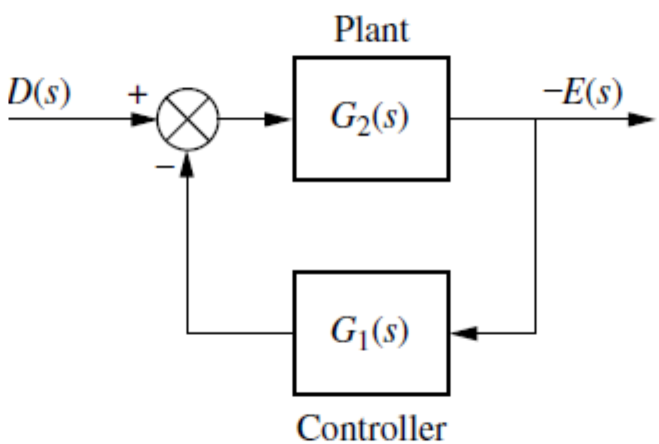
$$e_R(\infty) = \lim_{s\to 0}\frac{s}{1+G_1(s)G_2(s)}R(s)$$

$$e_D(\infty) = -\lim_{s\to 0}\frac{sG_2(s)}{1+G_1(s)G_2(s)}D(s)$$

# Erro em regime para perturb. (cont.)




$$e_D(\infty) = -\lim_{s\to 0} \frac{sG_2(s)}{1+G_1(s)G_2(s)} D(s)$$



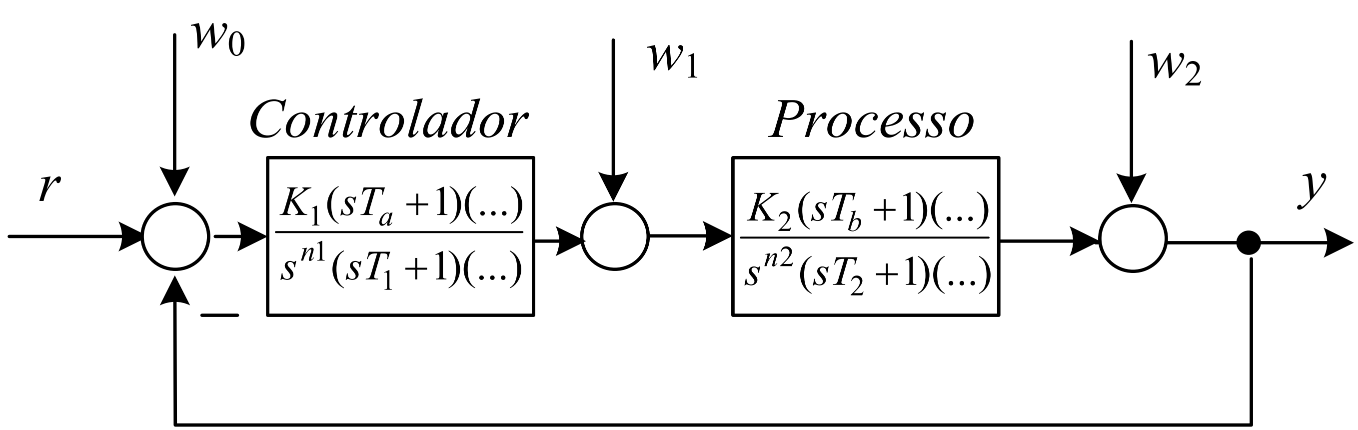

Perturbação como "entrada" do sistema

$$e_D(\infty) = -\frac{1}{\lim\limits_{s\to 0} \dfrac{1}{G_2(s)} + \lim\limits_{s\to 0} G_1(s)}$$

Boa rejeição de perturbações ocorre com $G_1\uparrow$ ou $G_2\downarrow$

Integrador em $G_1 \rightarrow e_D(\infty)=0$

# Múltiplas perturbações




- "Rejeição: Devem haver integradores à esquerda da perturbação"

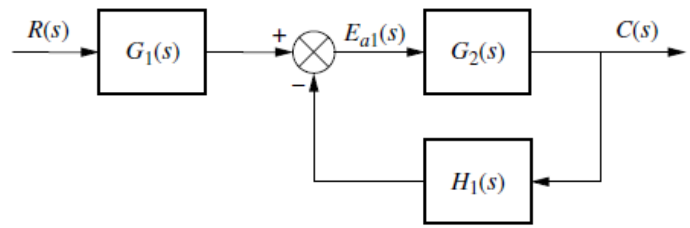
- O sistema é do tipo 0, $n1$, ($n1$+$n2$)
em relação às entradas de perturbação $w_0$,$w_1$ e $w_2$

- Perturbações na entrada ($w_0$) não são rejeitadas,
pois não é possível distinguí-las da entrada $r$

# Realimentação não unitária

a) Calcular Direto

$$\frac{C(s)}{R(s)} = \frac{G_1(s)G_2(s)}{1 + \ G_2(s)H_1(s)}$$

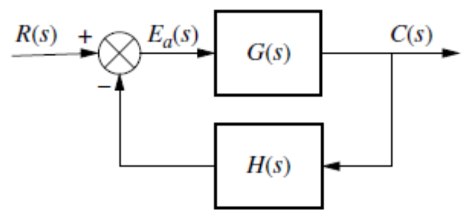



(a)

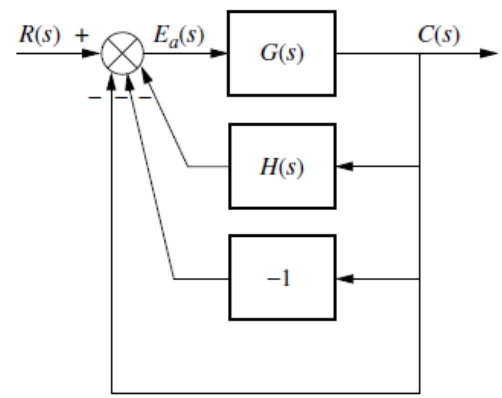
$$E(s) = R(s) - C(s)$$

$$E(s) = R(s) - \frac{G_1(s)G_2(s)R(s)}{1 + \ G_2(s)H_1(s)}$$

$$E(s) = \left(1 - \frac{G_1(s)G_2(s)}{1 + \ G_2(s)H_1(s)}\right)R(s)$$

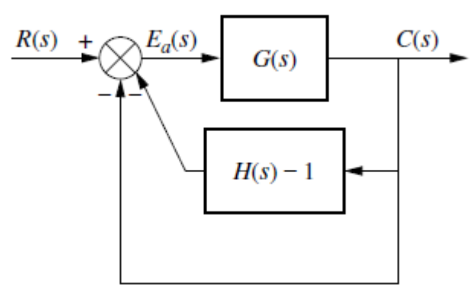



(b)

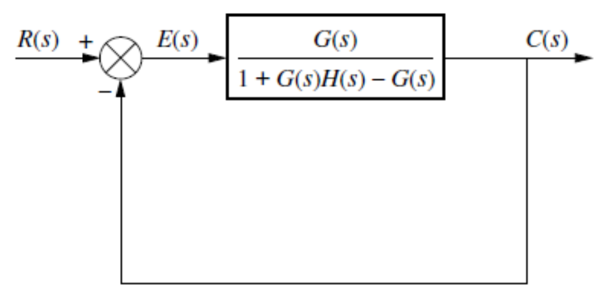



(c)

b) Rearranjo do diagrama de blocos

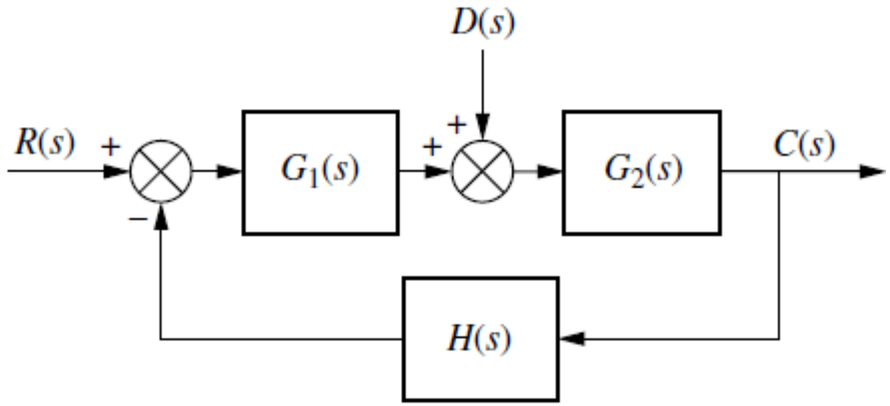



(d)



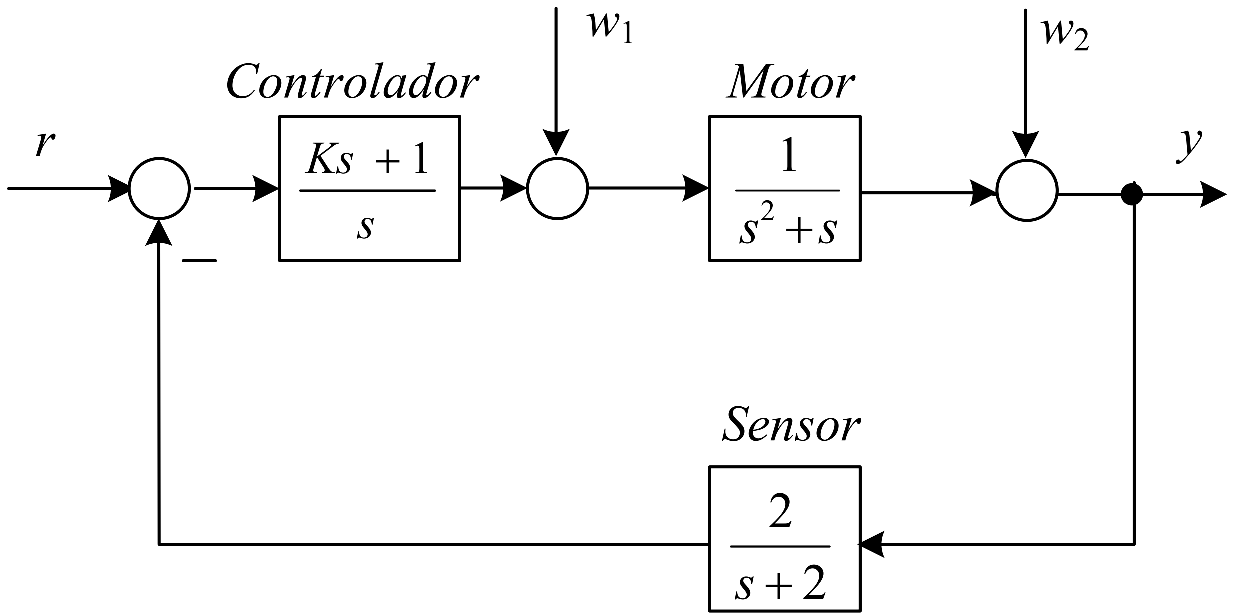

(e)

# Realimentação não unitária




$$e(\infty) = \lim_{s\to 0} sE(s) = \lim_{s\to 0} s\left\{ \left[1 - \frac{G_1(s)G_2(s)}{1+G_1(s)G_2(s)H(s)}\right]R(s) - \left[\frac{G_2(s)}{1+G_1(s)G_2(s)H(s)}D(s)\right]\right\}$$

# Exercício Extra 1- $e_{ss}$




Preencha a tabela (K=1).

| Entrada\ Sinal | $r$ | $w_1$ | $w_2$ |
|---|---|---|---|
| *Degrau* | 0 | 0 | 0 |
| *Rampa* | -1/2 | -1/K | 0 |
| *Parábola* | -∞ | -∞ | -1/K |

# Ex. Extra 1- $e_{ss}$

$$\frac{Y}{R} = G = \frac{K(s+2)}{s^2(s+2)+2K}$$

$e = r - y;$

$e_r = \lim_{s\to 0} s(R - G\,R);$ $\;$ *erro considerando* $r$ $(w = 0)$.

(r degrau)

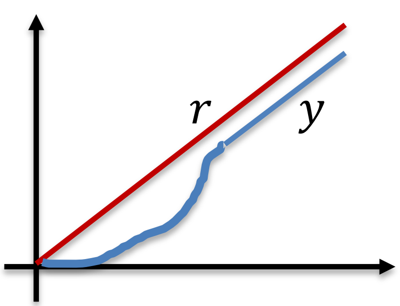
$e_{rd} = \lim_{s\to 0} s\left(\frac{1}{s} - \frac{K(s+2)}{s^2(s+2)+2K}\frac{1}{s}\right)$ =0.

(r rampa)

$e_{rr} = \lim_{s\to 0} s\left(\frac{1}{s^2} - \frac{K(s+2)}{s^2(s+2)+2K}\frac{1}{s^2}\right) = \lim_{s\to 0} s\left(\frac{s^2(s+2)+2K-K(s+2)}{s^2(s+2)+2K}\frac{1}{s^2}\right) = \lim_{s\to 0} s\left(\frac{s^2(s+2)-sK}{s^2(s+2)+2K}\frac{1}{s^2}\right)$

$= \lim_{s\to 0} s\left(\frac{s^2(s+2)-sK}{s^2(s+2)+2K}\frac{1}{s^2}\right) = \lim_{s\to 0}\left(\frac{s^2(s+2)-sK}{s^2(s+2)+2K}\frac{1}{s}\right) = \lim_{s\to 0}\left(\frac{s(s+2)-K}{s^2(s+2)+2K}\right)$ = -1/2

Obs: $r \to \infty$; $y \to \infty$; porém $e = r - y$ é finito!




(r parábola)

$e_{rp} = \lim_{s\to 0} s\left(\frac{1}{s^3} - \frac{K(s+2)}{s^2(s+2)+2K}\frac{1}{s^3}\right)$ = -∞

# Ex. Extra 1- $e_{ss}$

$r = 0;\ e = r - y;$

$$e_{w1} = \lim_{s\to 0} sG_{w1}\, W_1;$$

$$\frac{Y}{W_1} = \frac{s(s+2)}{s^2(s+1)(s+2)+2K(s+1)}$$

$$e_{w1d} = \lim_{s\to 0} s\,\frac{s(s+2)}{s^2(s+1)(s+2)+2K(s+1)}\,\frac{1}{s} = 0$$

$$e_{w1r} = \lim_{s\to 0} s\,\frac{s(s+2)}{s^2(s+1)(s+2)+2K(s+1)}\,1/s^2 = 1/\mathrm{K}$$

$$e_{w1p} = \lim_{s\to 0} s\,\frac{s(s+2)}{s^2(s+1)(s+2)+2K(s+1)}\,1/s^3 = \infty$$

$$e_{w2} = \lim_{s\to 0} sG_{w2}\, W_2;$$

$$\frac{Y}{W_2} = \frac{s^2\,(s+1)(s+2)}{s^2(s+1)(s+2)+2K(s+1)}$$

$$e_{w2d} = \lim_{s\to 0} s\,\frac{s^2\,(s+1)(s+2)}{s^2(s+1)(s+2)+2K(s+1)}\,\frac{1}{s} = 0$$

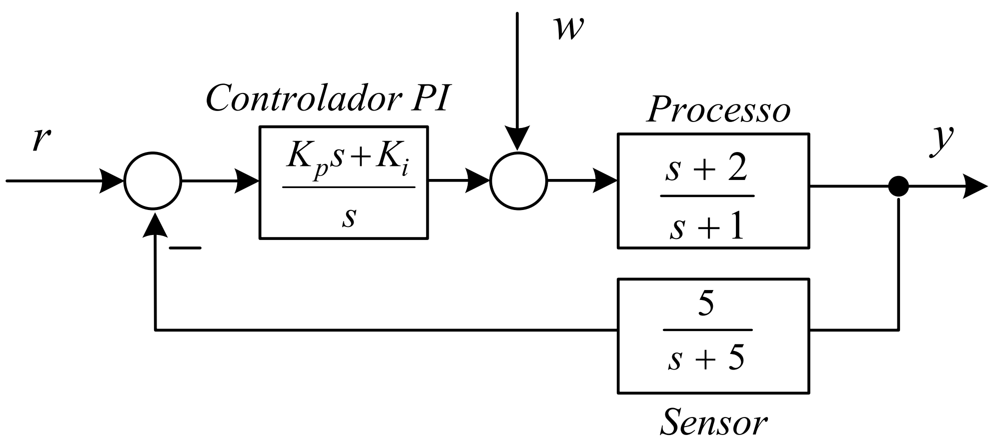
$$e_{w2r} = \lim_{s\to 0} s\,\frac{s^2\,(s+1)(s+2)}{s^2(s+1)(s+2)+2K(s+1)}\,1/s^2 = 0$$

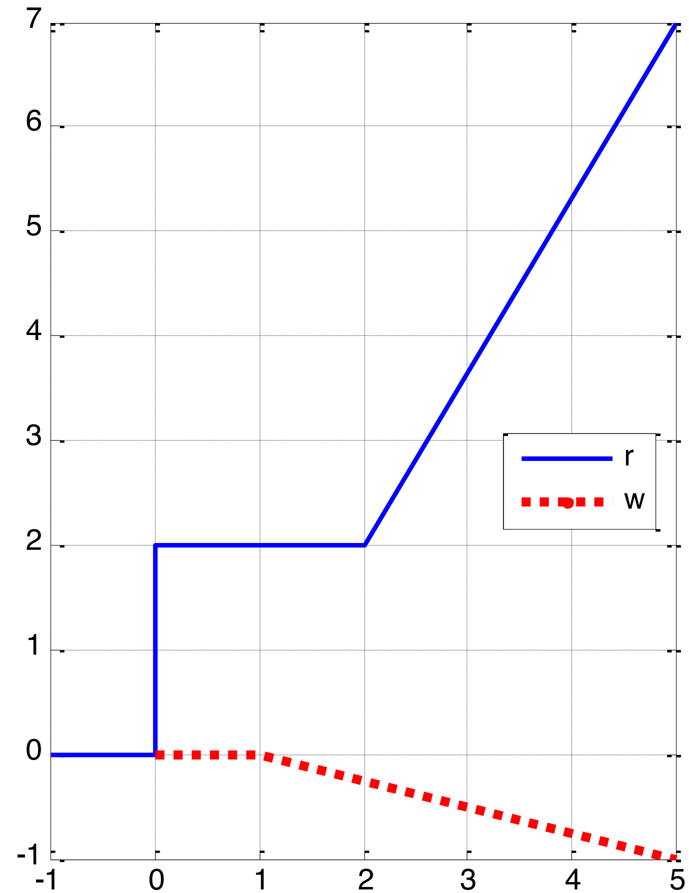
$$e_{w2p} = \lim_{s\to 0} s\,\frac{s^2\,(s+1)(s+2)}{s^2(s+1)(s+2)+2K(s+1)}\,1/s^3 = 1/\mathrm{K}$$

# Exercício Extra 2- $e_{ss}$

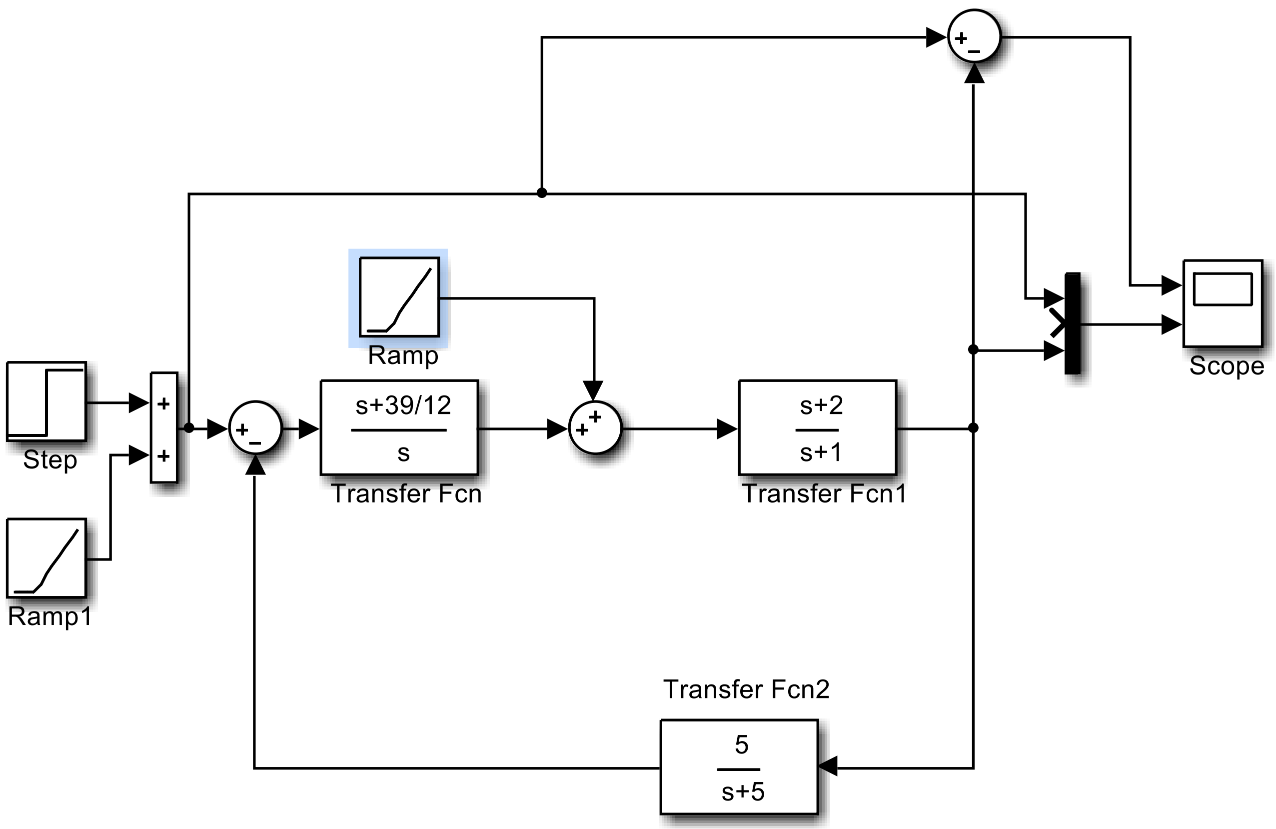
Considerando os sinais de referência e perturbação mostrados no gráfico, obtenha o erro em regime permanente para:

$K_i$ = 0,1*último dígito matrícula + 1;
$K_p$ = 10*penúltimo dígito matrícula + 1;

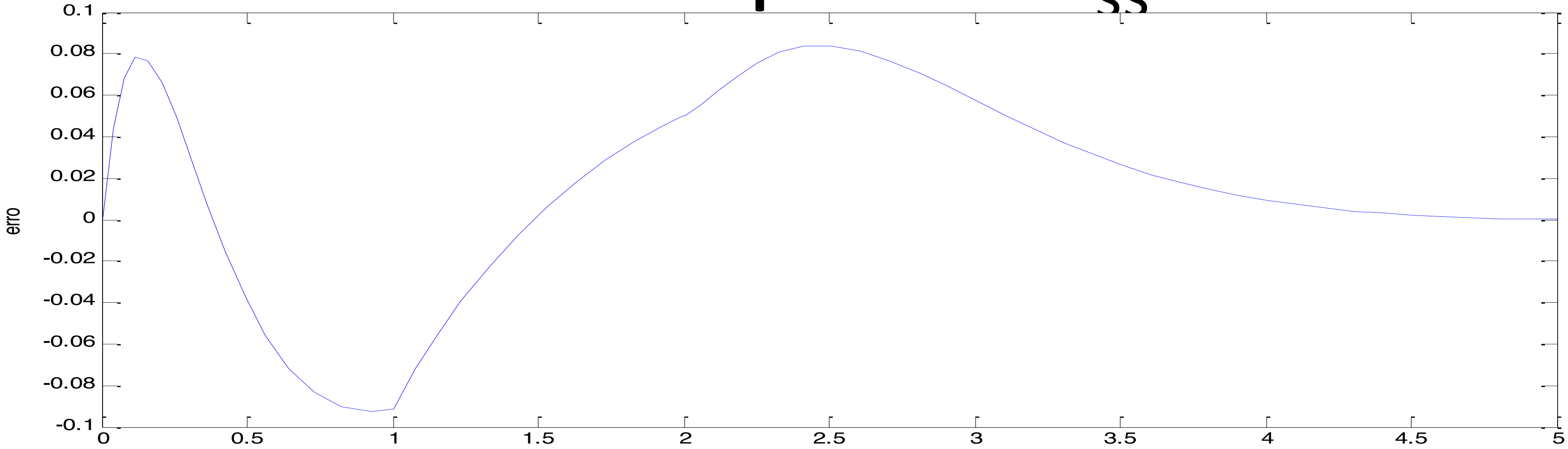


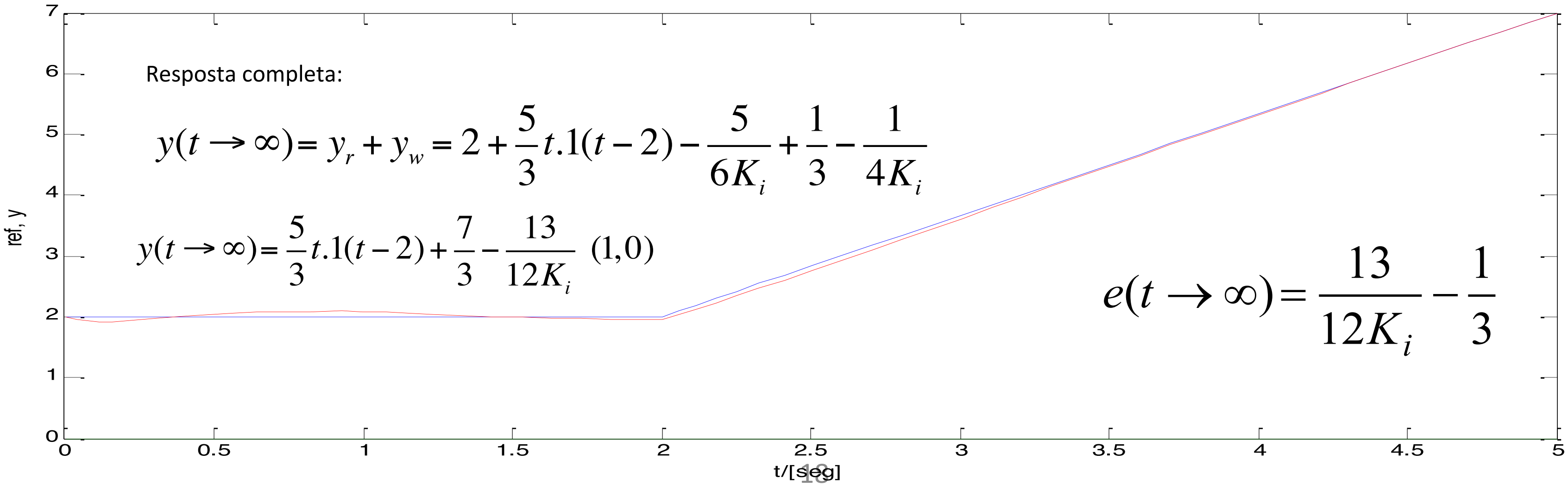



# Exemplo Extr3 - $e_{ss}$



# Exemplo 3 - $e_{ss}$




Resposta completa:

$$y(t \to \infty) = y_r + y_w = 2 + \frac{5}{3}t.1(t-2) - \frac{5}{6K_i} + \frac{1}{3} - \frac{1}{4K_i}$$

$$y(t \to \infty) = \frac{5}{3}t.1(t-2) + \frac{7}{3} - \frac{13}{12K_i} \quad (1,0)$$

$$e(t \to \infty) = \frac{13}{12K_i} - \frac{1}{3}$$