Controle de Sistemas Dinâmicos

CSD3 - Linearização

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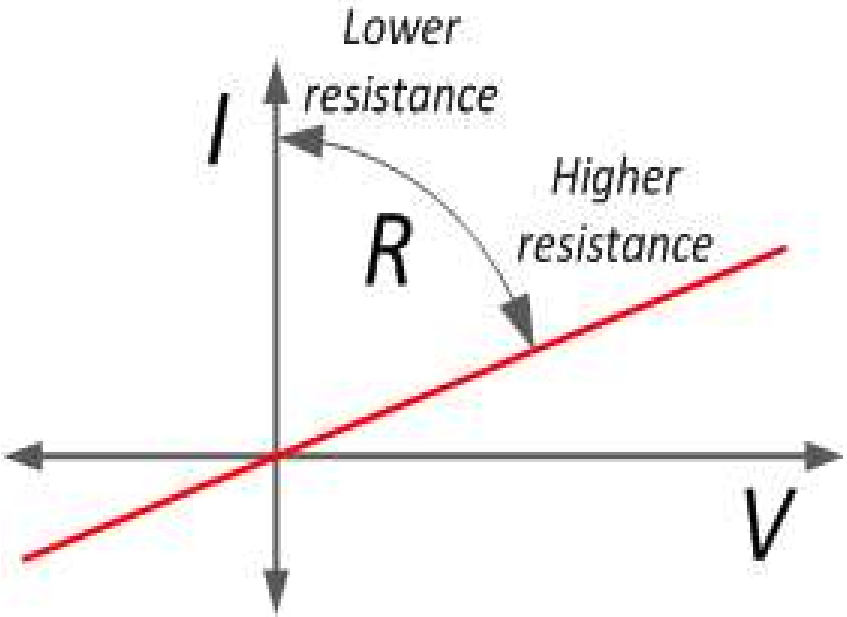
ENE/UnB

(Material de aula *Complementar,* adaptado parcialmente de Nise – Eng. de Sist. de Controle)

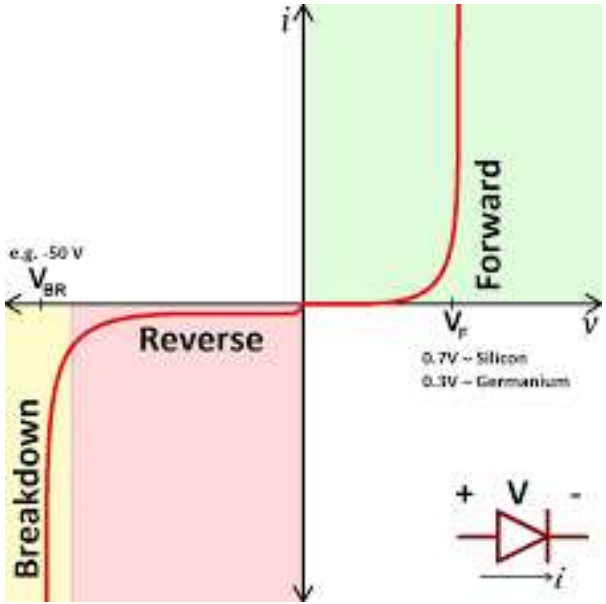
Sistema Linear

Superposição: $r_1(t) + r_2(t)$; produzirá uma saída $c_1(t) + c_2(t)$.

Homogeneidade: A $r_1(t)$ produzirá uma saída A $c_1(t)$.



a. Sistema linear;



b. sistema não-linear

Fig. 2.46

Algumas não-linearidades físicas

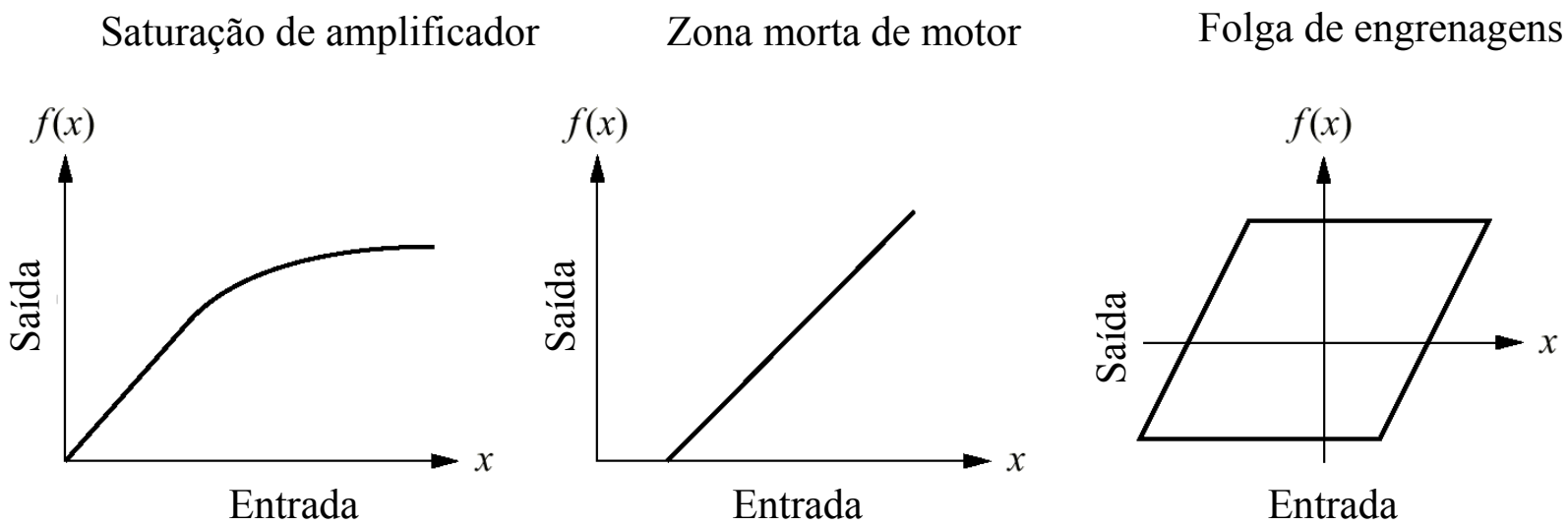


Fig. 2.47

Linearização em torno de ponto de operação (*A*)

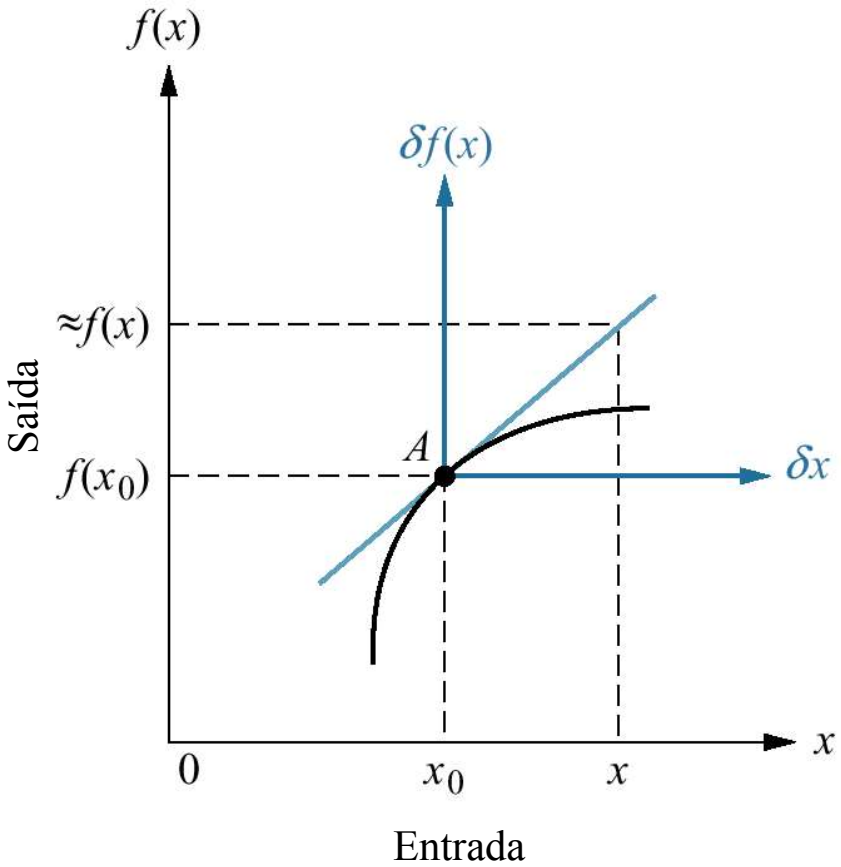
Etapas:

1. Escrever a equação diferencial não-linear para o componente não-linear.
2. Linearizar a equação para pequenos sinais de entrada em torno da solução de estado estacionário, ou *equilíbrio.*
3. Separar variáveis de Entrada, Saída e Intermediárias
4. Calcular a Função de Transferência a partir das equações linearizadas

$$[f(x) - f(x_0)] \approx m_a(x - x_0) \qquad \delta f(x) \approx m_a \delta x$$

$$f(x) = f(x_0) + \left.\frac{df}{dx}\right|_{x=x_0} \frac{(x - x_0)}{1!} + \left.\frac{d^2 f}{dx^2}\right|_{x=x_0} \frac{(x - x_0)^2}{2!} + \cdots$$

(Taylor)



$$\boxed{f(x) - f(x_0) \approx \left.\frac{df}{dx}\right|_{x=x_0} (x - x_0)}$$

$$\boxed{\delta f(x) \approx m|_{x=x_0}\, \delta x}$$

Fig. 2.48

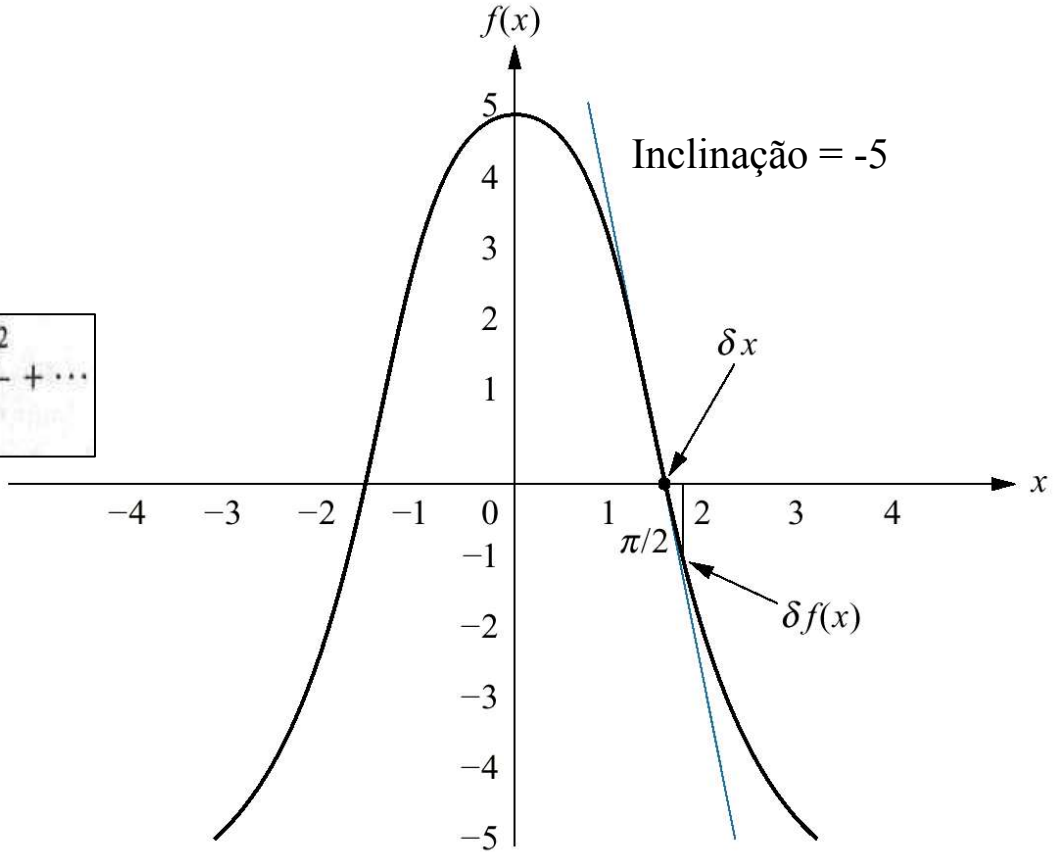
Linearização de 5 $\cos x$
em torno de $x = \pi / 2$

$$f(x) = f(x_0) + \left.\frac{df}{dx}\right|_{x=x_0} \frac{(x-x_0)}{1!} + \left.\frac{d^2f}{dx^2}\right|_{x=x_0} \frac{(x-x_0)^2}{2!} + \cdots$$

$$f(x) - f(x_0) \approx \left.\frac{df}{dx}\right|_{x=x_0} (x - x_0)$$

$$\delta f(x) \approx m|_{x=x_0}\, \delta x$$

$$\delta f(x) \approx -5\,\delta x$$



Linearização da uma eq. dif. não linear

$$\frac{d^2x}{dt^2} + 2\frac{dx}{dt} + \cos x = 0 \quad \text{em torno de } x_0 = \pi/4$$

Solução: Para linearizar a equação em torno de $x = \pi/4$, fazemos $x = \delta x + \pi/4$, onde δx é uma pequena excursão em torno de $\pi/4$, e substituímos x na eq. original:

$$\frac{d^2\left(\delta x + \frac{\pi}{4}\right)}{dt^2} + 2\frac{d\left(\delta x + \frac{\pi}{4}\right)}{dt} + \cos\left(\delta x + \frac{\pi}{4}\right) = 0$$

$$\frac{d^2\left(\delta x + \frac{\pi}{4}\right)}{dt^2} = \frac{d^2\delta x}{dt^2} \qquad \frac{d\left(\delta x + \frac{\pi}{4}\right)}{dt} = \frac{d\delta x}{dt}$$

$$f(x) - f(x_0) \approx \left.\frac{df}{dx}\right|_{x=x_0} (x - x_0)$$

$$\cos\left(\delta x + \frac{\pi}{4}\right) - \cos\left(\frac{\pi}{4}\right) = \left.\frac{d\cos x}{dx}\right|_{x=\frac{\pi}{4}} \delta x = -\operatorname{sen}\left(\frac{\pi}{4}\right)\delta x$$

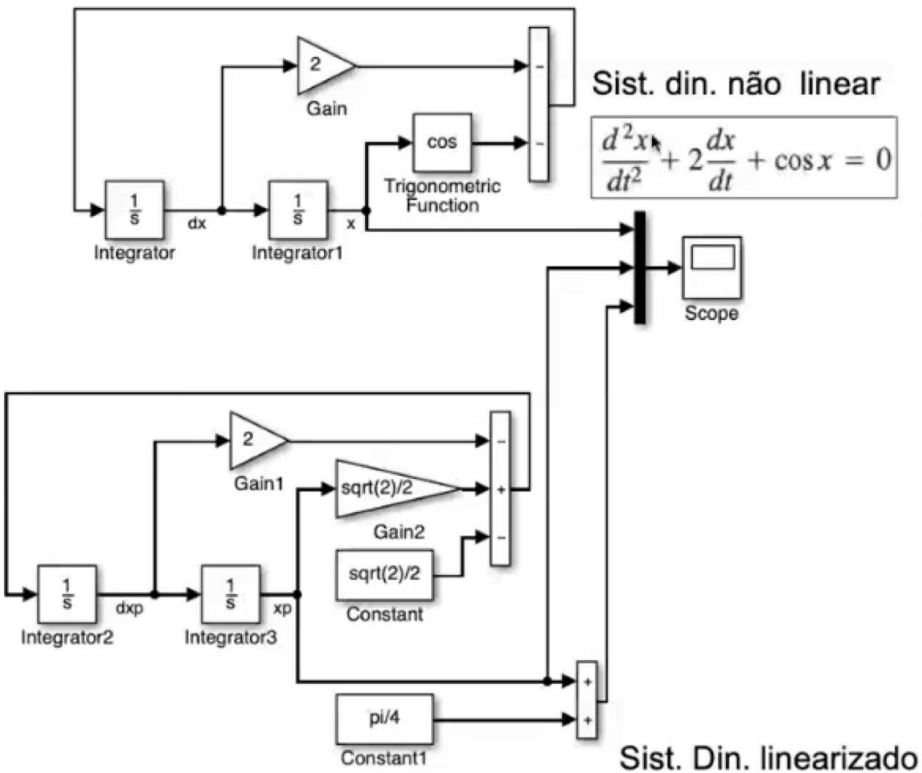
$$\cos\left(\delta x + \frac{\pi}{4}\right) = \cos\left(\frac{\pi}{4}\right) - \operatorname{sen}\left(\frac{\pi}{4}\right)\delta x = \frac{\sqrt{2}}{2} - \frac{\sqrt{2}}{2}\delta x$$

Substituindo, resulta a seguinte equação diferencial linearizada:

$$\boxed{\frac{d^2\delta x}{dt^2} + 2\frac{d\delta x}{dt} - \frac{\sqrt{2}}{2}\delta x = -\frac{\sqrt{2}}{2}}$$

Comparação: Sistema **Não** Linear x Linearizado

Sistema autônomo (sem entrada) em torno de $x = \pi/4$.

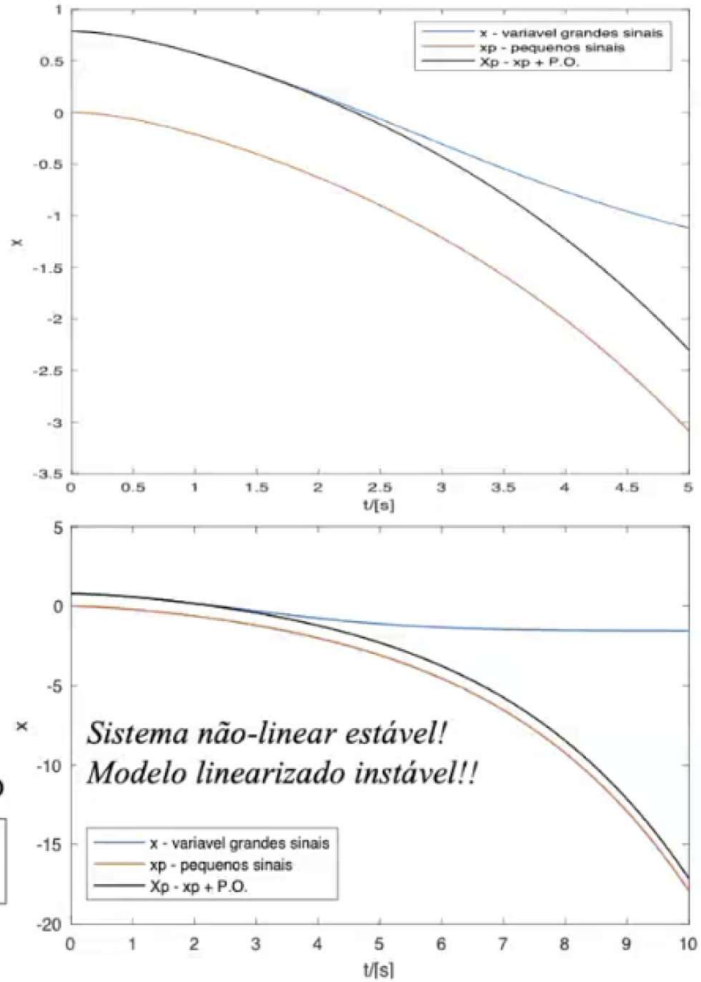


Sist. din. não linear

$$\frac{d^2x}{dt^2} + 2\frac{dx}{dt} + \cos x = 0$$

Sist. Din. linearizado

$$\frac{d^2\delta x}{dt^2} + 2\frac{d\delta x}{dt} - \frac{\sqrt{2}}{2}\delta x = -\frac{\sqrt{2}}{2}$$

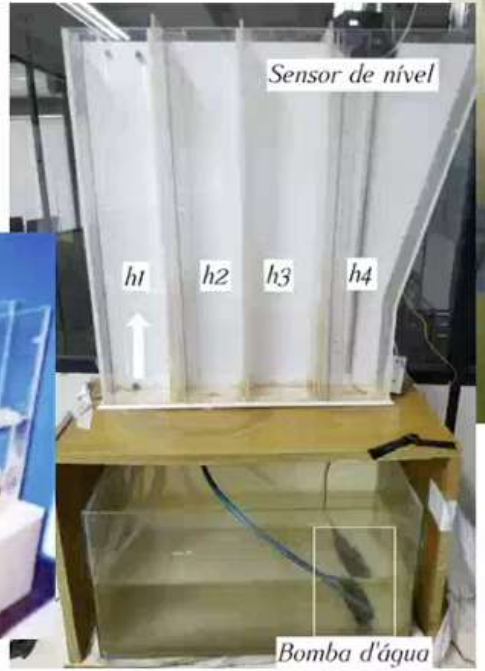


SISTEMAS DE NÍVEL DE LÍQUIDO

2ª ordem

3ª ordem

4ª ordem cascata



4ª ordem
Fase não-mínima

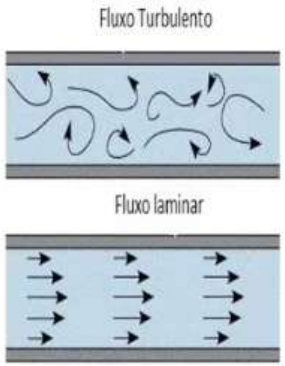
SISTEMAS DE NÍVEL DE LÍQUIDO

Regimes de fluxo :

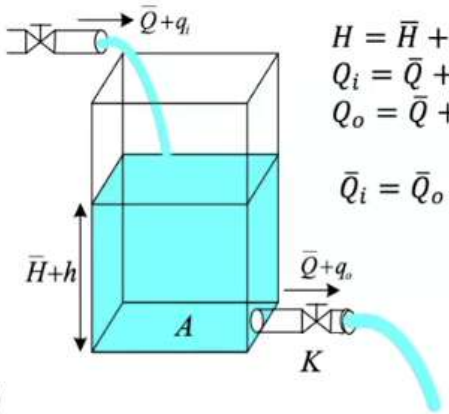
Reynolds > 3.000 ~ 4.000 ⇒ fluxo turbulento, $Q = K\sqrt{H}$, K [m$^{2.5}$/s]

Reynolds < 2000 ⇒ fluxo laminar, $Q = KlH$

Processos industriais geralmente são turbulentos



Grandes Sinais =
Ponto de Operação + Pequenos Sinais



$$H = \bar{H} + h$$
$$Q_i = \bar{Q} + q_i$$
$$Q_o = \bar{Q} + q_o$$

$$\bar{Q}_i = \bar{Q}_o = \bar{Q}$$

Balanço de massa (= volume, água):

$$A\,\frac{dH}{dt} = Q_i \quad - \quad K\sqrt{H} \quad (Q_o = K\sqrt{H})$$

$$A\,\frac{dh}{dt} = \bar{Q} + q_i - K\sqrt{\bar{H}} - \frac{K}{2\sqrt{\bar{H}}}h$$

Linearização

$$f(X) = f(\bar{X}) + \left.\frac{df}{dx}\right|_{\bar{X}} (X - \bar{X})$$

$$\sqrt{H} = \sqrt{\bar{H}} + \frac{1}{2\sqrt{\bar{H}}}h$$

$$A\,\frac{dh}{dt} + \frac{K}{2\sqrt{\bar{H}}}h = q_i$$

Laplace

$$AsH(s) + \frac{K}{2\sqrt{\bar{H}}}H(s) = Q_i(s)$$

$$\boxed{\frac{H(s)}{Q_i(s)} = \frac{1}{As + \dfrac{K}{2\sqrt{\bar{H}}}}}$$

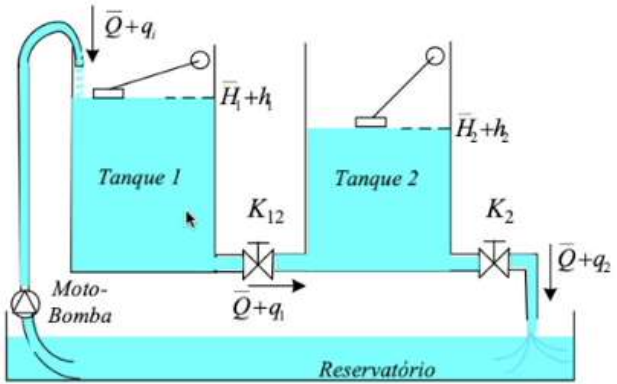
$$\boxed{\frac{H(s)}{Q_i(s)} = \frac{R}{RCs + 1}}$$

$$C = A;\ R = \frac{2\sqrt{\bar{H}}}{K}$$

SISTEMAS DE NÍVEL DE LÍQUIDO COM INTERAÇÃO

Balanço de vazão mássica – Fluxo Turbulento

$$A_1 \frac{dH_1}{dt} = Q_i - K_{12}\sqrt{H_1 - H_2}$$
$$A_2 \frac{dH_2}{dt} = K_{12}\sqrt{H_1 - H_2} - K_2\sqrt{H_2}$$



Linearização

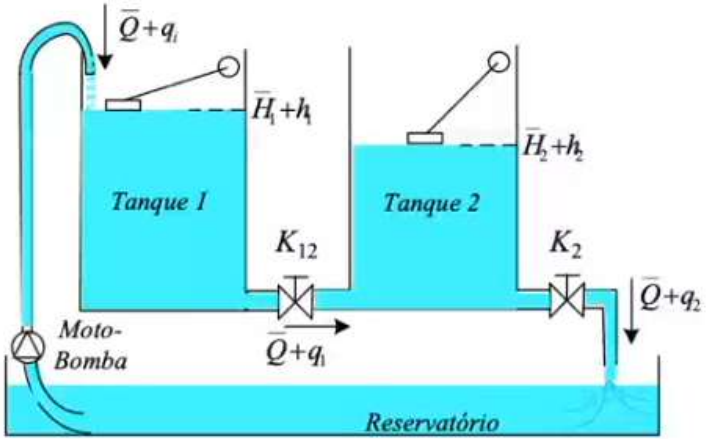
$$f(X) = f(\bar{X}) + \left.\frac{df}{dx}\right|_{\bar{X}} (X - \bar{X})$$

$$\sqrt{H_2} = \sqrt{\bar{H}_2} + \frac{1}{2\sqrt{\bar{H}_2}} h_2$$

$$\sqrt{H_1 - H_2} = \sqrt{\Delta} = \sqrt{\bar{\Delta}} + \frac{1}{2\sqrt{\bar{\Delta}}}(\Delta - \bar{\Delta})$$
$$= \sqrt{\bar{H}_1 - \bar{H}_2} + \frac{1}{2\sqrt{\bar{H}_1 - \bar{H}_2}}(H_1 - H_2 - \bar{H}_1 + \bar{H}_2)$$
$$= \sqrt{\bar{H}_1 - \bar{H}_2} + \frac{1}{2\sqrt{\bar{H}_1 - \bar{H}_2}}(h_1 - h_2)$$

Balanço de vazão mássica – Fluxo Turbulento

$$A_1 \frac{dH_1}{dt} = Q_i - K_{12}\sqrt{H_1 - H_2}$$
$$A_2 \frac{dH_2}{dt} = K_{12}\sqrt{H_1 - H_2} - K_2\sqrt{H_2}$$



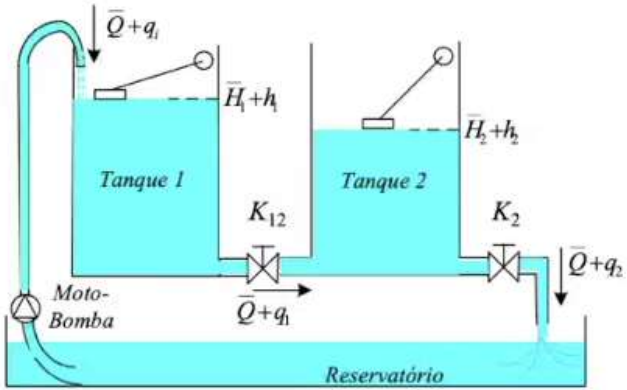
$$A_1 \frac{dh_1}{dt} = q_i + \bar{Q} - K_{12}\sqrt{\bar{H}_1 - \bar{H}_2} - \frac{K_{12}}{2\sqrt{\bar{H}_1 - \bar{H}_2}}(h_1 - h_2)$$
$$A_2 \frac{dh_2}{dt} = K_{12}\sqrt{\bar{H}_1 - \bar{H}_2} + \frac{K_{12}}{2\sqrt{\bar{H}_1 - \bar{H}_2}}(h_1 - h_2) - K_2\sqrt{\bar{H}_2} - \frac{K_2}{2\sqrt{\bar{H}_2}}h_2$$

$$A_1 \frac{dh_1}{dt} = q_i - \frac{K_{12}}{2\sqrt{\bar{H}_1 - \bar{H}_2}}(h_1 - h_2)$$
$$A_2 \frac{dh_2}{dt} = \frac{K_{12}}{2\sqrt{\bar{H}_1 - \bar{H}_2}}(h_1 - h_2) - \frac{K_2}{2\sqrt{\bar{H}_2}}h_2$$

$$\bar{Q} = K_{12}\sqrt{\bar{H}_1 - \bar{H}_2} = K_2\sqrt{\bar{H}_2}$$

Modelo linearizado

$$A_1 \frac{dh_1}{dt} = q_e - ah_1 + ah_2$$
$$A_2 \frac{dh_2}{dt} = ah_1 - (a+b)h_2$$
$$a = \frac{K_{12}}{2\sqrt{\bar{H}_1 - \bar{H}_2}} = \frac{1}{R_1}; b = \frac{K_2}{2\sqrt{\bar{H}_2}} = \frac{1}{R_2}$$



Transformada de Laplace

$$A_1 s H_1(s) = Q_i(s) - aH_1(s) + aH_2(s)$$
$$A_2 s H_2(s) = aH_1(s) - (a+b)H_2(s)$$

$$(A_1 s + a)H_1(s) - aH_2(s) = Q_i(s)$$
$$-aH_1(s) + (A_2 s + a + b)H_2(s) = 0$$

$$H_2(s) = \frac{\begin{vmatrix} A_1 s + a & Q_i(s) \\ -a & 0 \end{vmatrix}}{\begin{vmatrix} A_1 s + a & -a \\ -a & A_2 s + a + b \end{vmatrix}}$$

$$\boxed{\frac{H_{2(s)}}{Q_i(s)} = \frac{a}{A_1 A_2 s^2 + (A_2 a + A_1(a+b))s + ab}}$$

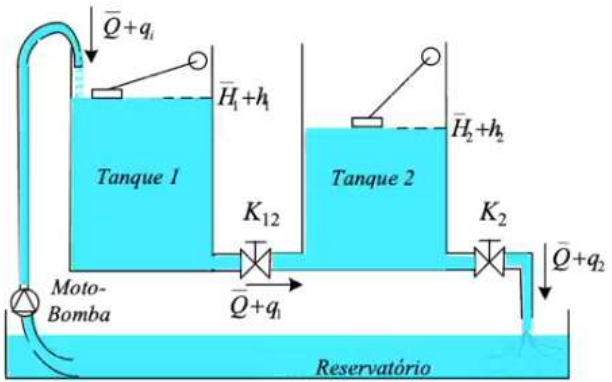
$$\boxed{\frac{H_{2(s)}}{Q_i(s)} = \frac{1/R_1}{A_1 A_2 s^2 + \left(A_2/R_1 + A_1 \frac{R_1 + R_2}{R_1 R_2}\right))s + \frac{1}{R_1 R_2}}}$$

$$\boxed{\frac{H_{2(s)}}{Q_i(s)} = \frac{R_2}{A_1 A_2 R_1 R_2 s^2 + (A_2 R_2 + A_1 R_1 + A_1 R_2))s + 1}}$$

Resposta no tempo em pequenos sinais:

$$\frac{H_{2(s)}}{Q_i(s)} = \frac{a}{A_1A_2s^2+(A_2a+A_1(a+b))s+ab}$$

$$a = \frac{K_{12}}{2\sqrt{\bar{H}_1 - \bar{H}_2}}; b = \frac{K_2}{2\sqrt{\bar{H}_2}}$$



Exemplo:

$Q_i = 2$; $K_{12} = 1$; $K_2 = 2$; $A_1 = 1$; $A_2 = 2$;

Obtem-se os níveis $\bar{H}_2 = (\frac{Q_e}{K_2})^2 = 1$. $\bar{H}_1 = (\frac{Q_i}{K_{12}})^2 + 1 = 5$;

$$a = \frac{1}{4}; b = 1; \quad 2s^2 + (3a+b)s + ab$$
$$= 2s^2 + 1{,}75s + 0{,}25$$

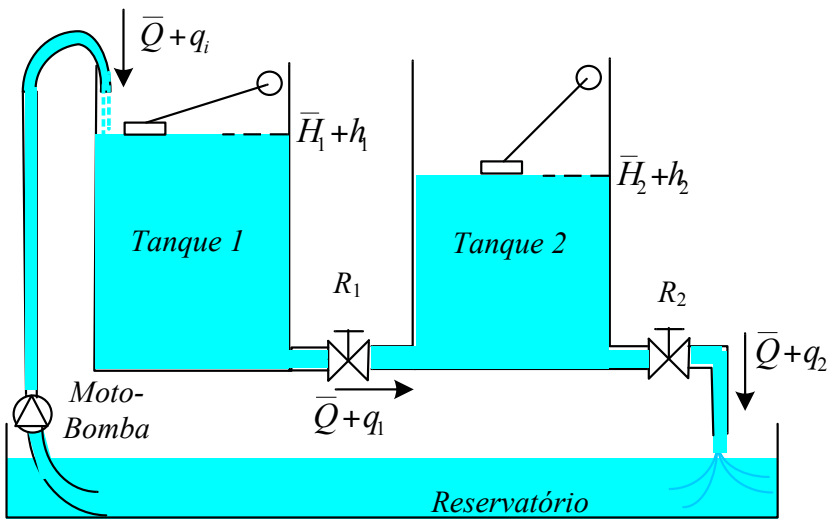
$$=> y(t) = K_0 + K_1e^{-t/0{,}6952} + K_2e^{-t/0{,}1798}$$

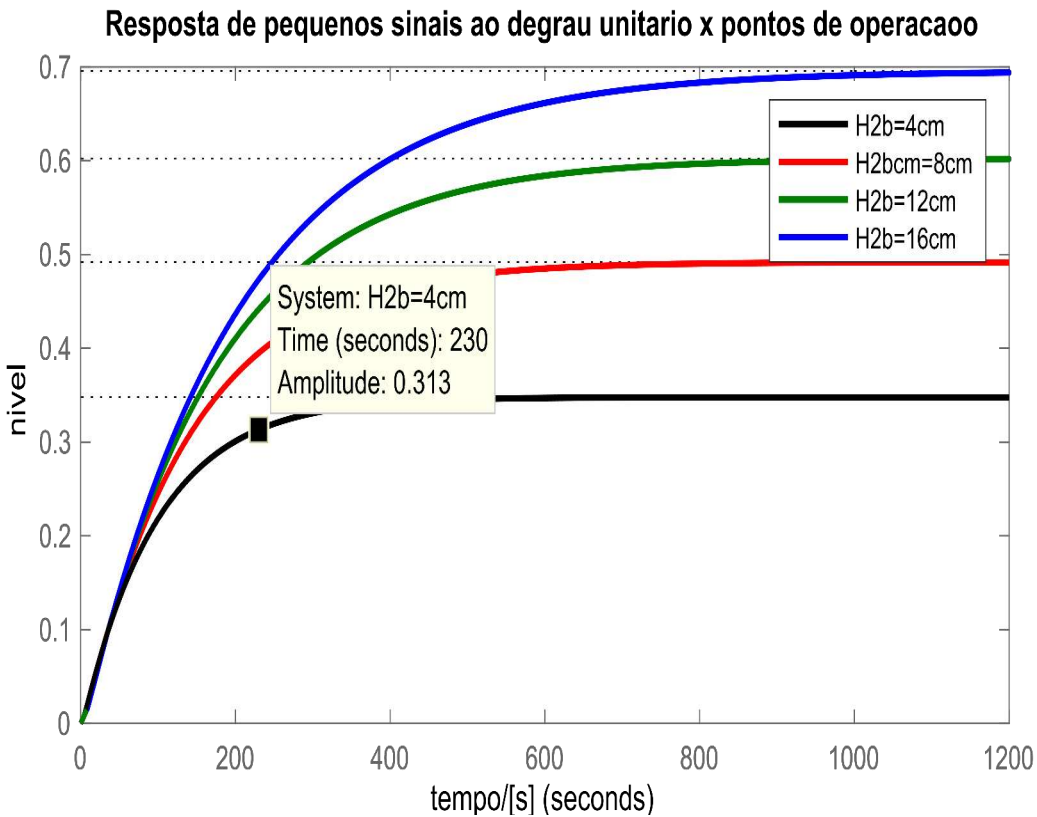
Modo dominante: $K_2e^{-t/0{,}6952}$

(o mais lento)

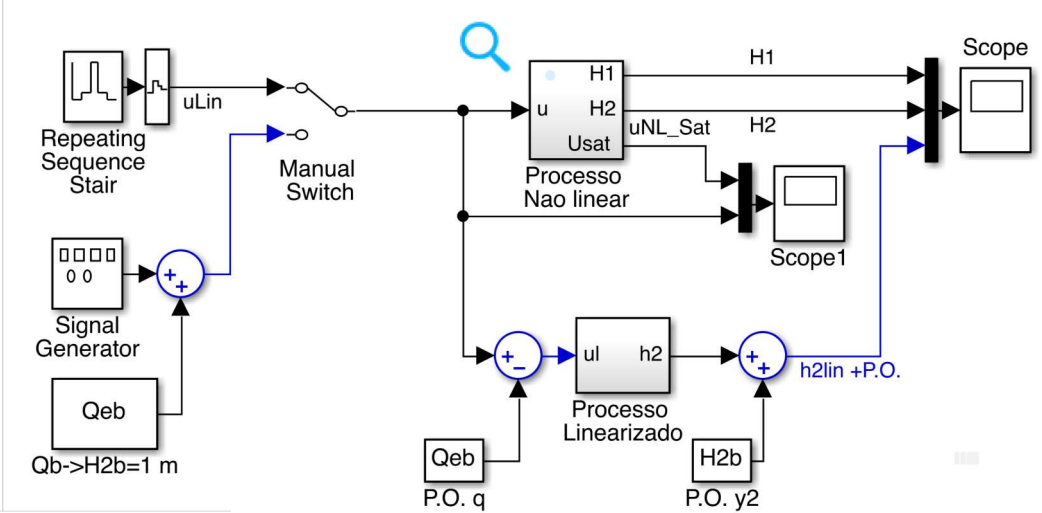
Parâmetros (Proc. LARA)

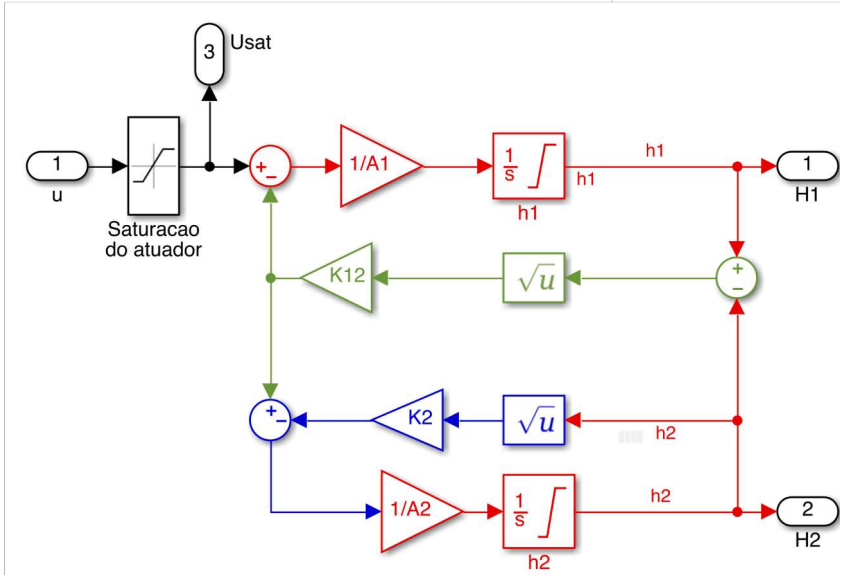
```
% Parametros do Processo
Ar=6*25.4;     % cm2 – seção transversal dos tanques
k12=38;        % válvula entre tanques
k2=11.5;       % parâmetro do furo do tanque de saída
hmax=16.5;     % altura máxima (ladrão)
qmax=90;       % vazão máxima da bomba +
qmin=-85;      % vazão máxima da bomba -
qb=25.7148;    % vazao que produz H2b=5cm
```



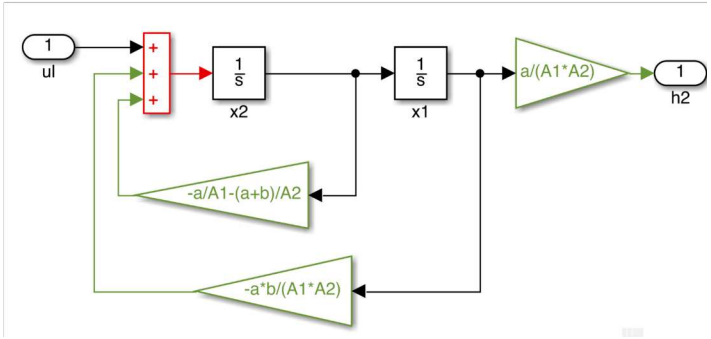


Simulação



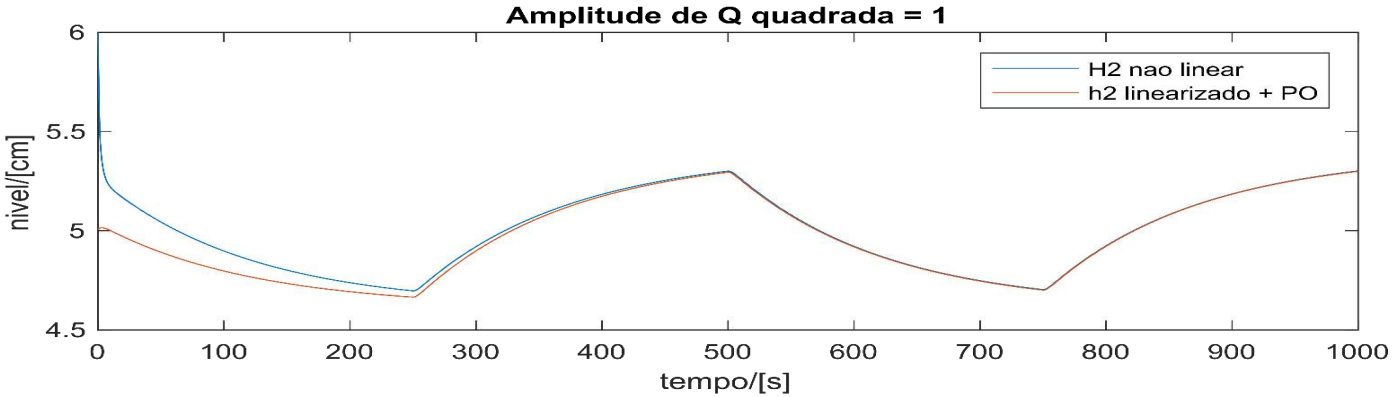


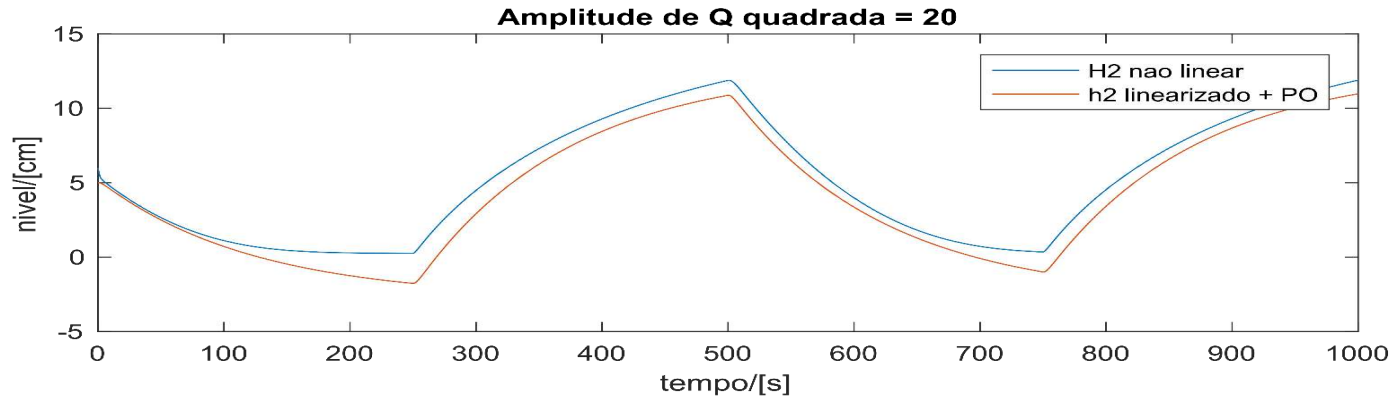
Processo não Linear

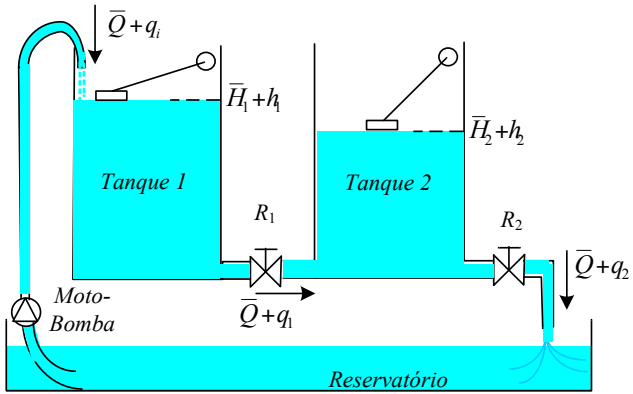


Processo Linearlizado

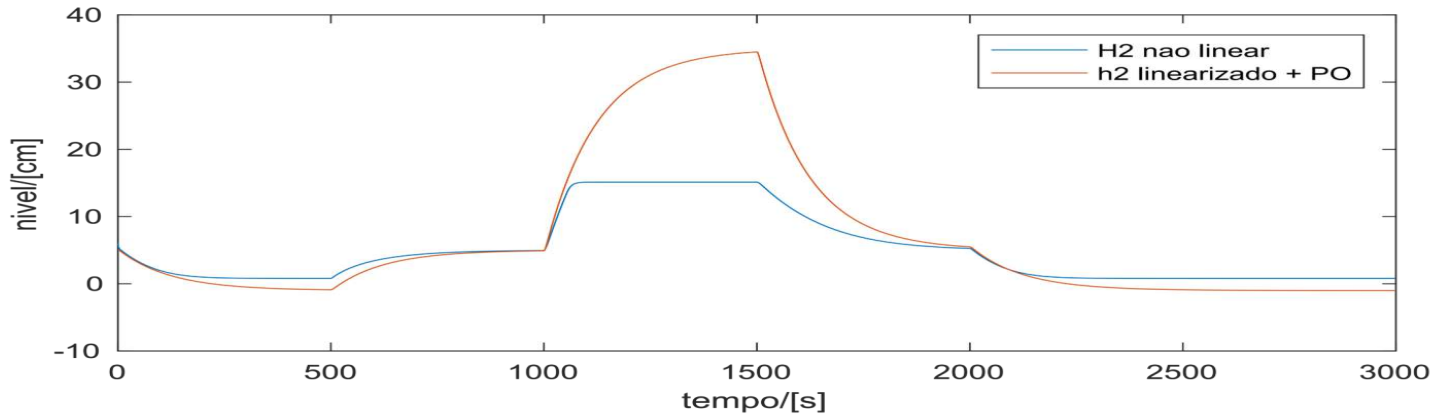
Simulação
- Pequenos Sinais?

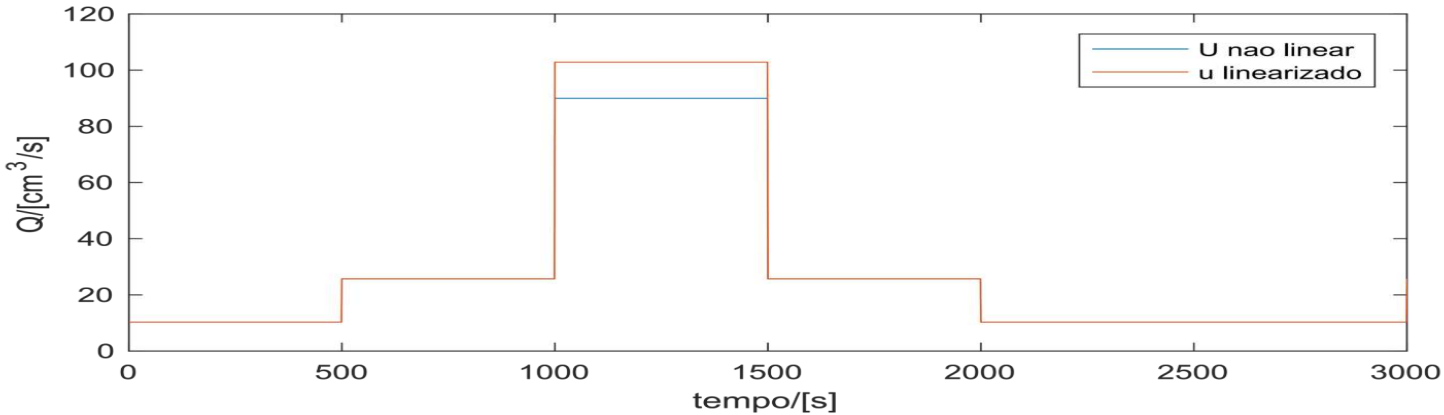


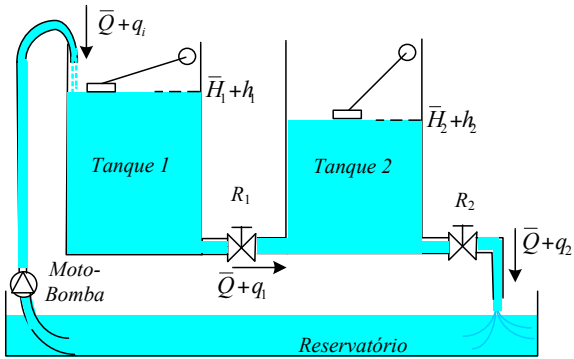




Simulação - Não Linearidades

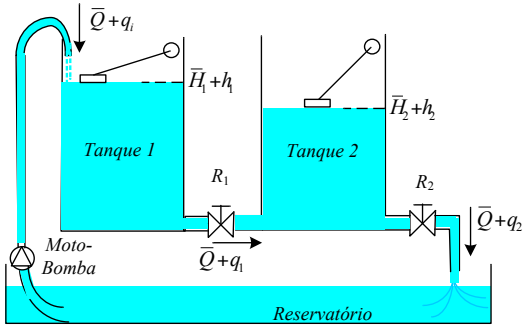




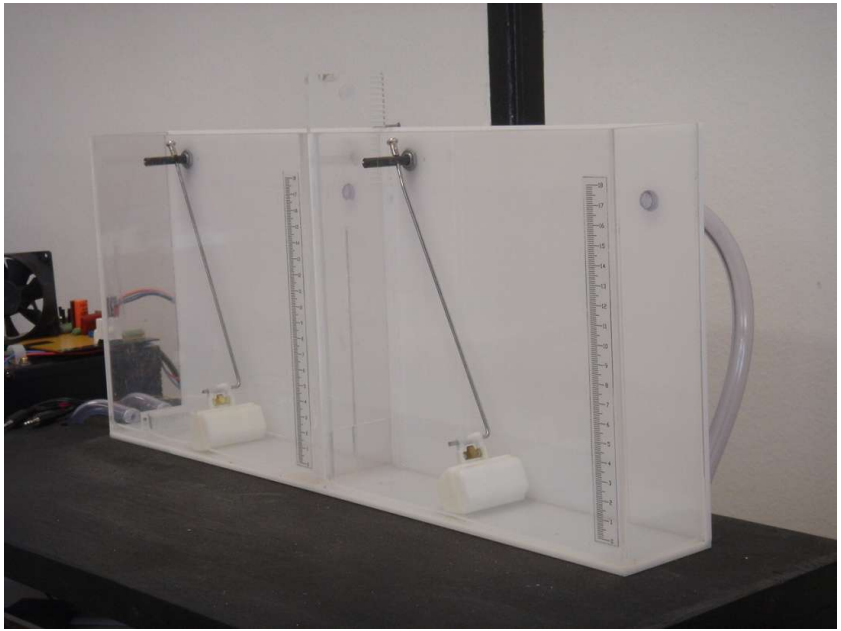


Vídeo

TG 2008 – LARA/ENE/UnB
- Adriano Peixoto Ramos
- Gabriel Lula Barros Wense



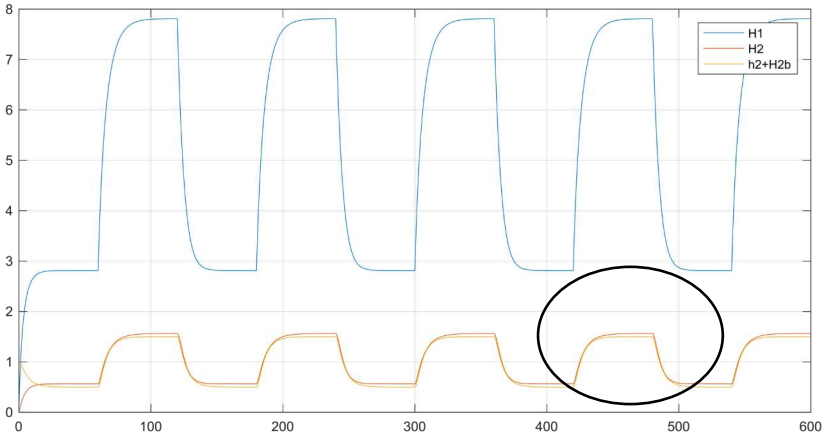
Sistema Didático de Nível de Líquidos

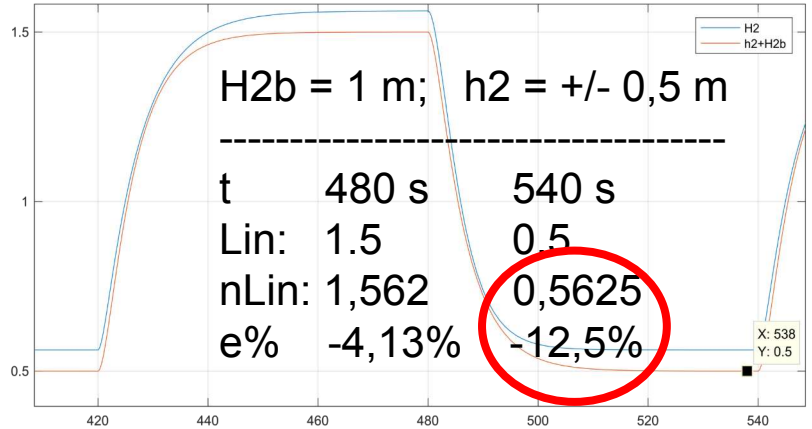


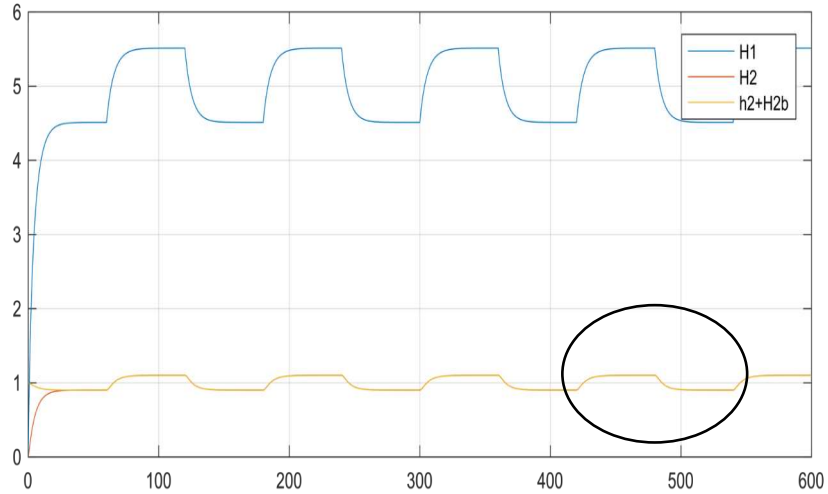
http://www.ene.unb.br/adolfo/Lectures/CSD/Liq2.avi

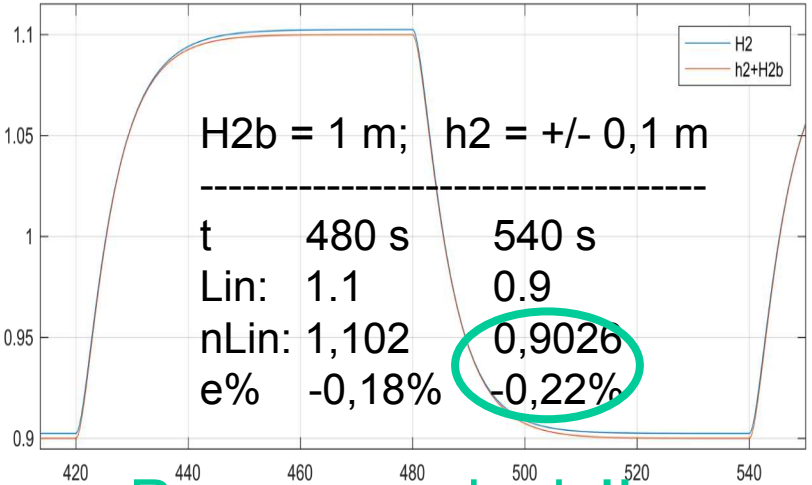
Linear?

Simulação: H1, H2 - não linear h2+H2b - linearizado









Pequenos sinais!!