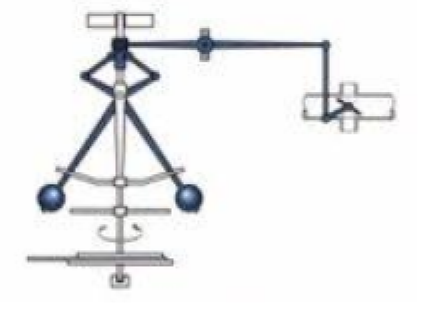
Controle de Sistemas Dinâmicos
CSD6-MAxMF_TipoEss



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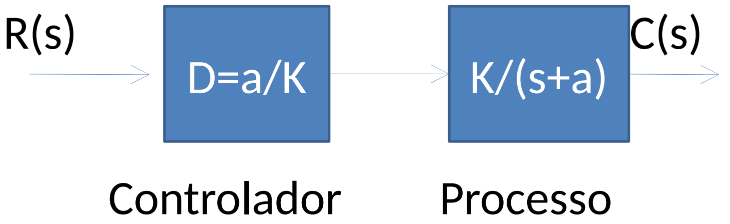
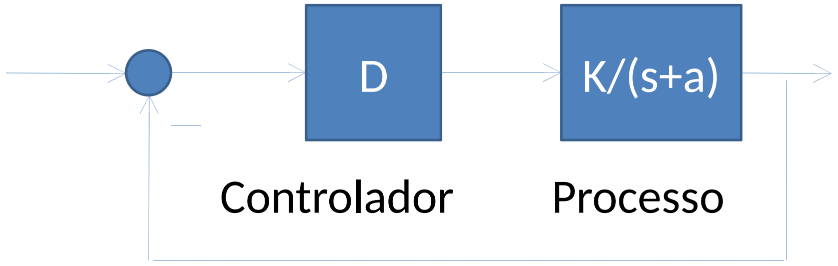
Teoria da Realimentação

Qual esquema de controle é melhor, MA ou MF?

Critérios:

1. Sensibilidade à variação de parâmetros
2. Rejeição de Perturbações
3. Acompanhamento de Sinais





1- Sensibilidade

$$S_{F:P} = \lim_{\Delta P \to 0} \frac{Variação\ \%\ de\ F}{Variação\ \%\ de\ P}$$

F- Função
P- Parâmetro

$$S_{F:P} = \lim_{\Delta P \to 0} \frac{\Delta F / F}{\Delta P / P}$$

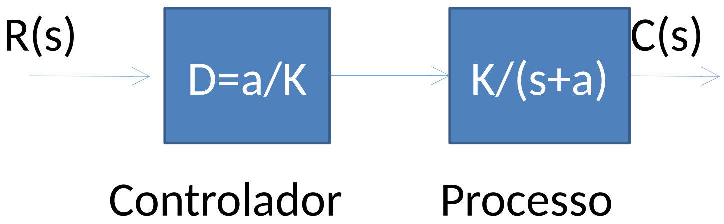
$$S_{F:P} = \lim_{\Delta P \to 0} \frac{P}{F} \frac{\Delta F}{\Delta P}$$

$$S_{F:P} = \frac{P}{F} \frac{\partial F}{\partial P}$$

Sensibilidade da função F em relação a variações do parâmetro P

$$S_{F:P} = \frac{P}{F}\frac{\partial F}{\partial P}$$

Exemplo: MA x MF



$$F_{MA} = \frac{DK}{s+a}$$

$$S_{F_{MA}:K} = \frac{K}{\frac{DK}{s+a}}\frac{D}{s+a} = 1$$

$$F_{MF} = \frac{\frac{DK}{s+a}}{1+\frac{DK}{s+a}} = \frac{DK}{s+a+DK}$$

$$S_{F_{MF}:K} = \frac{K}{\frac{DK}{s+a+DK}}\frac{D(s+a+DK)-DDK}{(s+a+DK)^2} = \frac{s+a}{s+a+DK}$$

Se DK↑ → S↓

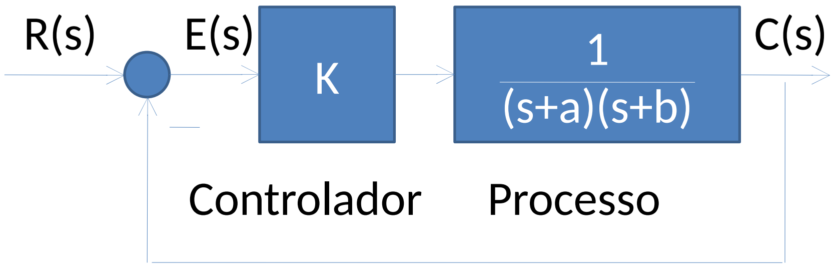
e.g. DK=99a → S=0,01

1% de variação em K
→ 0,01% de var. em F

DK = Ganho de Malha

Sensibilidade do erro(∞) com entrada degrau

Kp=K/ab



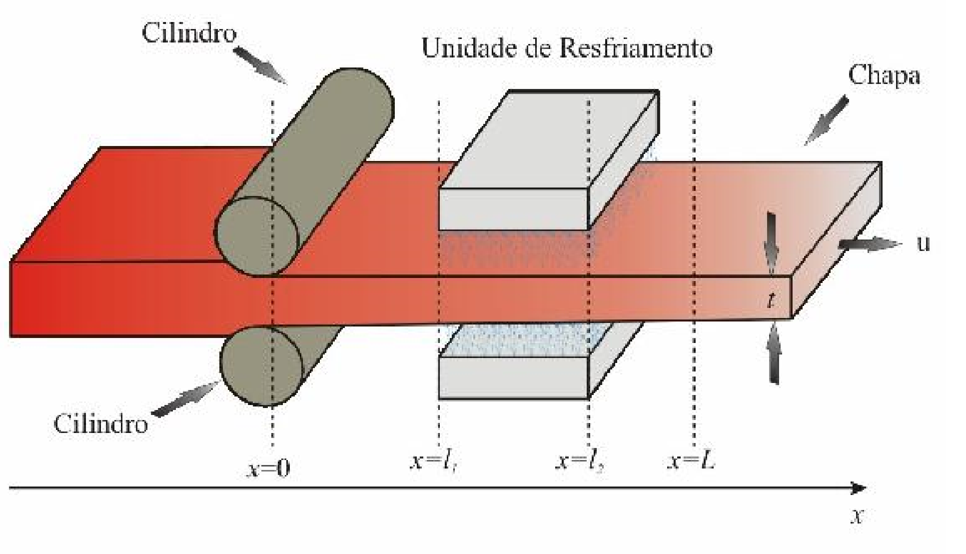
$$e(\infty) = \frac{1}{1+K_p} = \frac{1}{1+\frac{K}{ab}} = \frac{ab}{ab+K}$$

$$S_{e(\infty):a} = \frac{a}{e}\frac{\partial e}{\partial a} = \frac{a}{\frac{ab}{ab+K}}\frac{b(ab+K)-bab}{(ab+K)^2} = \frac{K}{ab+K}$$

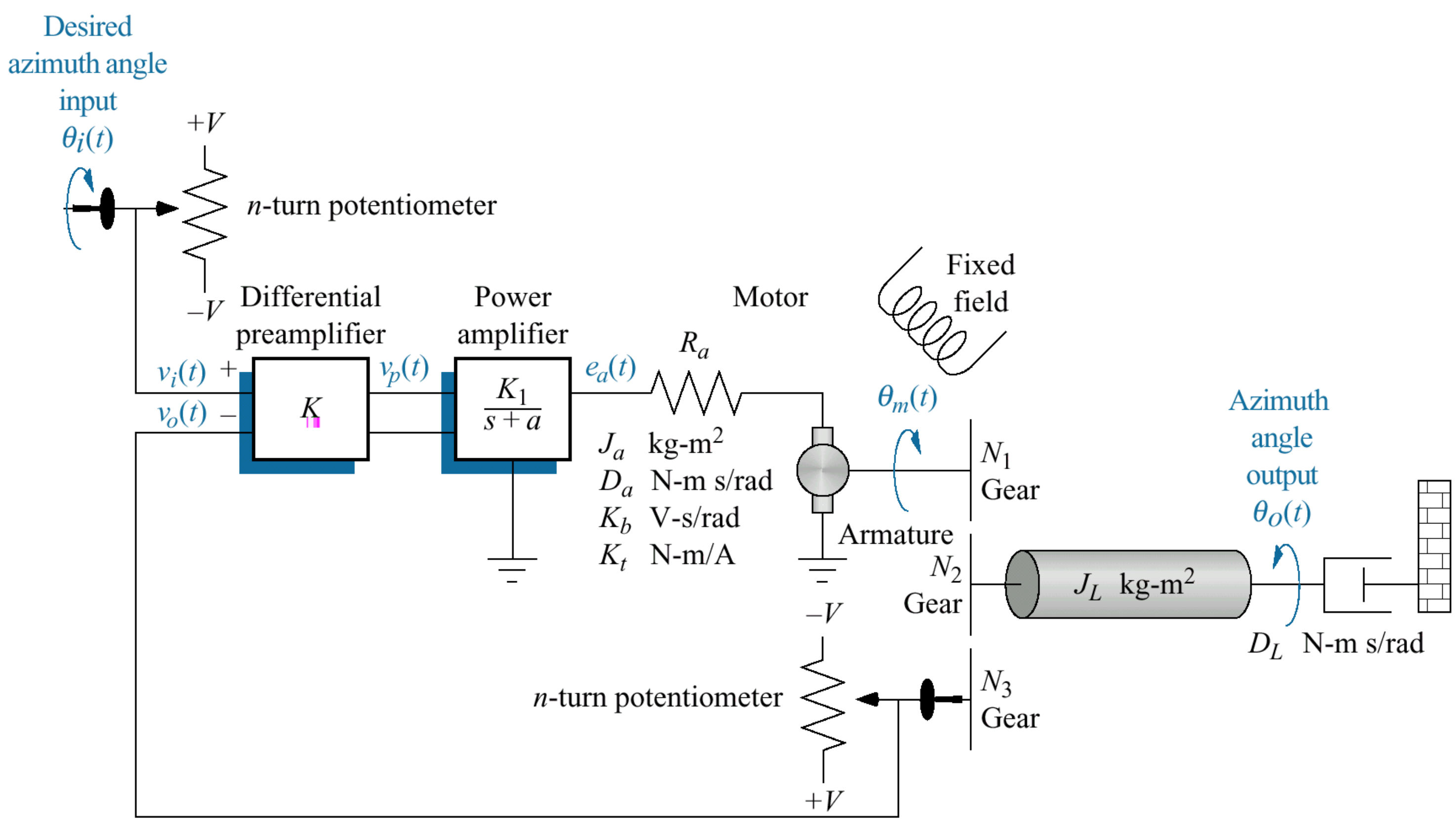
$$S_{e(\infty):K} = \frac{K}{e}\frac{\partial e}{\partial K} = \frac{K}{\frac{ab}{ab+K}}\frac{-ab}{(ab+K)^2} = \frac{-K}{ab+K}$$

2 – Rejeição de Perturbações (MA x MF)

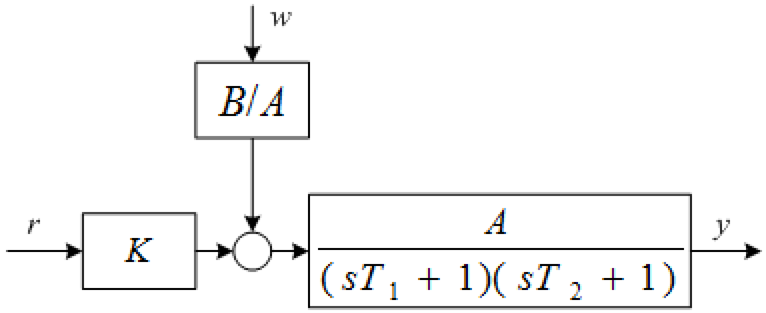
Laminação a quente

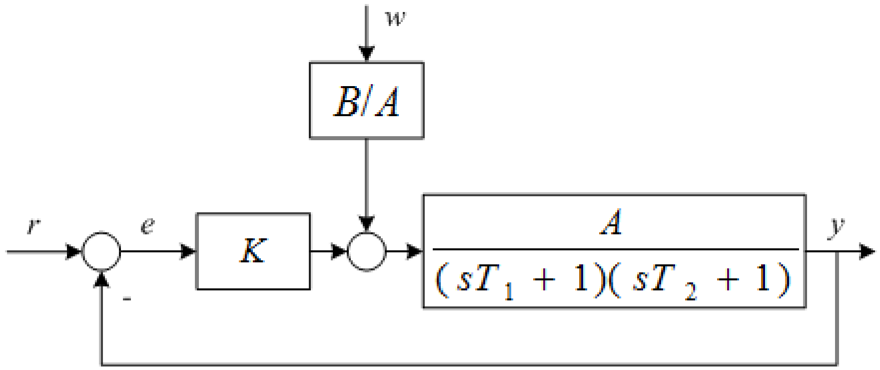


Motor CC em MF



Rejeição de Perturbações (MA x MF)





$$Y(s)=\frac{A}{(sT_1+1)(sT_2+1)}V_a(s)+\frac{B}{(sT_1+1)(sT_2+1)}W(s)$$

$$v_a = K(r-y)$$

Controlador Proporcional $K = 1/A$

$$y_{ss} = Av_a + Bw = AKr + Bw$$

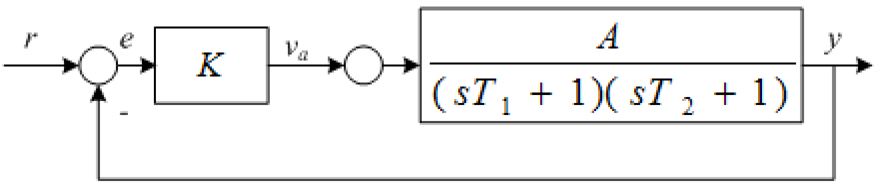
$$Y(s)=\frac{AKR(s)+BW(s)}{(sT_1+1)(sT_2+1)+AK}$$

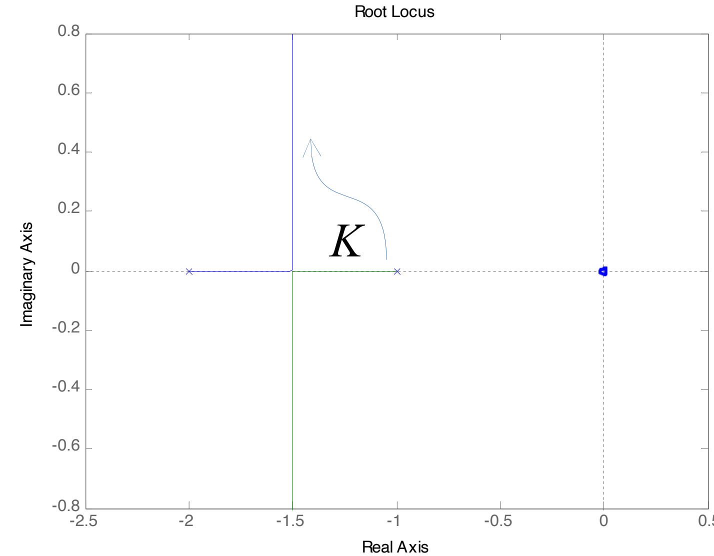
$$y_{ss} = r + Bw$$

$$y_{ss} = \frac{AK}{1+AK}r + \frac{B}{1+AK}w$$

3 - Acompanhamento de Sinais (MA x MF)

Se o sistema é lento
MA → novo projeto
MF → ajuste da resposta dinâmica

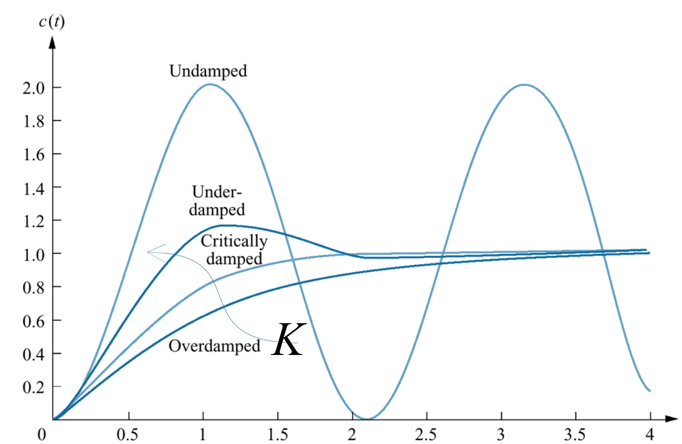




eq. caracteristica

$$T_1T_2s^2 + (T_1 + T_2)s + 1 + AK = 0$$

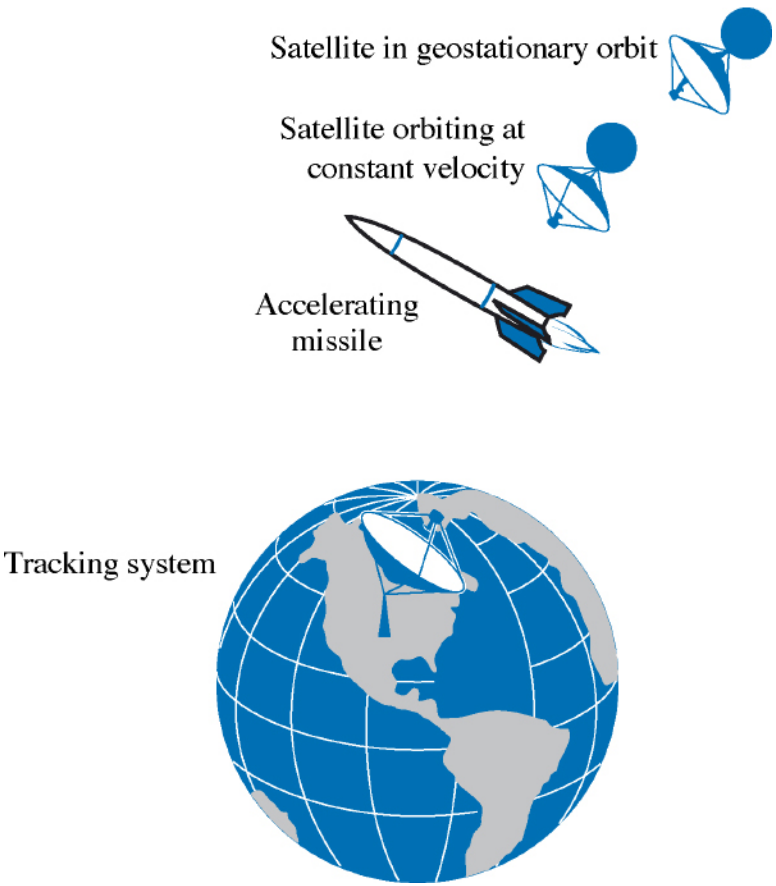
$$s = \frac{-(T_1 + T_2) \pm \sqrt{(T_1 + T_2)^2 - 4T_1T_2(1 + AK)}}{2T_1T_2}$$

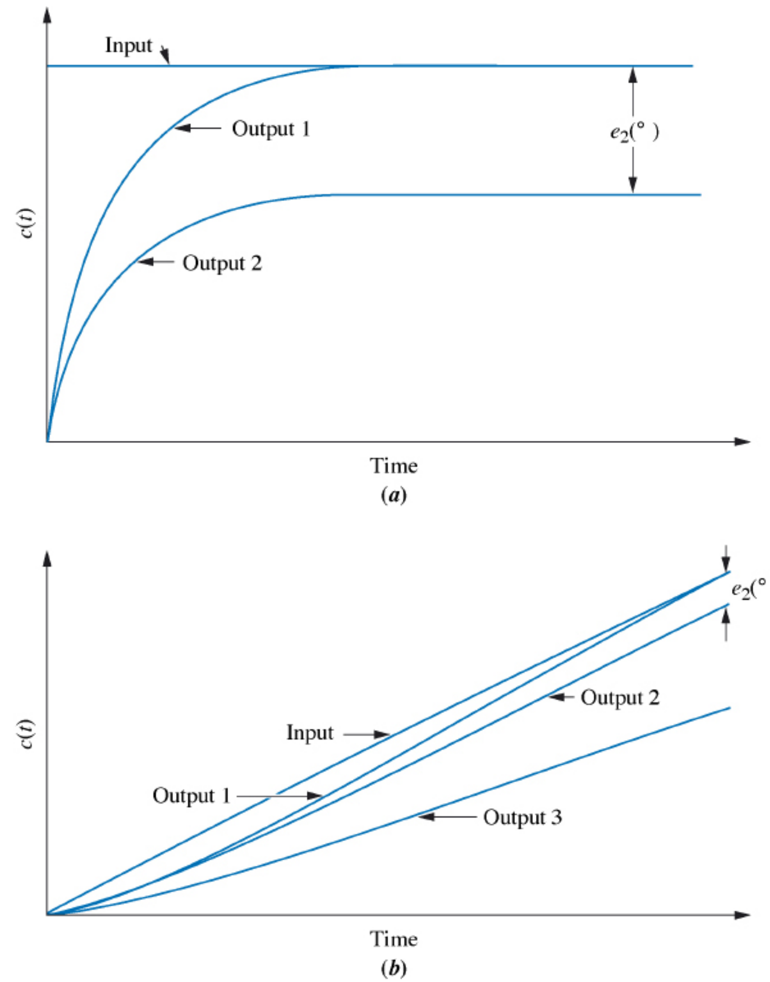


Resposta em Regime Permanente

Erro em Regime: e(t→∞)

Mede a capacidade de sistemas de acompanhar sinais em regime permanente

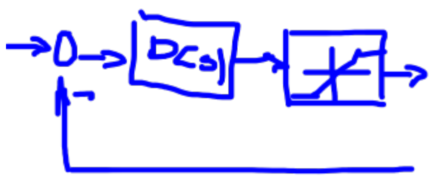


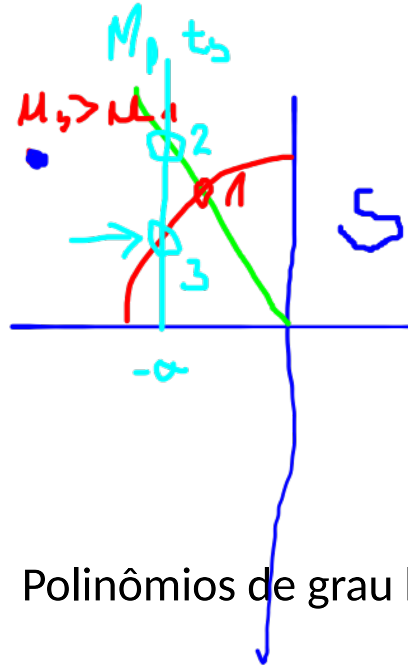


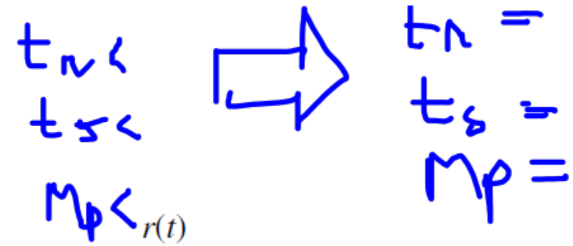
Resposta em Regime Permanente

Classificação de sistemas de controle:
tipo 0,1,2,...

Quanto à habilidade de seguir entradas:
degrau, rampa, parábola,...





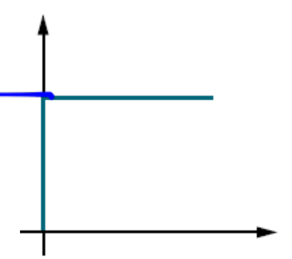


$$r(t) = \frac{t^k}{k!} 1(t) \Leftrightarrow R(s) = \frac{1}{s^{k+1}}$$

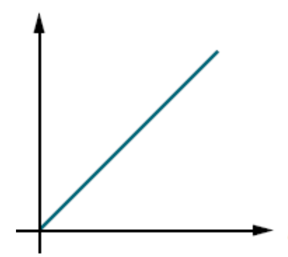
Polinômios de grau k

erro $\overset{\Delta}{=}$ saída desejada – saída real

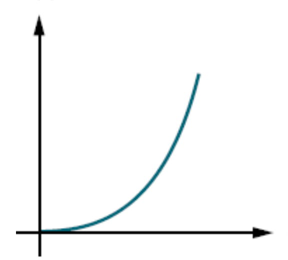
e = r - y



1 $\qquad \frac{1}{s}$

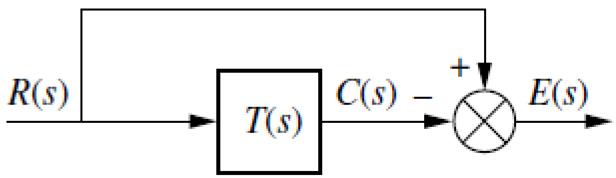


t $\qquad \frac{1}{s^2}$

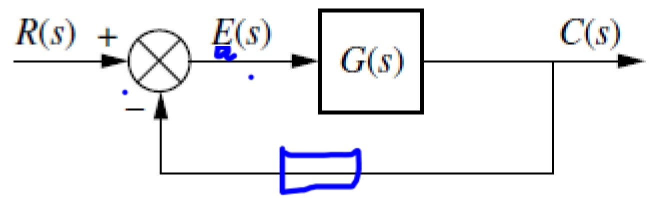


$\frac{1}{2}t^2$ $\qquad \frac{1}{s^3}$

Erro em malha fechada



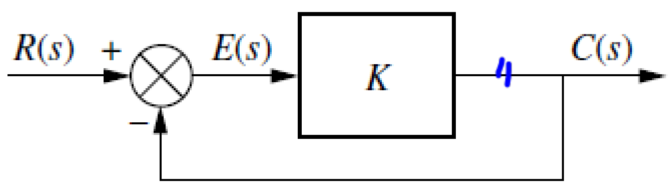
(a)



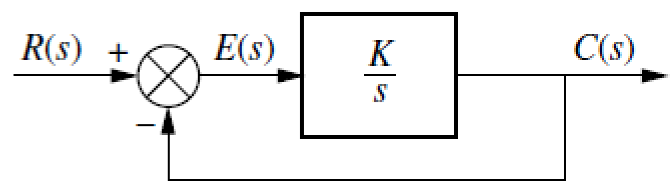
(b)

Erro genérico

Erro para realimentação unitária



(a)



(b)

e_{ss} finito para degrau de referência

e_{ss} = 0 para degrau de referência

Erro em função de G(s)

$$E(s) = R(s) - C(s)$$

$$C(s) = E(s)G(s)$$

$$E(s) = \frac{R(s)}{1 + G(s)}$$

$$e(\infty) = \lim_{s \to 0} \frac{sR(s)}{1 + G(s)}$$

$$com \; R(s) = \frac{1}{s}$$

$$e(\infty) = e_{\text{step}}(\infty) = \lim_{s \to 0} \frac{s(1/s)}{1 + G(s)} = \frac{1}{1 + \lim_{s \to 0} G(s)}$$

K_p

$$G(s) \equiv \frac{(s + z_1)(s + z_2) \cdots}{s^n (s + p_1)(s + p_2) \cdots}$$

$$e_{ss} \to 0 \;\; \text{se} \;\; \lim_{s \to 0} G(s) = \infty$$

Erro em função de G(s)

$$G(s) \equiv \frac{(s+z_1)(s+z_2)\cdots}{s^n(s+p_1)(s+p_2)\cdots}$$

$Rampa: R(s) = \dfrac{1}{s^2}$

$$e(\infty) = e_{\text{ramp}}(\infty) = \lim_{s\to 0}\frac{s(1/s^2)}{1+G(s)} = \lim_{s\to 0}\frac{1}{s+sG(s)} = \frac{1}{\lim_{s\to 0} sG(s)}$$

K_v

$e_{ss} \to 0$ se $sG(s) \to \infty$

$Parábola: R(s) = \dfrac{1}{s^3}$

$$e(\infty) = e_{\text{parabola}}(\infty) = \lim_{s\to 0}\frac{s(1/s^3)}{1+G(s)} = \lim_{s\to 0}\frac{1}{s^2+s^2G(s)} = \frac{1}{\lim_{s\to 0} s^2G(s)}$$

K_a

$e_{ss} \to 0$ se $s^2G(s) \to \infty$

Constantes de Erro

$$e(\infty) = e_{\text{step}}(\infty) = \frac{1}{1 + \lim_{s \to 0} G(s)}$$

Coeficiente de erro de posição:

$$K_p = \lim_{s \to 0} G(s)$$

$$e(\infty) = e_{\text{ramp}}(\infty) = \frac{1}{\lim_{s \to 0} sG(s)}$$

Coeficiente de erro de velocidade:

$$K_v = \lim_{s \to 0} sG(s)$$

$$e(\infty) = e_{\text{parabola}}(\infty) = \frac{1}{\lim_{s \to 0} s^2 G(s)}$$

Coeficiente de erro de aceleração:

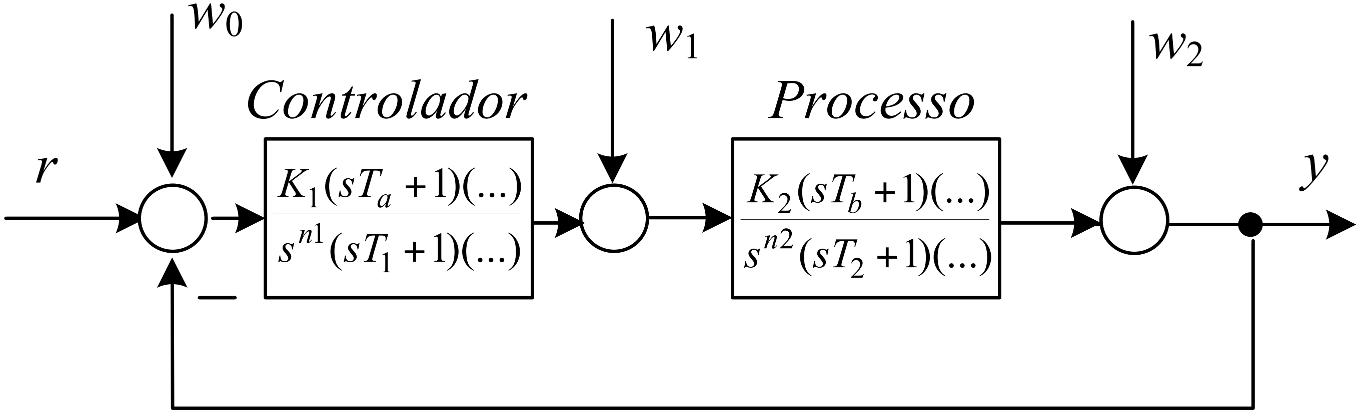
$$K_a = \lim_{s \to 0} s^2 G(s)$$

...Coeficiente de erro da derivada da aceleração

Tabela
erro x tipo sistema x coef. erro x e_{ss}

		Type 0		Type 1		Type 2	
Input	Steady-state error formula	Static error constant	Error	Static error constant	Error	Static error constant	Error
Step, $u(t)$	$\frac{1}{1+K_p}$	$K_p = \text{Constant}$	$\frac{1}{1+K_p}$	$K_p = \infty$	0	$K_p = \infty$	0
Ramp, $tu(t)$	$\frac{1}{K_v}$	$K_v = 0$	∞	$K_v = \text{Constant}$	$\frac{1}{K_v}$	$K_v = \infty$	0
Parabola, $\frac{1}{2}t^2u(t)$	$\frac{1}{K_a}$	$K_a = 0$	∞	$K_a = 0$	∞	$K_a = \text{Constant}$	$\frac{1}{K_a}$

Múltiplas perturbações



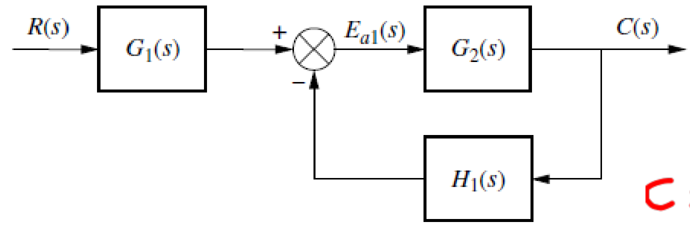
- "Rejeição: Devem haver integradores à esquerda da perturbação"

- O sistema é do tipo 0, *n*1, (*n*1+*n*2)
em relação às entradas de perturbação w_0,w_1 e w_2

- Perturbações na entrada (w_0) não são rejeitadas,
pois não é possível distinguí-las da entrada *r*

Realimentação não unitária

a) Calcular Direto



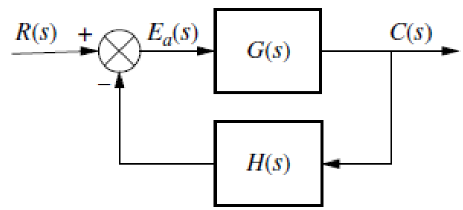
(a)

$\frac{1}{s}$

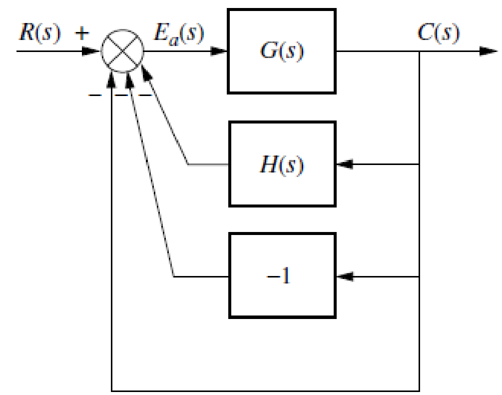
$E = R - C$

$C = R \frac{G_1 G_2}{1 + G_1 G_2}$

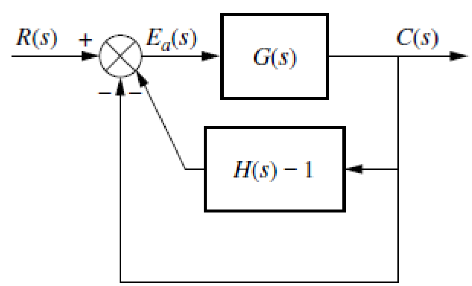
b) Rearranjo do diagrama de blocos



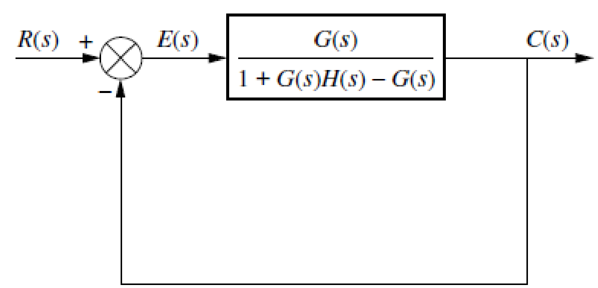
(b)



(c)

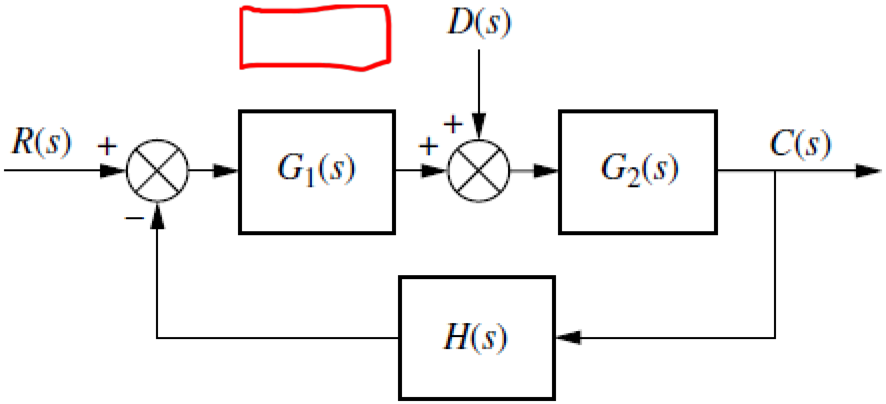


(d)



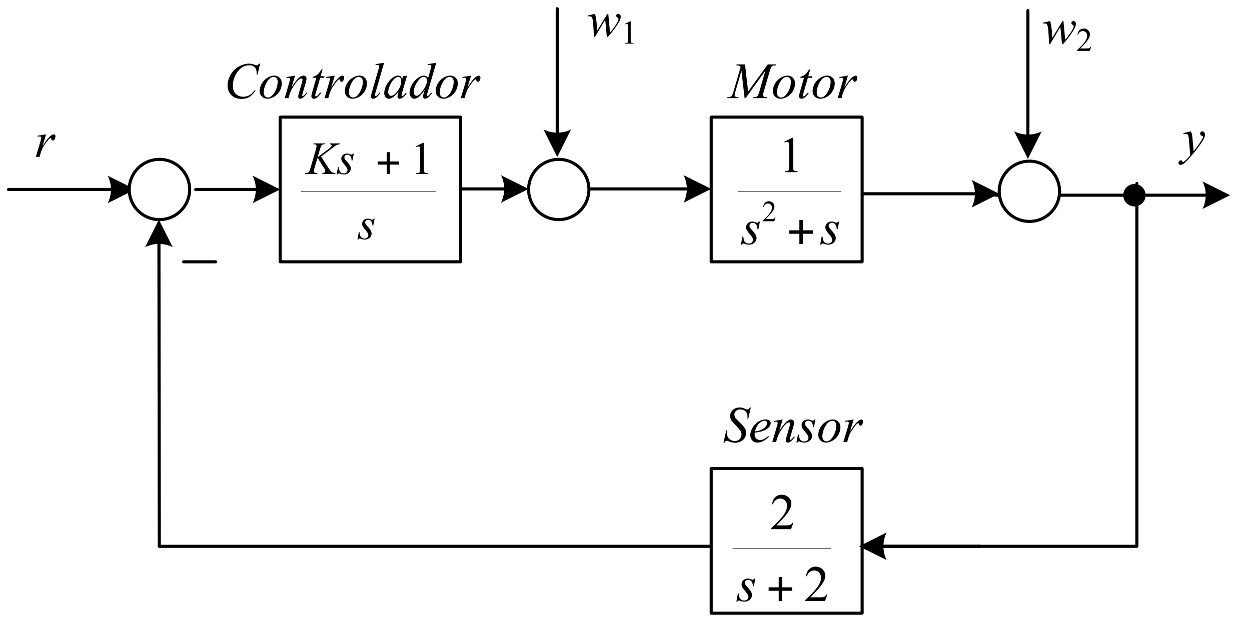
(e)

Realimentação não unitária



$$e(\infty) = \lim_{s\to 0} sE(s) = \lim_{s\to 0} s\left\{ \left[1 - \frac{G_1(s)G_2(s)}{1+G_1(s)G_2(s)H(s)}\right]R(s) - \left[\frac{G_2(s)}{1+G_1(s)G_2(s)H(s)}D(s)\right]\right\}$$

Exercício Extra 1- e_{ss}



Preencha a tabela *com* e_{ss}(K=1)

Entrada\Sinal	r	w_1	w_2
Degrau	0	0	0
Rampa	-1/2	-1/K	0
Parábola	-∞	-∞	-1/K

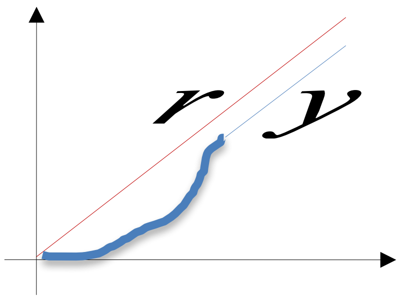
Ex. Extra 1- e_{ss}

(r degrau)
e_{ss} = 0

(r rampa)
y →oo
e_{ss} = -1/2

Obs: r,y → oo porém erro é finito!

(r parábola)
e_{ss}= -∞

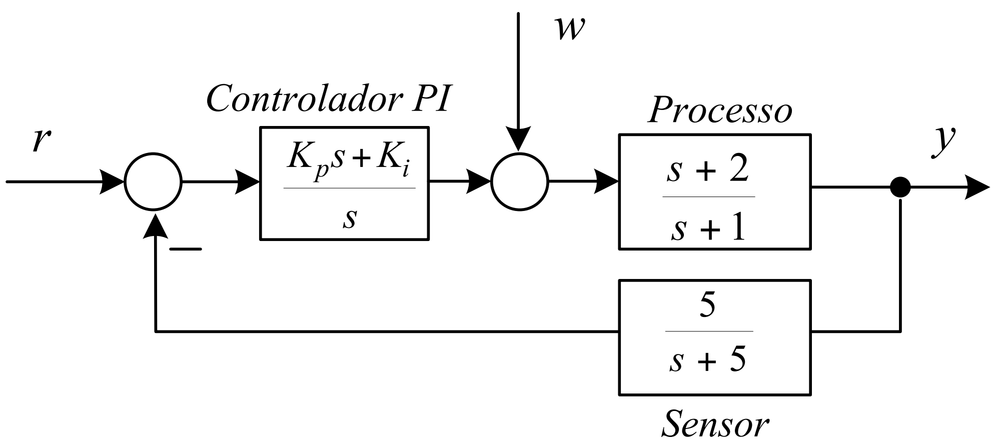


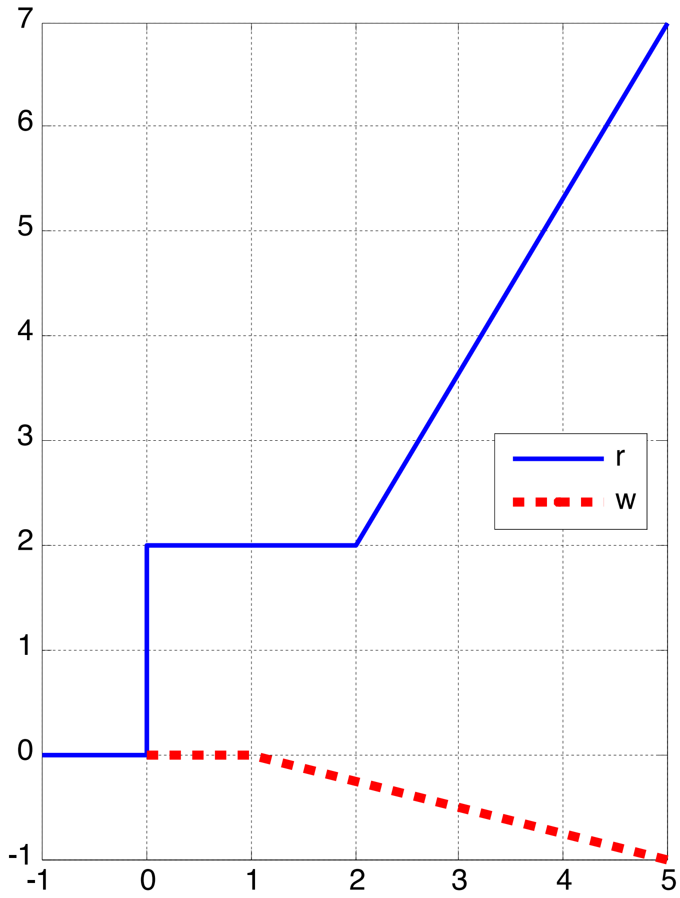
w1	w2
=0	=0
= 1/K	= 0
= ∞	= 1/K

Exercício Extra 2- e_{ss}

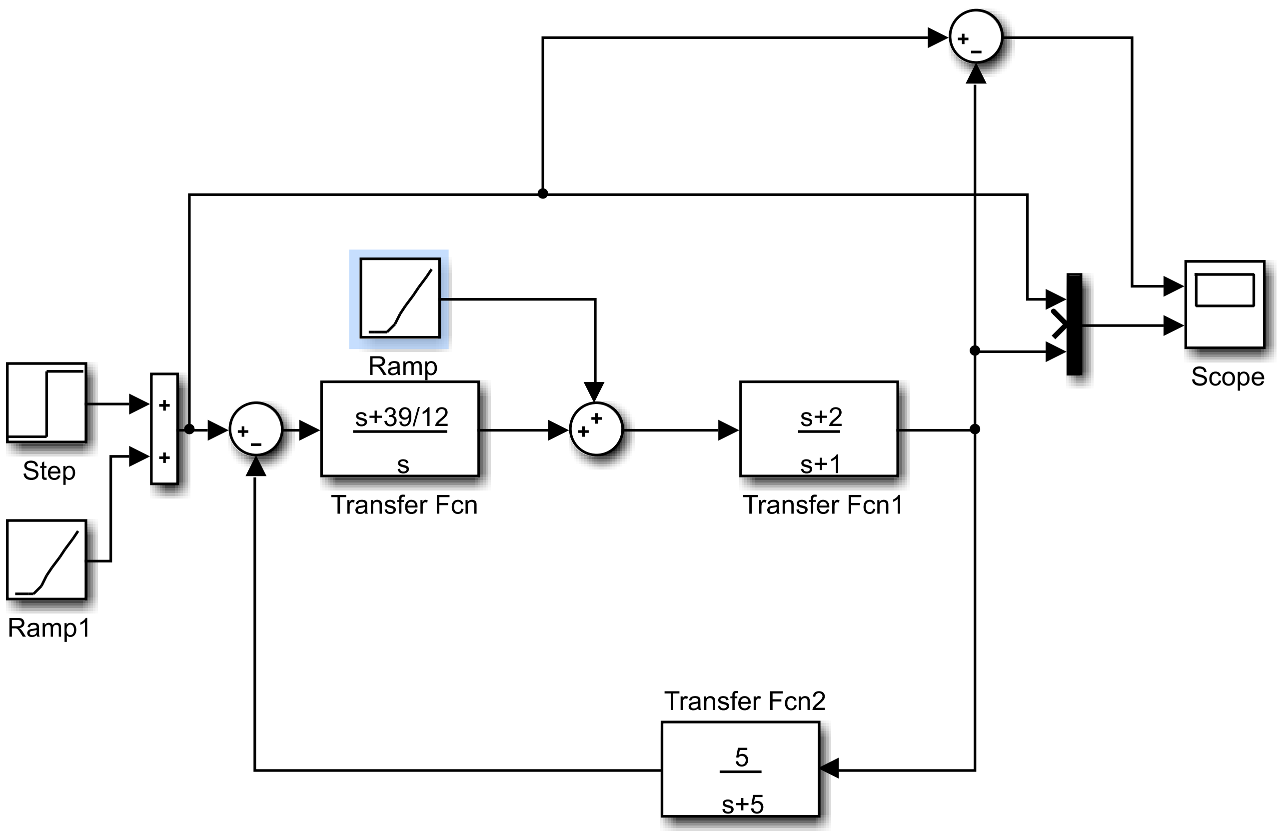
Considerando os sinais de referência e perturbação mostrados no gráfico, obtenha o erro em regime permanente para:

K_i = 0,1*último dígito matrícula + 1;
K_p = 10*penúltimo dígito matrícula + 1;

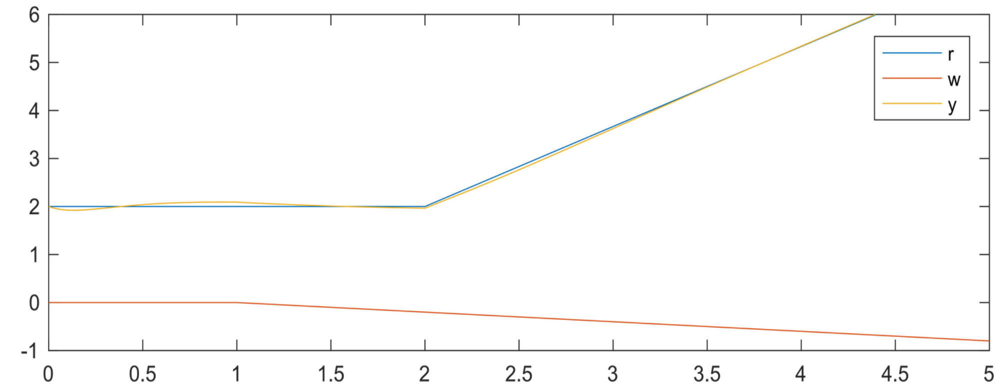


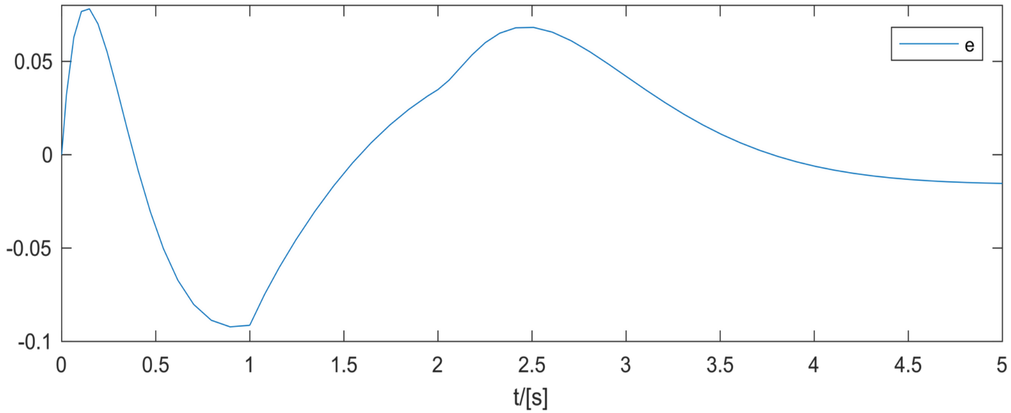


Exemplo Extra 3 - e_{ss}



Exemplo 3 - e_{ss}





Resposta completa:

$$y(t \to \infty) = y_r + y_w = 2 + \frac{5}{3}t.1(t-2) - \frac{5}{6K_i} + \frac{1}{3} - \frac{1}{4K_i}$$

$$y(t \to \infty) = \frac{5}{3}t.1(t-2) + \frac{7}{3} - \frac{13}{12K_i} \quad (1{,}0)$$

$$e(t \to \infty) = \frac{13}{12K_i} - \frac{1}{3}$$