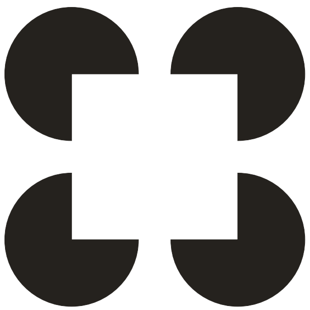
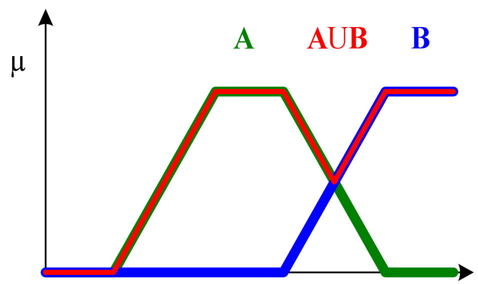
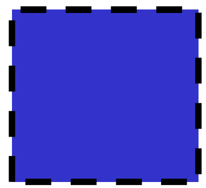
Part 3 –Fuzzy Logic and Fuzzy Systems

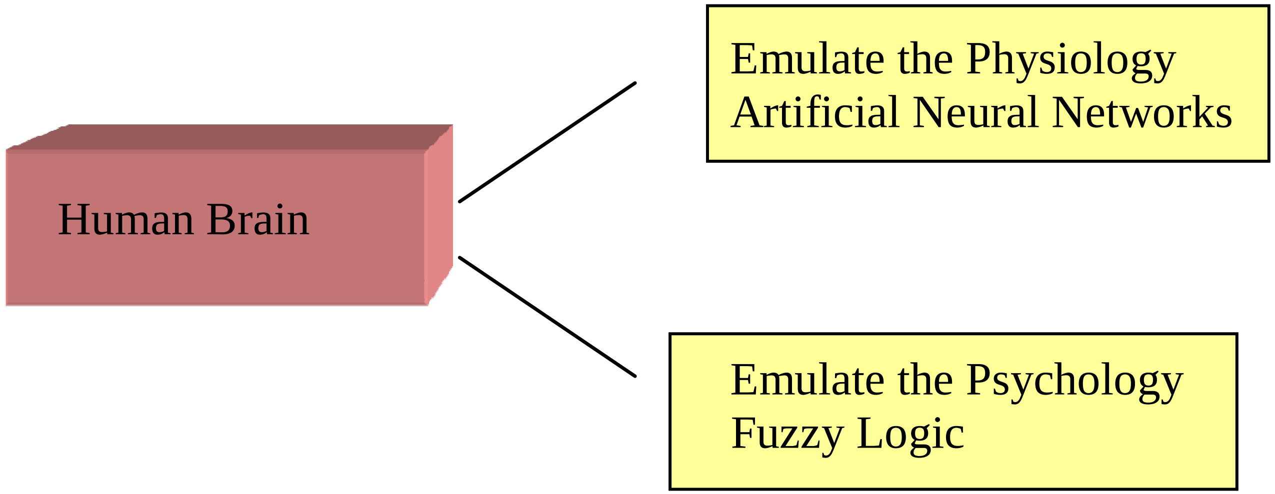






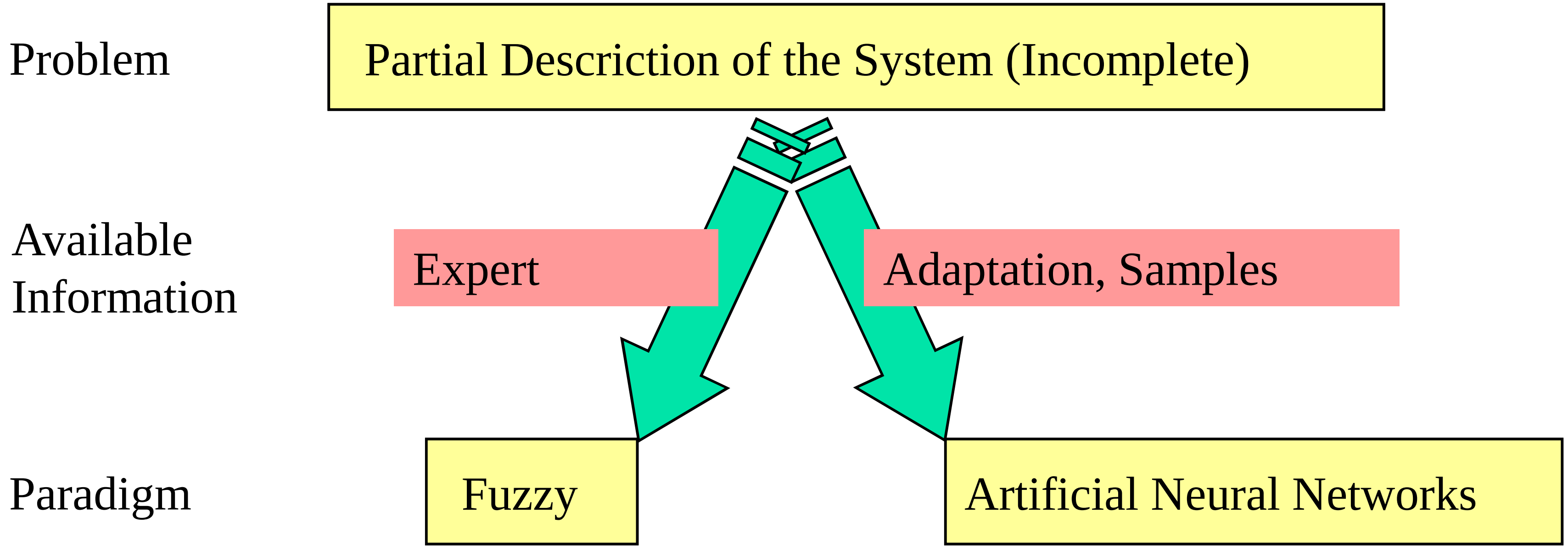
Fuzzy Logic

- The fuzzy set theory was proposed by Lotfi Zadeh in 1965.
- Was long misunderstood.
- In the mid-80s used to design Mamdani fuzzy controllers

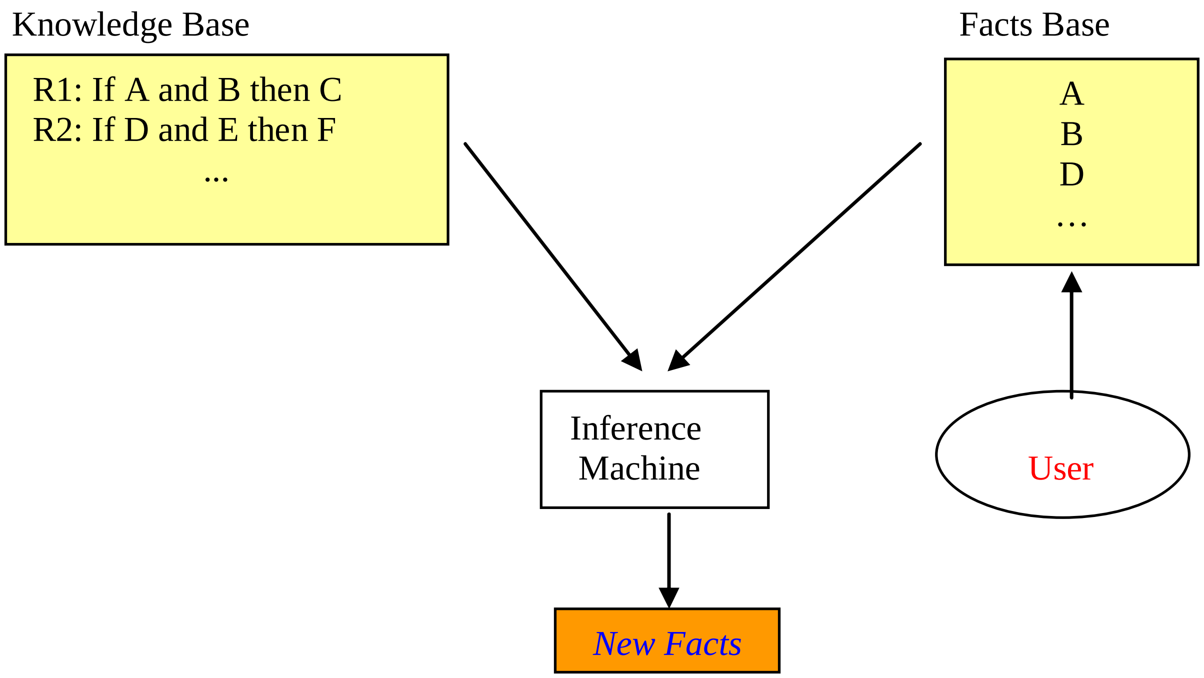


Fuzzy Logic

According to the availability of
an ***expert*** Or ***samples*** of a system
the fuzzy or the RNA paradigm is indicated.

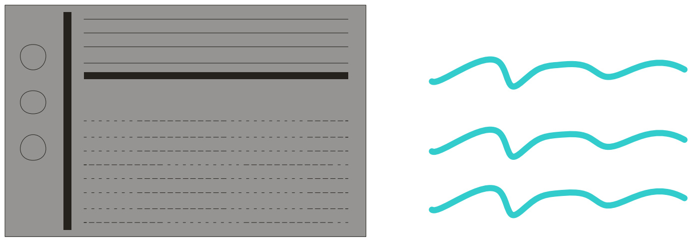


Expert Systems

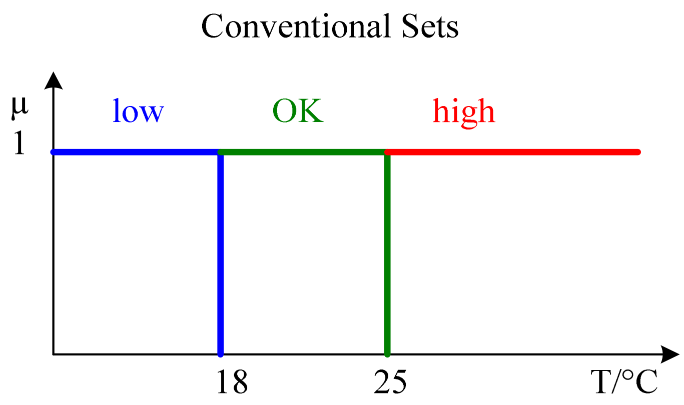


1. Extraction of Knowlegde of Expert (build knowledge base)
2. I.T. expert creates the environment (shell)
3. "Normal" human presents facts and questions (over and over)
4. Response of ES similar to the human expert!

Example of “If-Then” Rules



Air Conditioner



If temperature is **low**

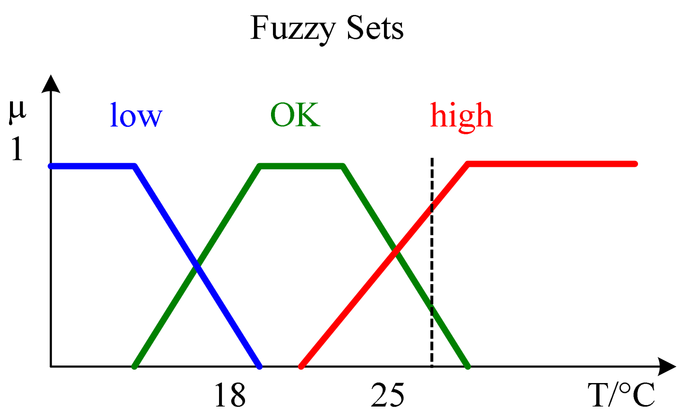
Then reduce air conditioning

If temperature is **OK**

Then do nothing

If temperature is **high**

Then increase air conditioning power



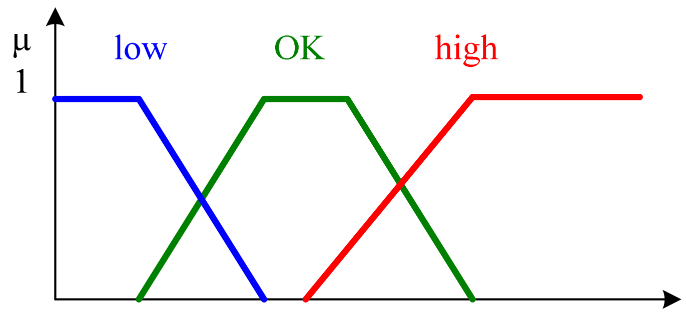
Partial membership to both linguistic concepts!

Fuzzy Sets – Membership Fuction

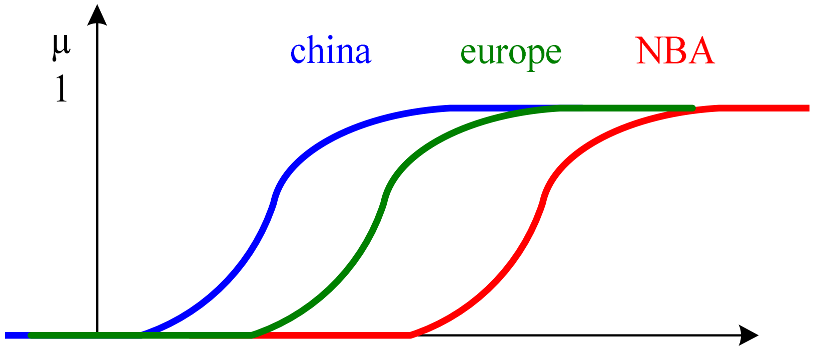
$$\mu_A(x): X \rightarrow [0,1]$$

$$\mu_A(x) = \begin{cases} 1 & \text{if } x \text{ is a fully member of } A \\ (0,1) & \text{if } x \text{ belongs partially to } A \\ 0 & \text{if } x \text{ is not a member of } A \end{cases}$$

Fuzzy Sets



Tal person:
Subjectiv concept and depends on the context



Extension of the Boolean Logic – Historical perspective:

~ 1930, Lukasiewicz : {0,1/2,1}, [0,1]
1937, Black : Membership fuction
1965, Lotfi Zadeh : Fuzzy Sets
Multivalent Set Theory
~ 1988, Commercial Products : "third wave" of interest

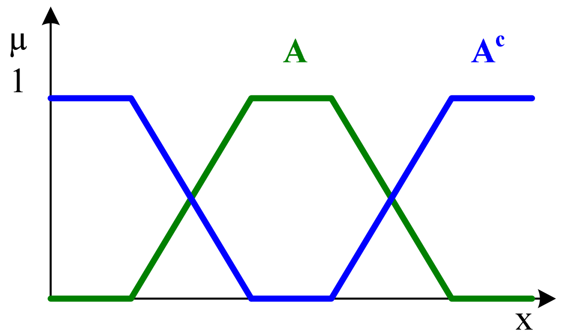
Fuzzy Sets – operations

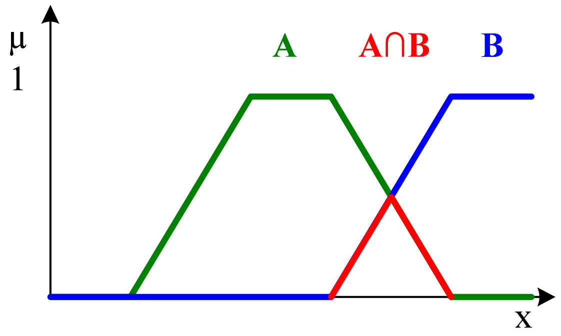
Ex.: Complement, Intersection and Union

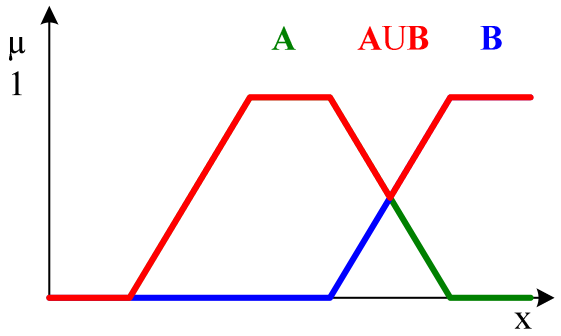
$$\mu A^c(x) = 1 - \mu A(x)$$

$$\mu A \cap B(x) = \min(\mu A(x), \mu B(x))$$

$$\mu A \cup B(x) = \max(\mu A(x), \mu B(x))$$







Fuzzy Sets – Properties

Involution	$(A^C)^C = A$	
Comutativity	$A \cup B = B \cup A$	$A \cap B = B \cap A$
Associativity	$A \cup (B \cup C) = (A \cup B) \cup C$	$A \cap (B \cap C) = (A \cap B) \cap C$
Distributivity	$A \cap (B \cup C) = (A \cap B) \cup (A \cap C)$	$A \cup (B \cap C) = (A \cup B) \cap (A \cup C)$
Idempotency	$A \cup A = A$	$A \cap A = A$
Absorption	$A \cup (A \cap B) = A$	$A \cap (A \cup B) = A$
Identity	$A \cup \Phi = A$	$A \cap \Omega = A$
Absorption by Ω e Φ	$A \cup \Omega = \Omega$	$A \cap \Phi = \Phi$
De Morgan's Law	$(A \cap B)^C = A^C \cup B^C$	$(A \cup B)^C = A^C \cap B^C$

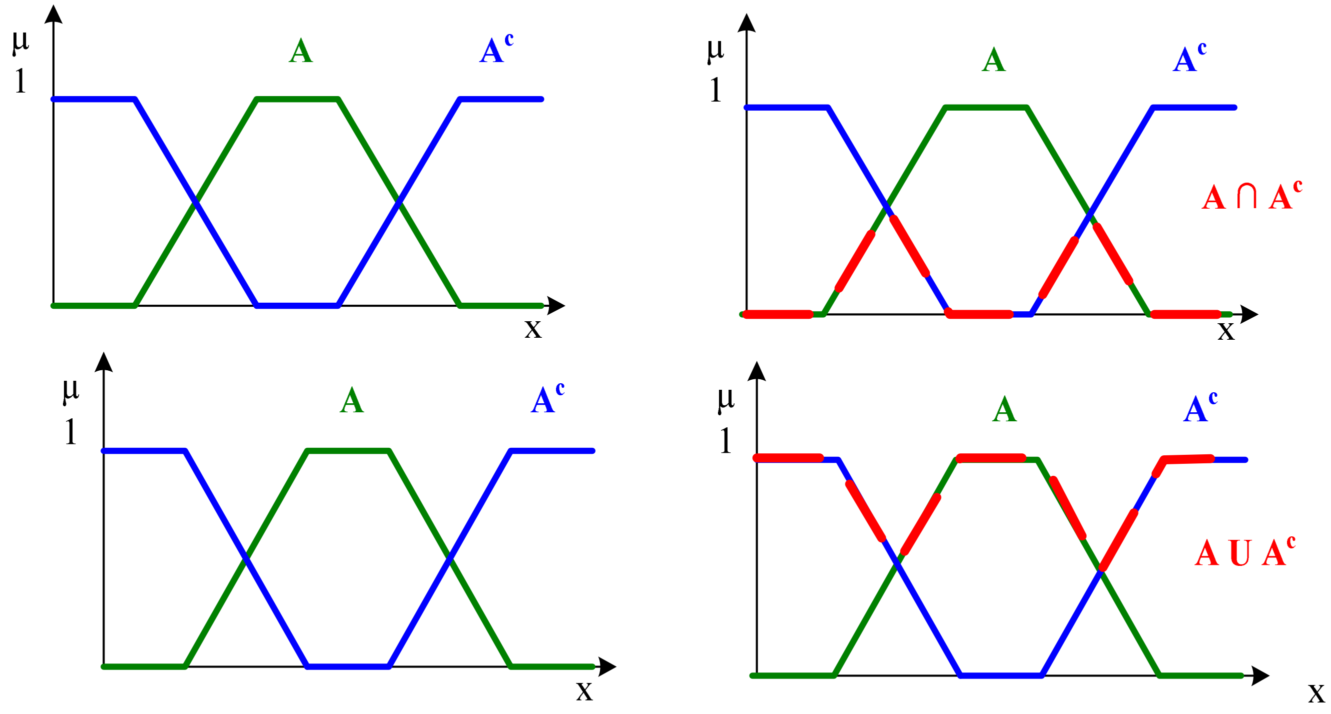
However:

$A \cap A^C \neq \Phi$	Does not satisfy the law of no-contradiction
$A \cup A^C \neq \Omega$	Does not satisfy the law of third excluded
$A \cup (A^C \cap B) \neq A \cup B$	Does not satisfy the absorption of the complement
$A \cap (A^C \cup B) \neq A \cap B$	Does not satisfy the absorption of the complement

Fuzzy Sets – Examples of “strange” behavior

$A \cap A^C \neq \Phi$ Does not satisfy the law of non-contradiction

$A \cup A^C \neq \Omega$ Does not satisfy of the third excluded



Sentence Calculus – Classical Logic

In Classical Logic, the truth values of propositions (sentence calculus) are obtained by the following truth table ("modus ponens" – afirmative modus).

A	B	$\neg A$	$A \wedge B$	$A \vee B$	$A \rightarrow B$ $(\neg A \vee B)$
0	0	1	0	0	1
0	1	1	0	1	1
1	0	0	0	1	0
1	1	0	1	1	1

Sentence Calculus – Fuzzy Logic

When the information is inaccurate, the inference engine implements the so-called approximate reasoning.

Fuzzy logic implements approximate reasoning in the context of fuzzy sets ("generalized modus ponens").

fact:	A`	Tomatoes are very red
rule:	$A \rightarrow B$	If tomatoes are red then they are mature
consequence	B`	The tomatoes are very ripe

$\neg A$	$= n(A)$	n – negation
$A \wedge B$	$= T(A,B)$	T – t -norm
$A \vee B$	$= S(A,B)$	S – t-conorm
$A \rightarrow B$	$= I(A,B)$	I – implication

Implication Operators

“If <premise> Then <conclusion>”

$I : [0,1]^2 \rightarrow [0,1]$, $\mu A : X \rightarrow [0,1]$, $\mu B : Y \rightarrow [0,1]$

$\mu A \rightarrow B\ (x,y) = I(\mu A(x), \mu B(y))$

Implication	Name
max (1-a,b)	Kleene-Dimes
min(1-a+b,1)	Lukasiewicz
→ min(a.b)	Mamdani
a.b	Larsen
…	

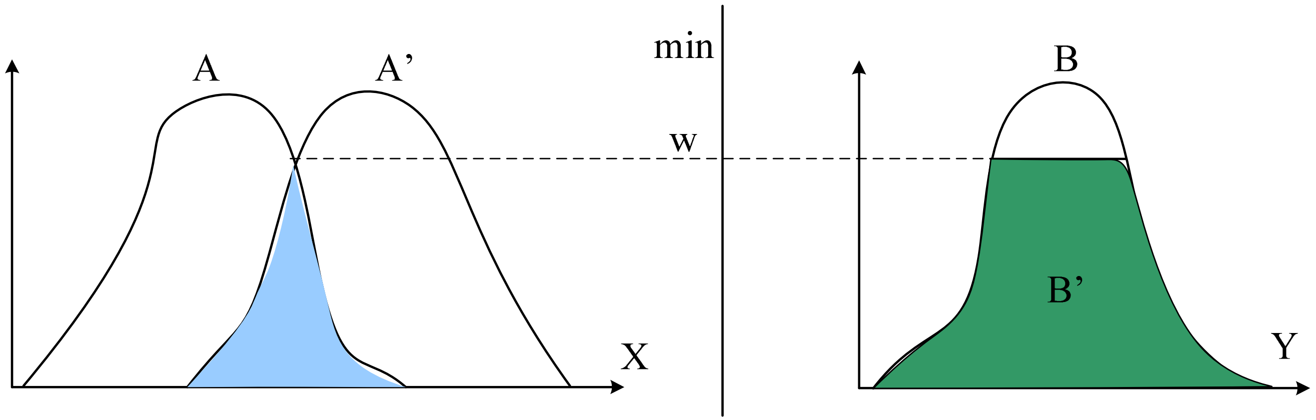
Fuzzy Reasoning
based on Max-Min composition

Definition: Let A, A' and B fuzzy sets on X, X and Y respectively. Assume that the fuzzy implication A → B is expressed by the fuzzy relation R on X x Y, then the fuzzy set B' is induced "x is A'" and the fuzzy rule "**if x is A then y is B**" is defined by:

$$\mu B'(y) = \max_x \min [\mu A'(x), \mu R(x,y)]$$
$$= \vee_x [\mu A'(x) \wedge \mu R(x,y)],$$

that means: $B' = A' \circ R = A' \circ (A \rightarrow B)$

One fuzzy rule with one antecedent

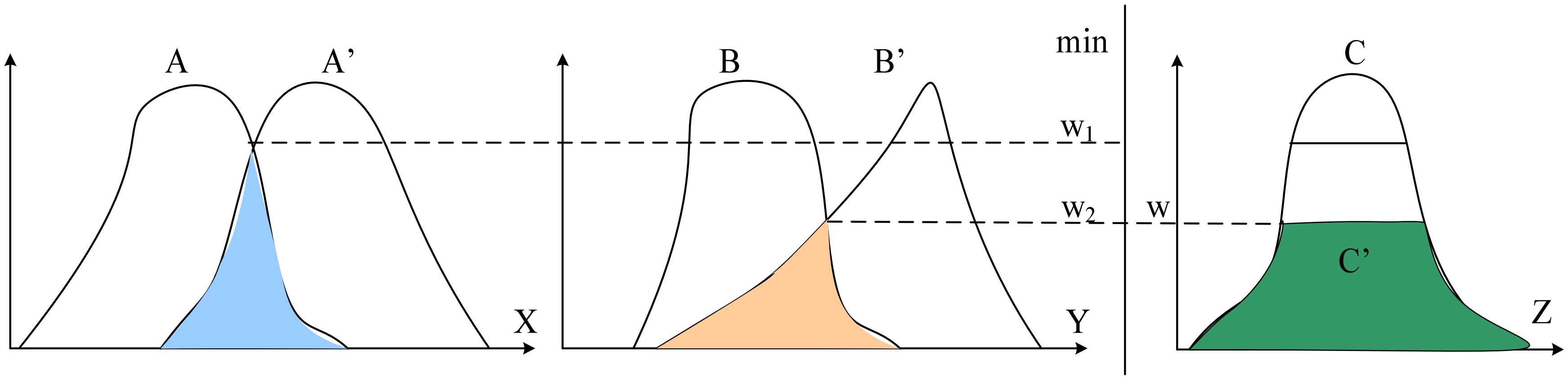


w→ degree of membership to the rule

Fuzzy Reasoning
based on Max-Min composition

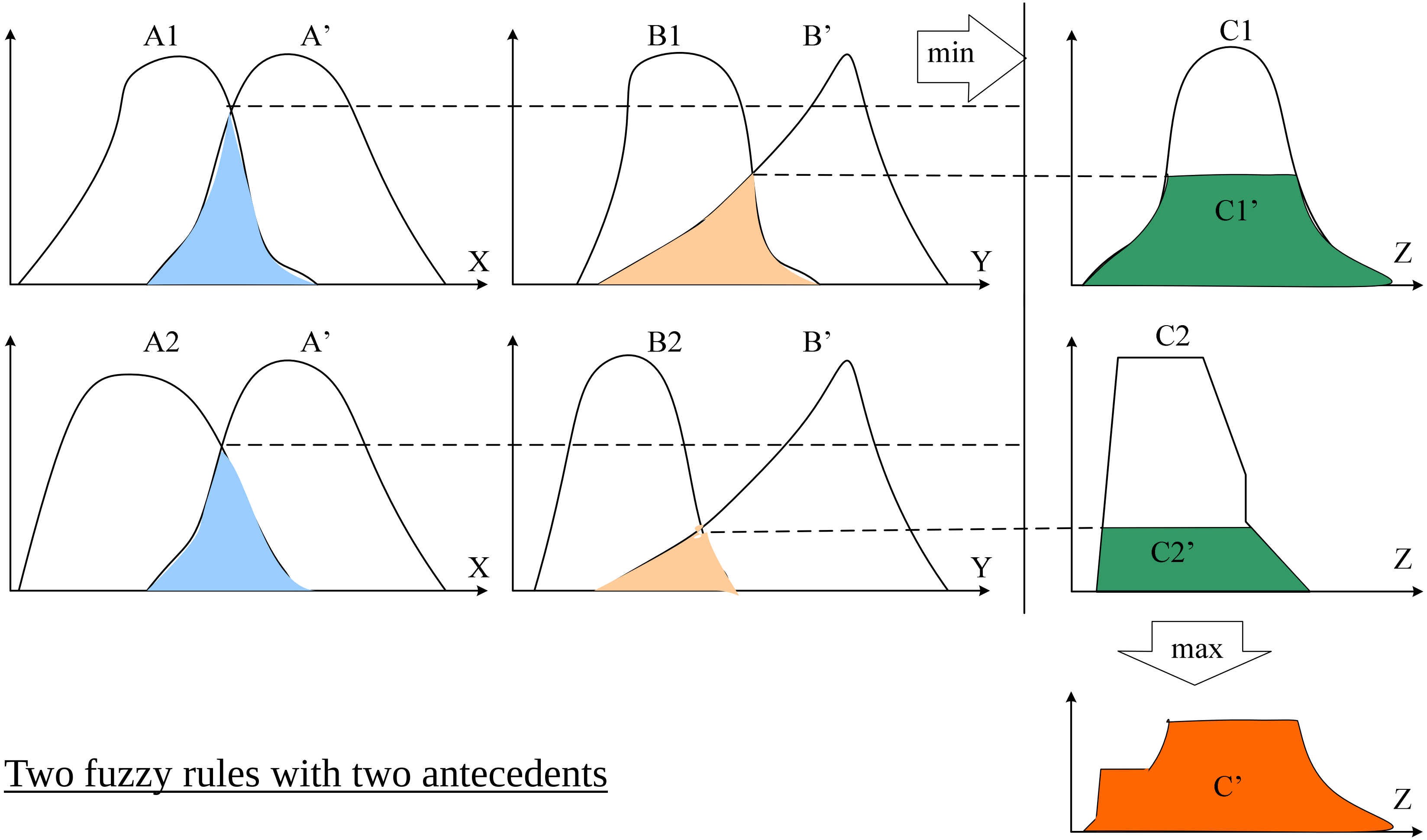
<u>One fuzzy rule with two antecedents</u>

“if x is A and y is B then z is C”



w_1, w_2 $\rightarrow$ degrees of membership to the respective rules.

Max-Min Fuzzy Reasoning



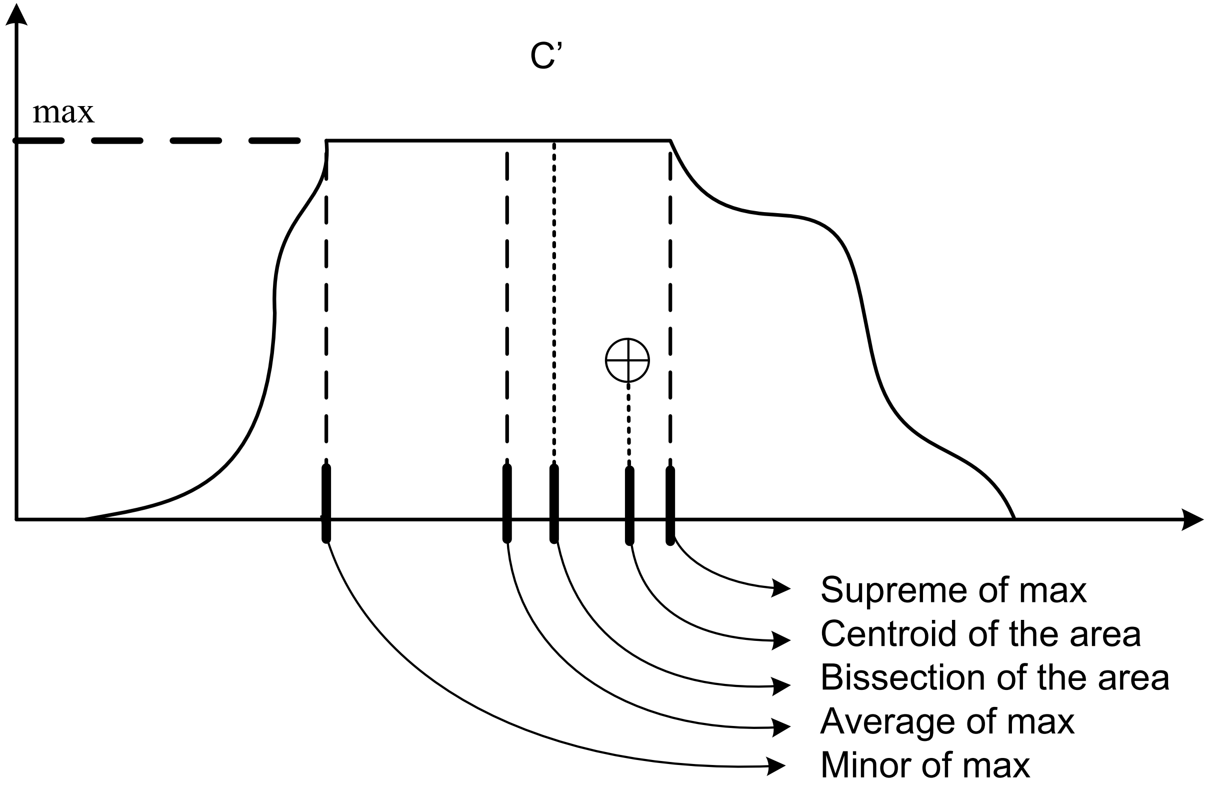
Two fuzzy rules with two antecedents

"If x is A1 and y is B1 then z is C1"
"If x is A2 and y is B2 then z is C2"

Result: C'

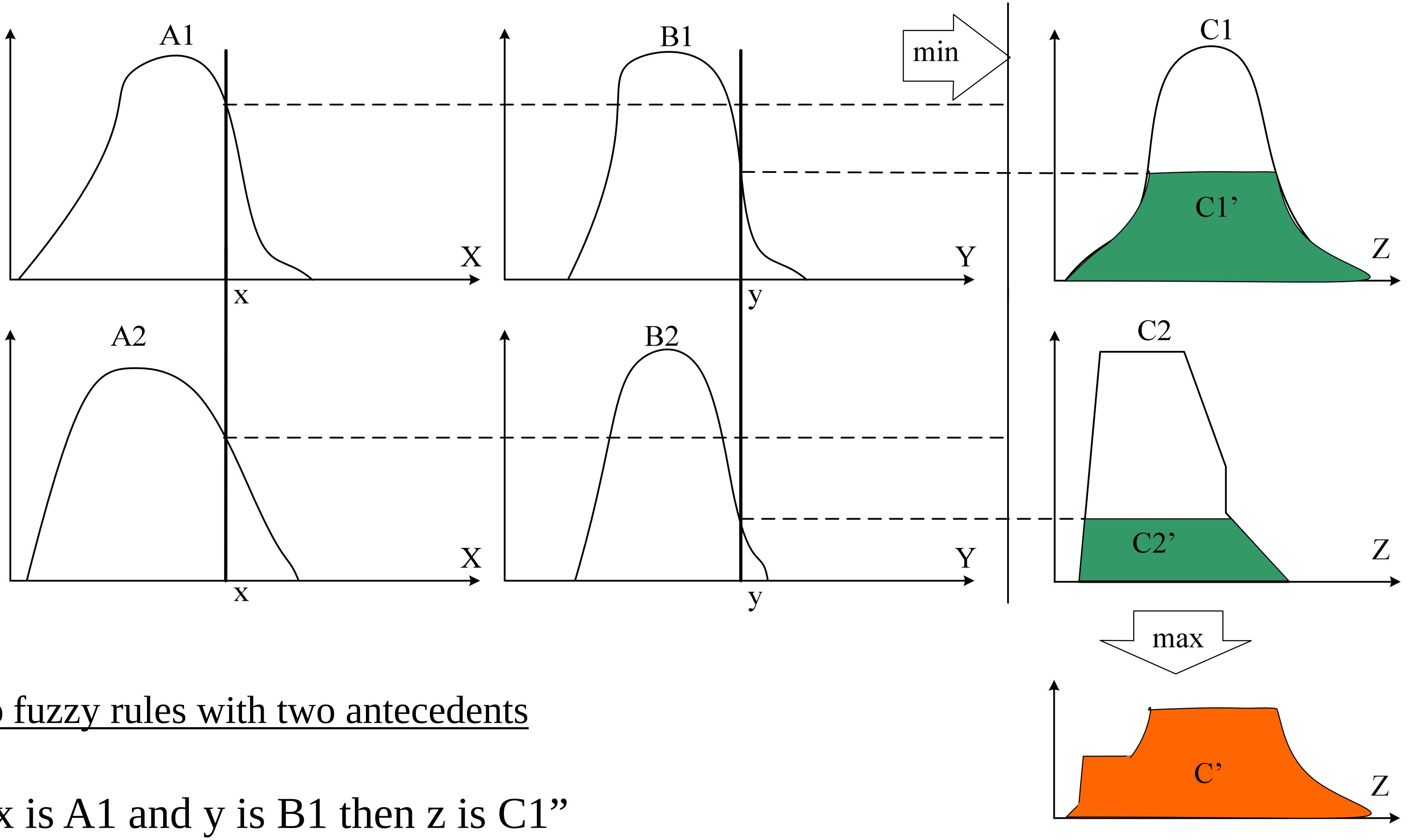
Defuzzyfication Schemes

Associate a numeric value
→ output of the fuzzy inference machine



Fuzzy Inference with exact A' ("crisp")

- Mamdani Model



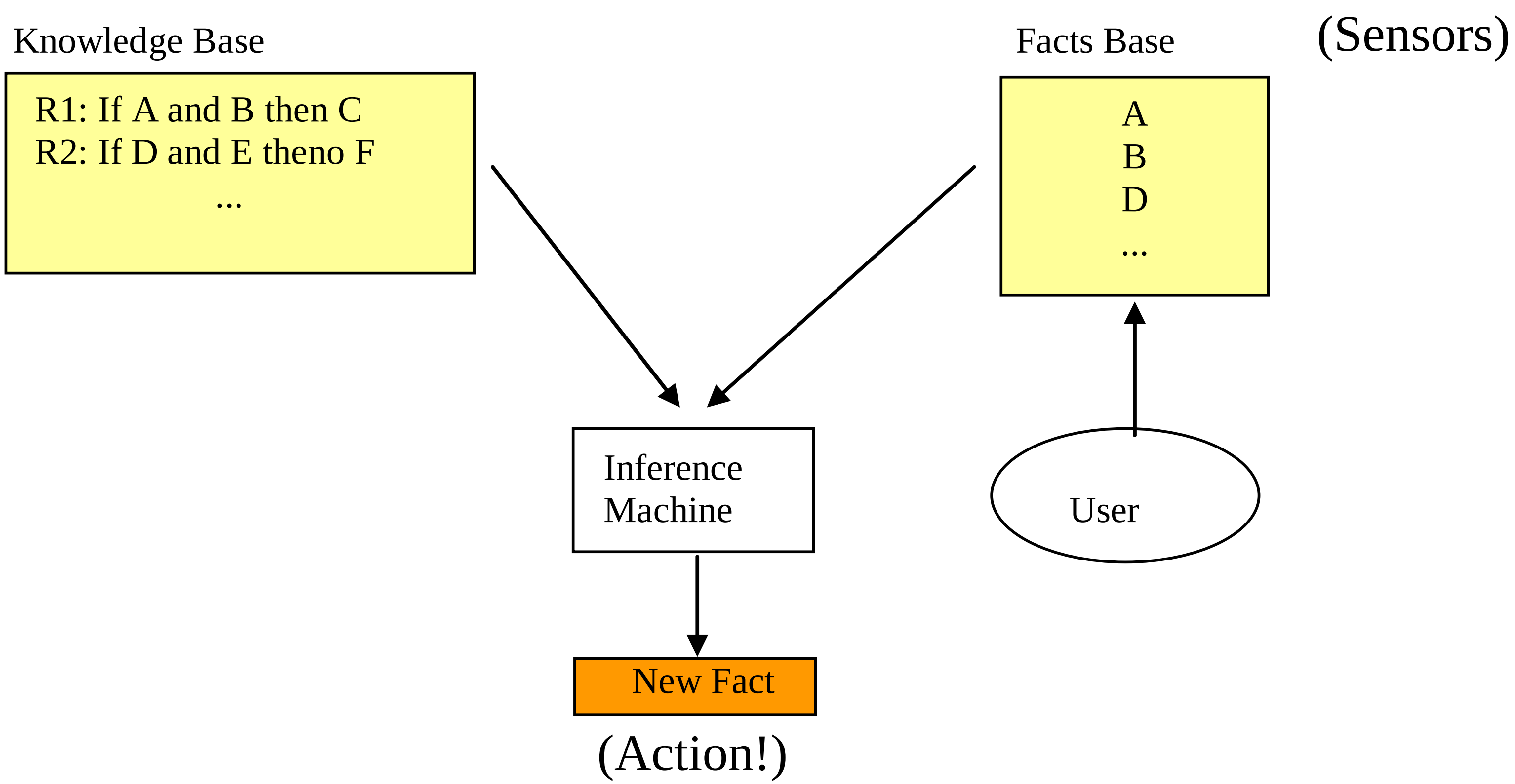
Two fuzzy rules with two antecedents

"if x is A1 and y is B1 then z is C1"
"if x is A2 and y is B2 then z is C2"

Result: C'

Fuzzy Inference Systems

• Fuzzy systems are knowledge-based systems (like Expert Systems).



Fuzzy Inference Machine

The fuzzy inference machine follows these steps to obtain the Inference Result given a set of facts:

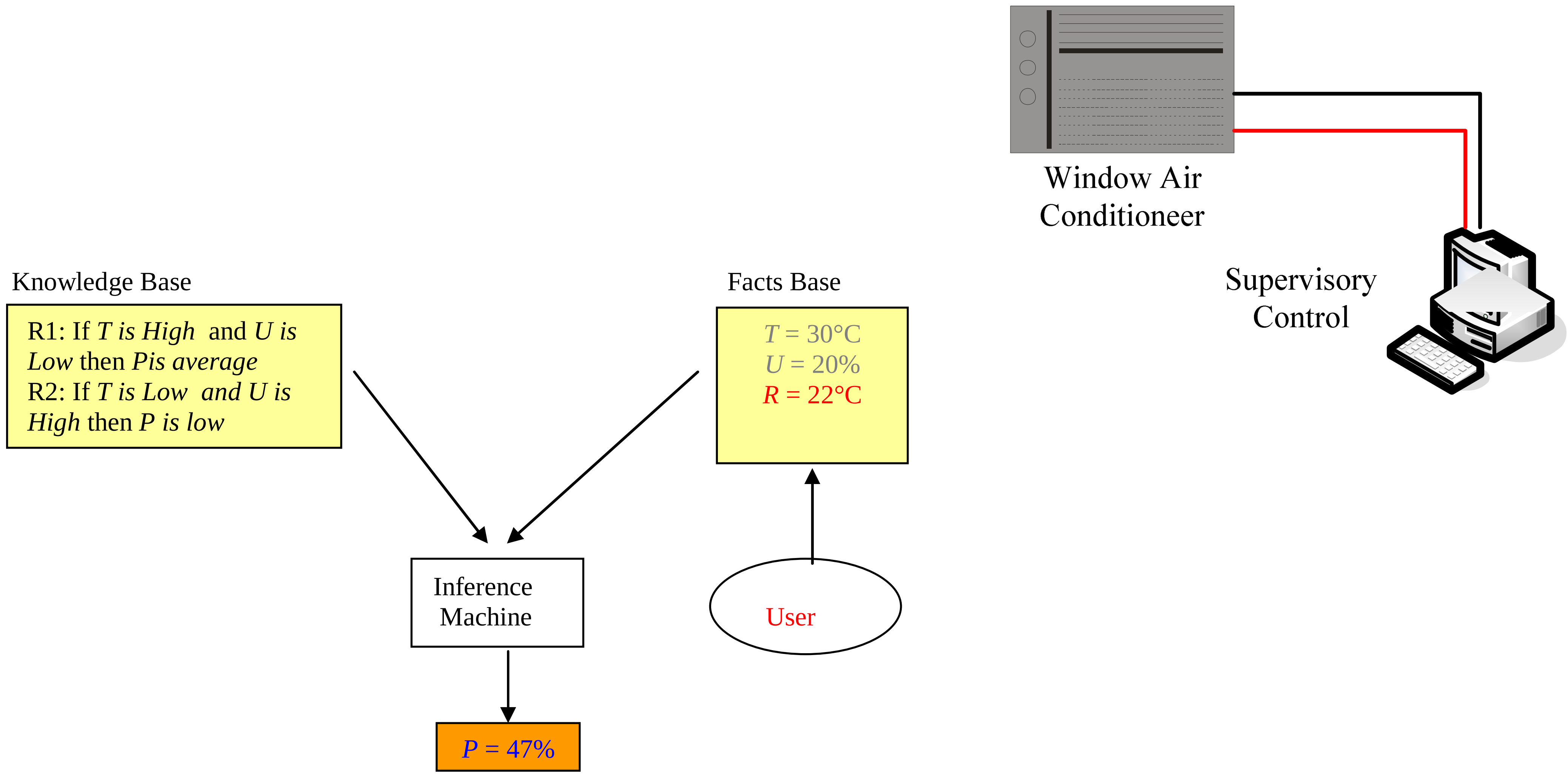
1. facts with premises (antecedents)
2. compatibility degree of each rule
3. belief in each rule
4. aggregation

For the aggregation four methods are popular:

a) Mamdani (Max-Min)
b) Larsen
c) Tsukamoto
d) Takagi-Sugeno

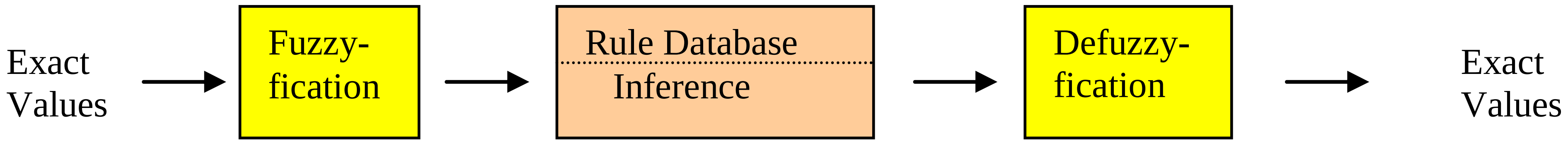
Exemple: Fuzzy Control (revisited)

– Air Conditioneer

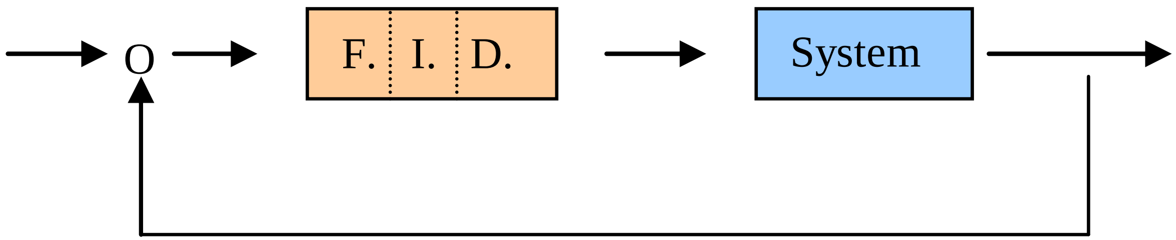


Interface with the Real World

- Fuzzyfication and Defuzzyfication



A feedback controller based on fuzzy logic (Intelligent Controller) would have the following structure, where F. I. D. means: Fuzzyfication, Inference and Defuzzyfication.

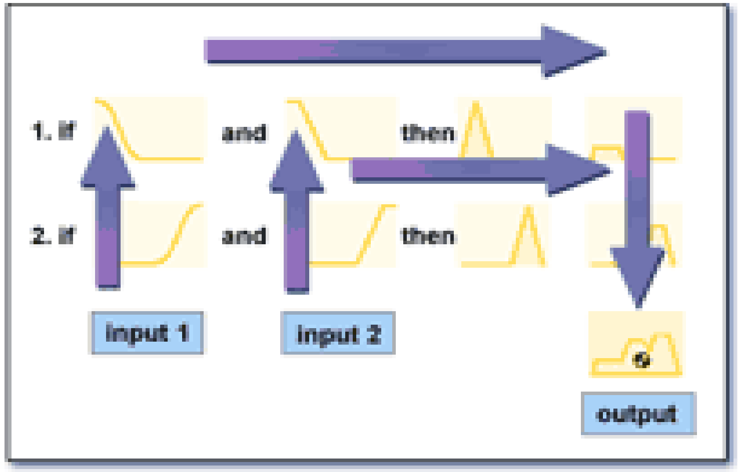


Computational Tools

http://www.imse.cnm.es/Xfuzzy/download.htm

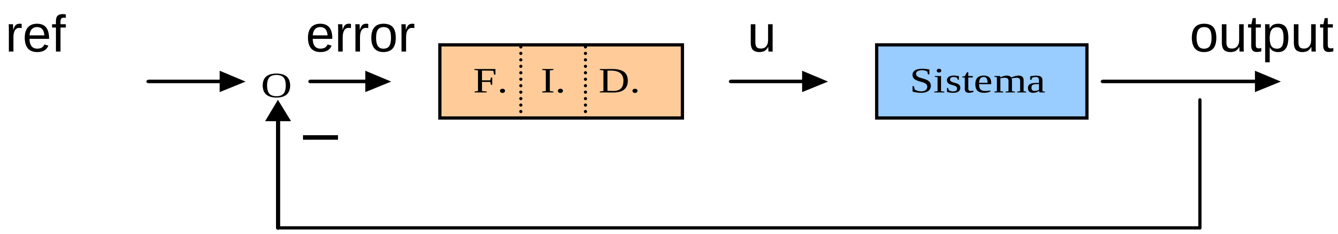
XFuzzy System for Unix developed by the
Instituto de Microelectrónica de Sevilla – Espanha

http://www.mathworks.com MatLab®

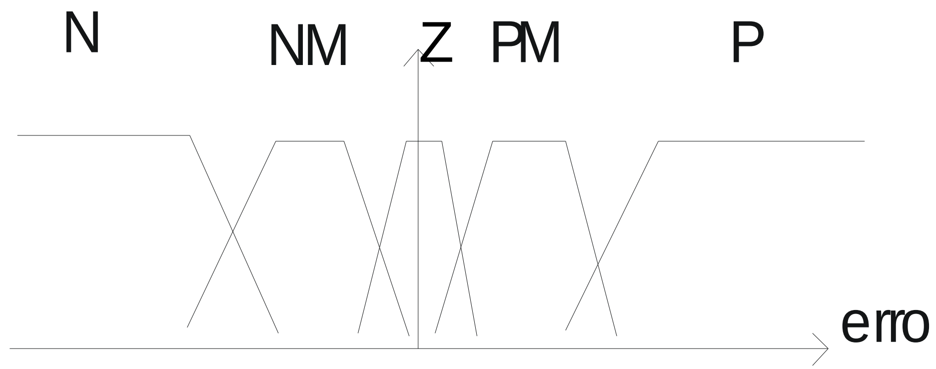


http://www-rocq.inria.fr/scilab/ SciLab

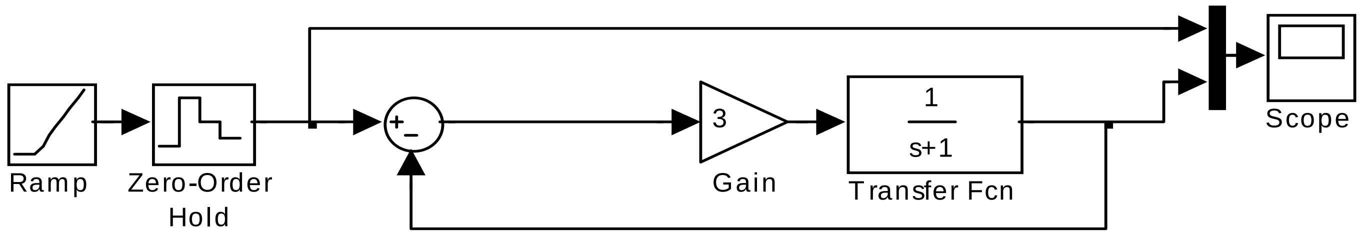
Ex: Fuzzy Proportional Control

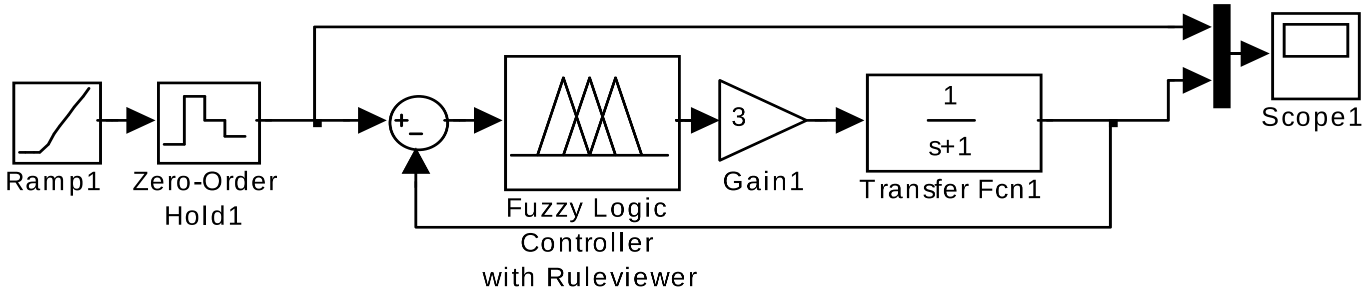


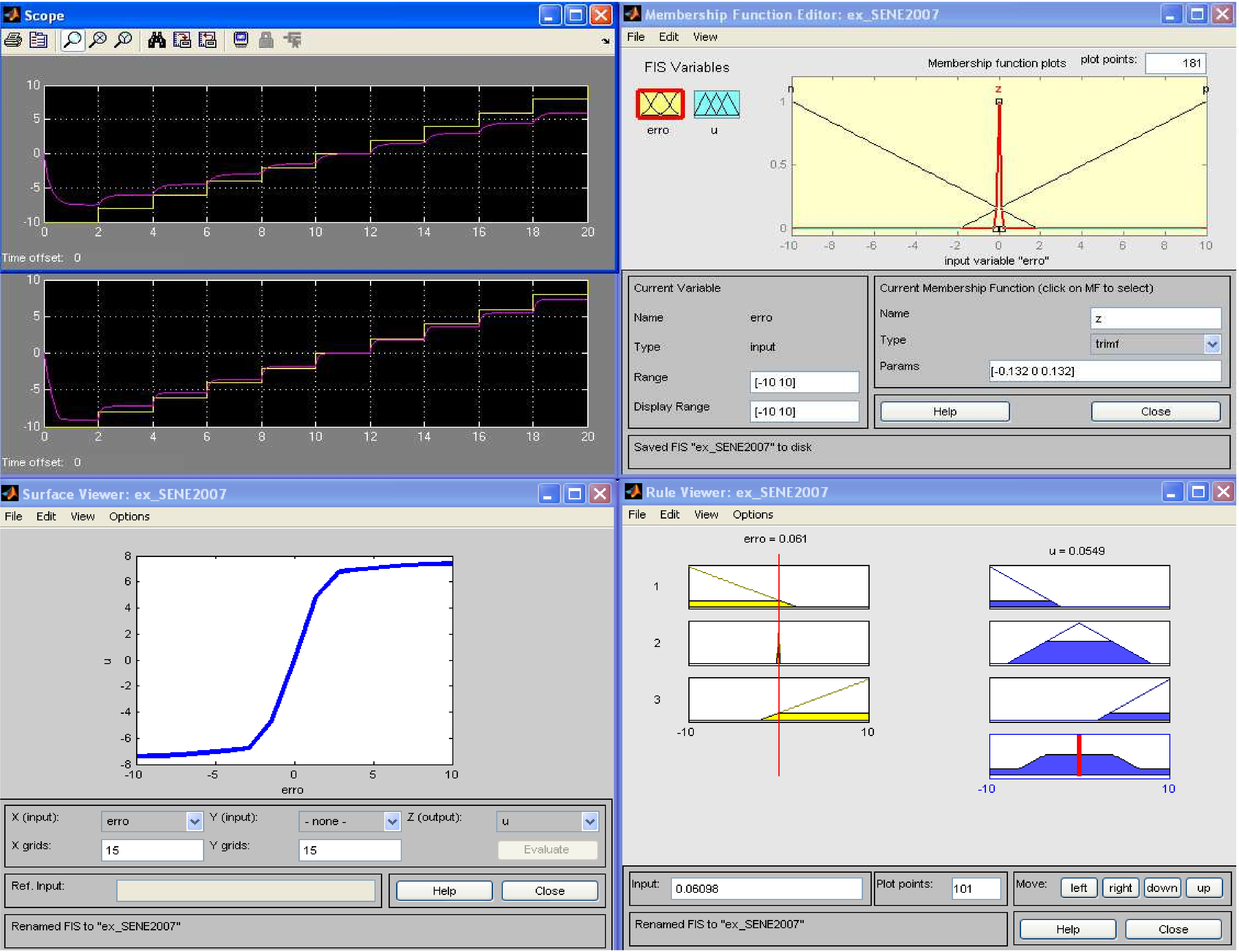
Closed Loop Control



Ex: Fuzzy Proportional Control






Scope
Time offset: 0
Time offset: 0
Membership Function Editor: ex_SENE2007
File Edit View
FIS Variables
erro
u
Membership function plots
plot points: 181
n
z
p
input variable "erro"
Current Variable
Name erro
Type input
Range [-10 10]
Display Range [-10 10]
Current Membership Function (click on MF to select)
Name z
Type trimf
Params [-0.132 0 0.132]
Help
Close
Saved FIS "ex_SENE2007" to disk
Surface Viewer: ex_SENE2007
File Edit View Options
u
erro
X (input): erro
Y (input): - none -
Z (output): u
X grids: 15
Y grids: 15
Evaluate
Ref. Input:
Help
Close
Renamed FIS to "ex_SENE2007"
Rule Viewer: ex_SENE2007
File Edit View Options
erro = 0.061
u = 0.0549
Input: 0.06098
Plot points: 101
Move: left right down up
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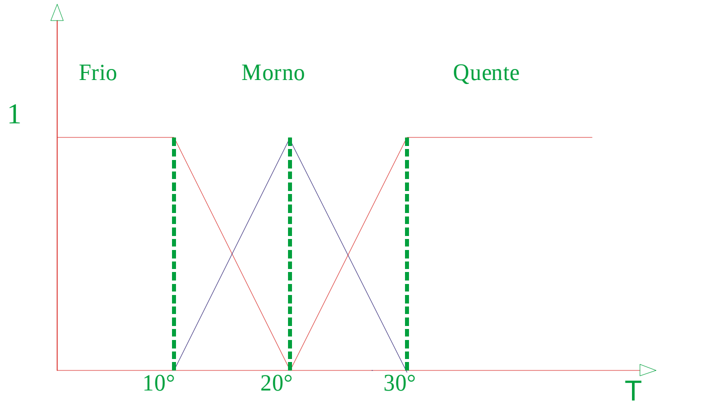
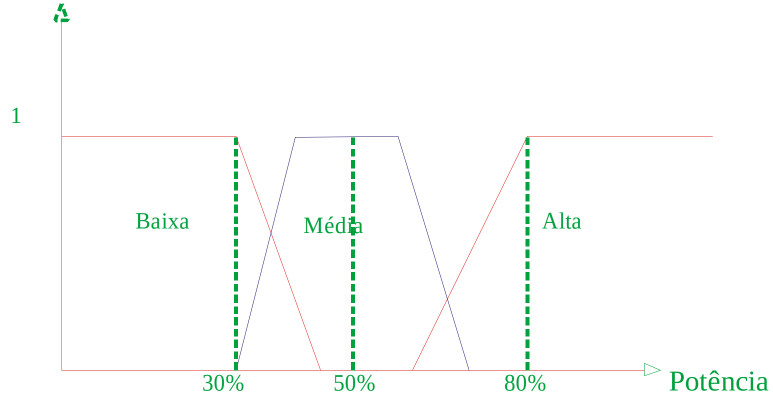
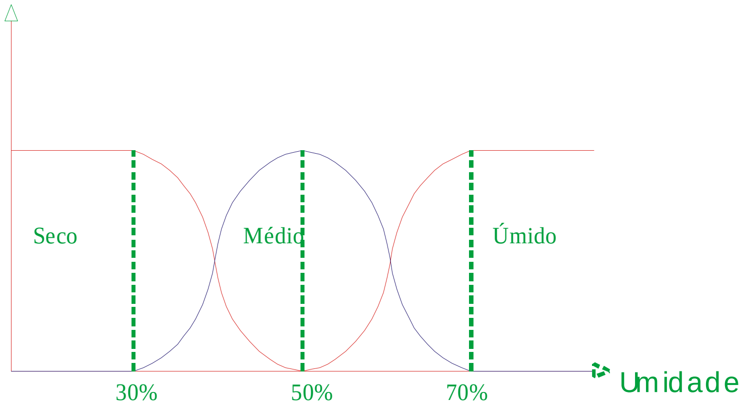
Help
Close

Ex: Multivariable *Fuzzy* Controller

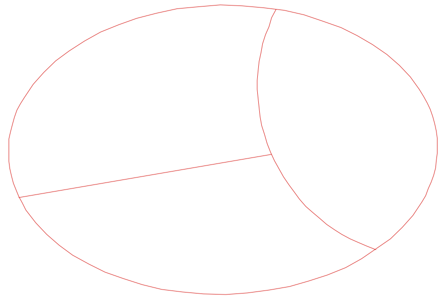
Air conditioning system for an office environment.







Conjunto Fuzzy
Partição do Universo de discurso



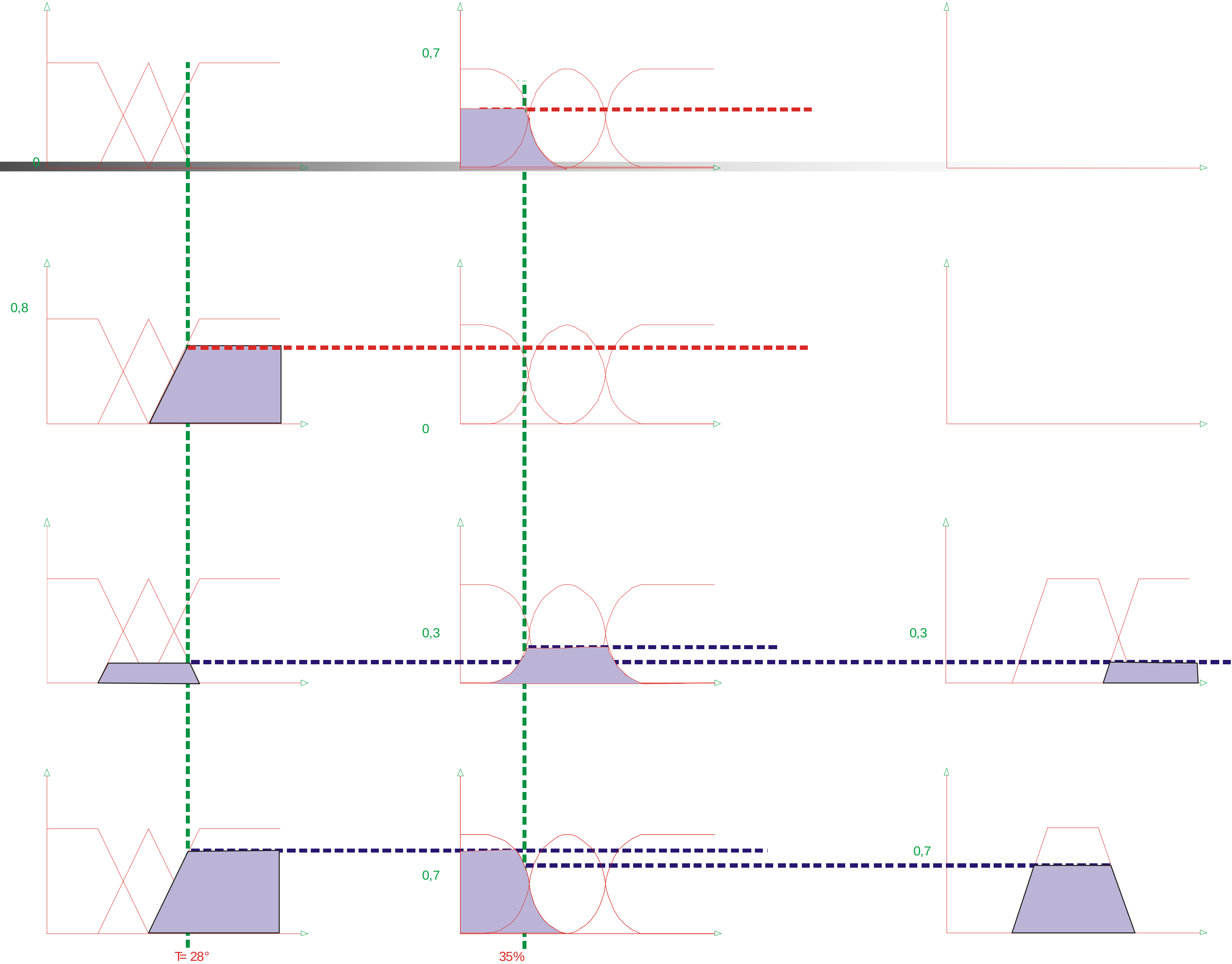
Membership functions used for the temperature control.

Technische Universität Kaiserslautern

Fuzzy Inference:

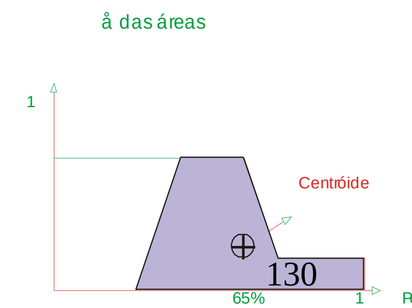
Temperature is 28º C
Relative humidity is 35%

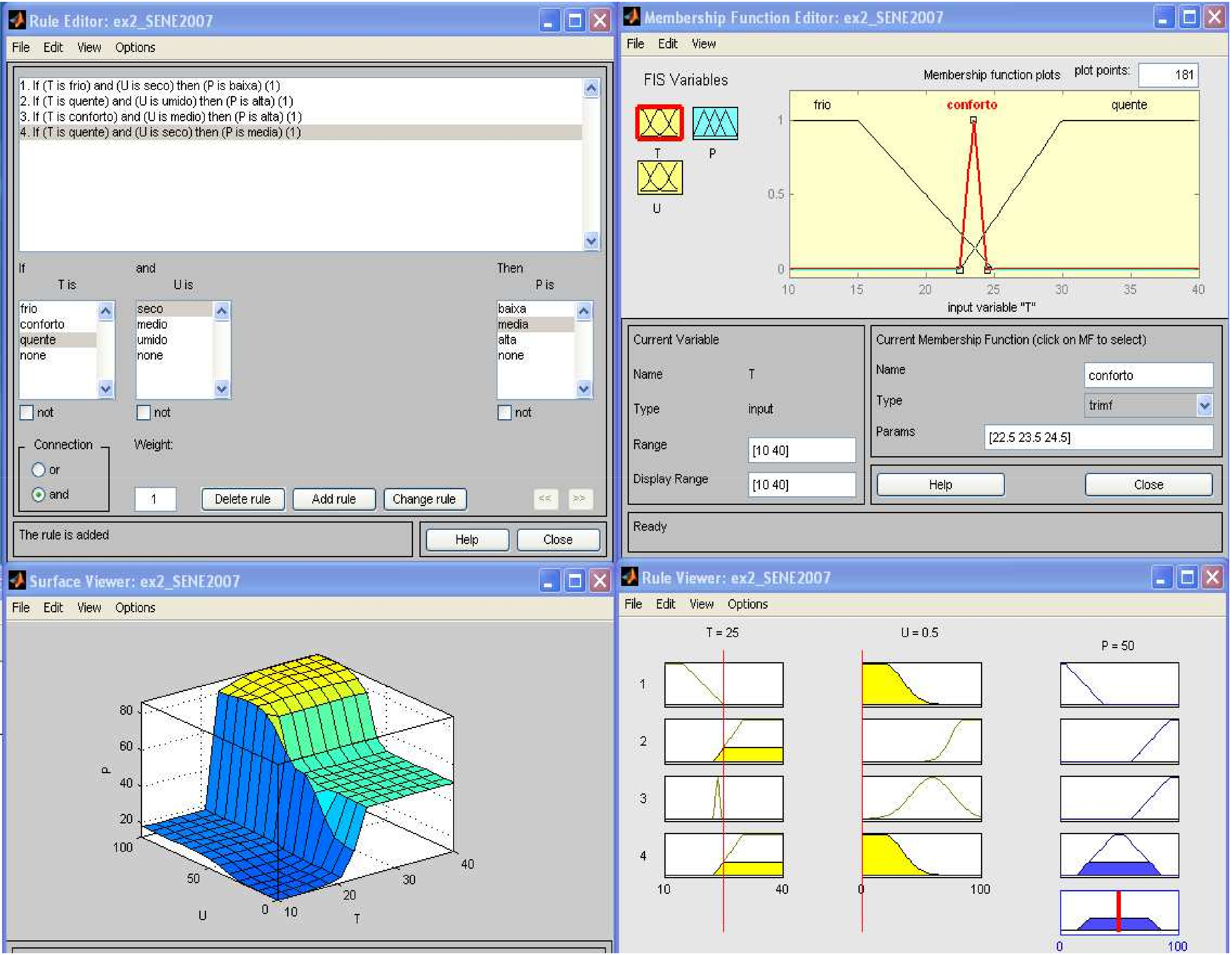
The calculated Power is 65%.



Rule Basis

If T is cold and U is dry then P is low
If T is hot and U is humid then P is high
If T is warm and U is average then P is high
If T is hot and U is dry then P is average




Rule Editor: ex2_SENE2007
File Edit View Options
1. If (T is frio) and (U is seco) then (P is baixa) (1)
2. If (T is quente) and (U is umido) then (P is alta) (1)
3. If (T is conforto) and (U is medio) then (P is alta) (1)
4. If (T is quente) and (U is seco) then (P is media) (1)
If
T is
frio
conforto
quente
none
not
and
U is
seco
medio
umido
none
not
Then
P is
baixa
media
alta
none
not
Connection
or
and
Weight:
1
Delete rule
Add rule
Change rule
The rule is added
Help
Close
Membership Function Editor: ex2_SENE2007
File Edit View
FIS Variables
T
P
U
Membership function plots
plot points:
181
frio
conforto
quente
input variable "T"
Current Variable
Name T
Type input
Range [10 40]
Display Range [10 40]
Current Membership Function (click on MF to select)
Name conforto
Type trimf
Params [22.5 23.5 24.5]
Help
Close
Ready
Surface Viewer: ex2_SENE2007
File Edit View Options
P
U
T
Rule Viewer: ex2_SENE2007
File Edit View Options
T = 25
U = 0.5
P = 50

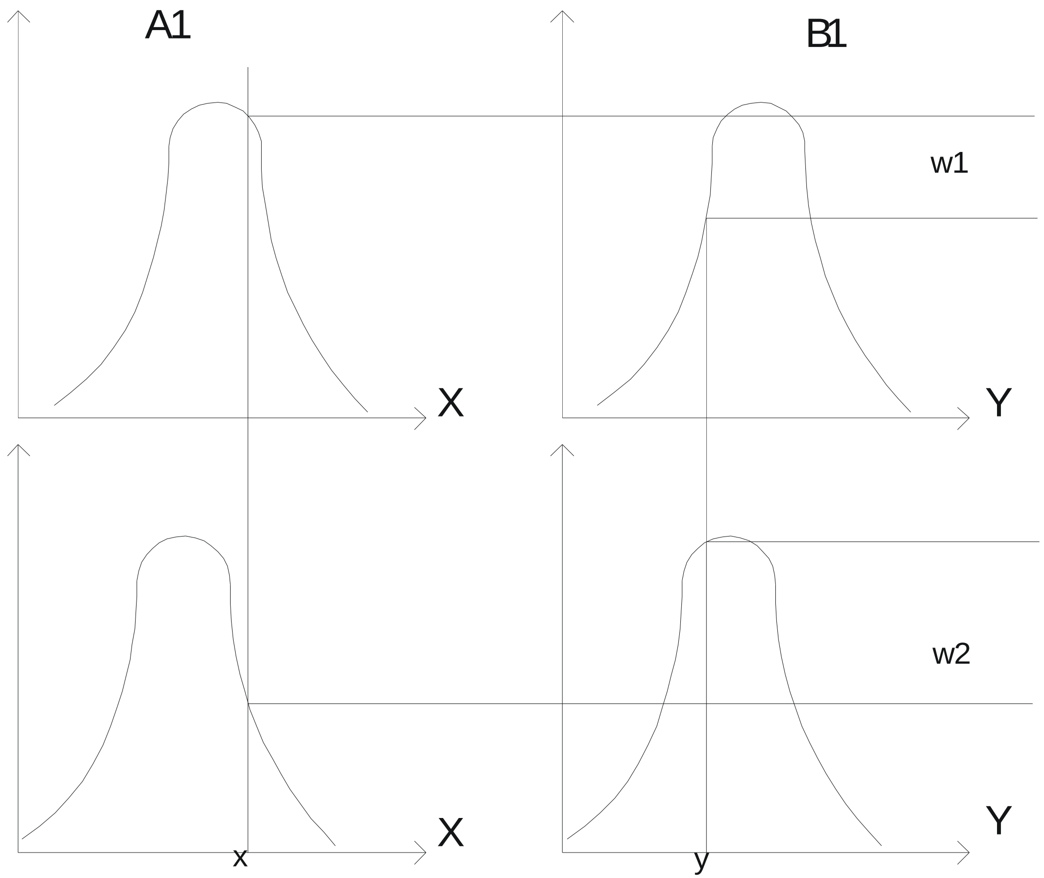
1^{st} Order Sugeno Fuzzy Inference System

R_1: IF x is A_1 AND y is B_1 THEN $f_1 = p_1x + q_1y + r_1$

R_2: IF x is A_2 AND y is B_2 THNE $f_2 = p_2x + q_2y + r_2$

Consequent: linear combination of the inputs

(pi,qi,ri) instead of output M.F.



f1= p1x+ q1y+ r1

f2= p2x+ q2y+ r2

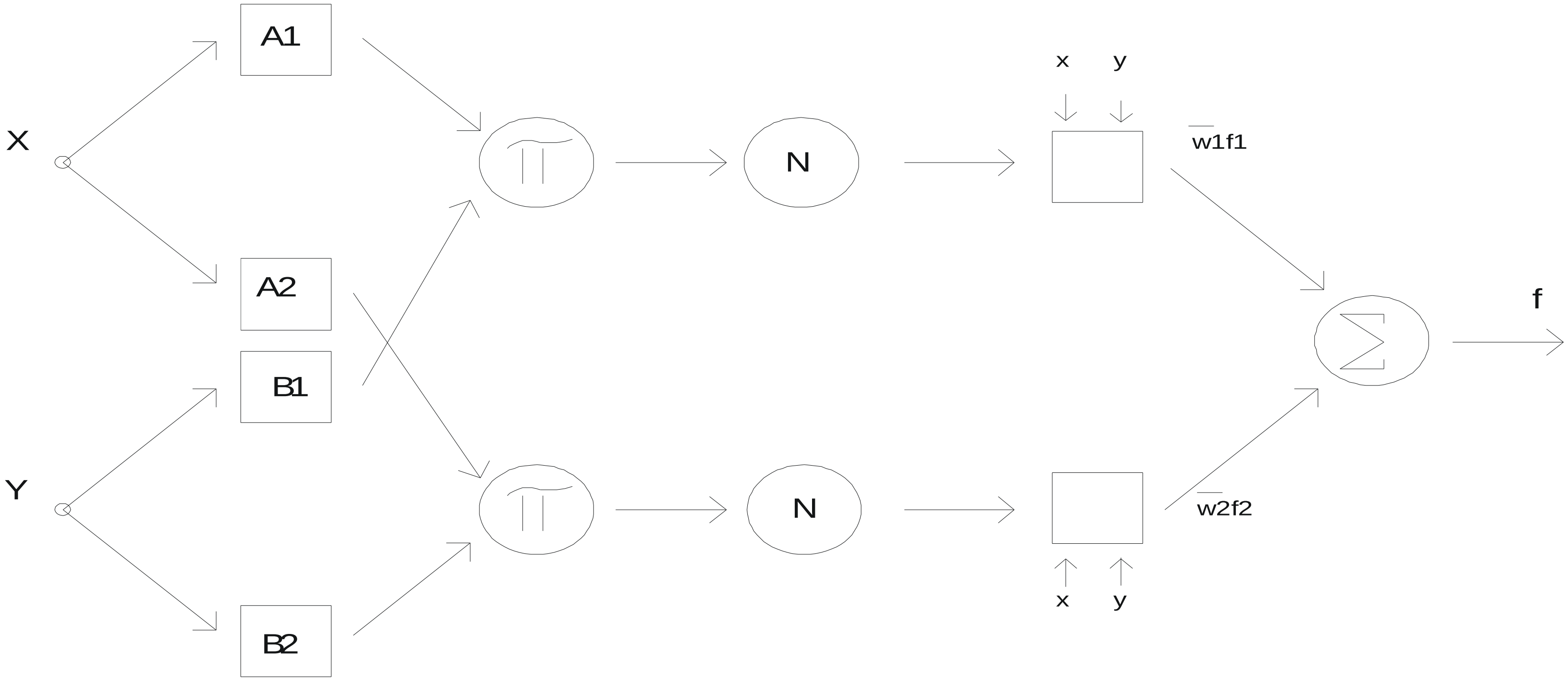
$$f = \frac{w1f1 + w2f2}{w1 + w2} = \overline{w1}f1 + \overline{w2}f2$$

Degrees of compatibility (w1,w2) weigth the rule interpolation

Adaptive Neuro Fuzzy Inference System (ANFIS)

R_1: IF x is A_1 AND y is B_1 THEN $f_1 = p_1x + q_1y + r_1$

R_2: IF x is A_2 AND y is B_2 THNE $f_2 = p_2x + q_2y + r_2$



Adaptive feed-forward network for the Sugeno fuzzy model

ANFIS Layers

Layer 1: Adaptive Nodes

$o_{1,i} = \mu_{Ai}(x)$, for i = 1, 2.
$o_{1,i} = \mu_{B\ i-2}(y)$, for i = 3, 4.

Ai – generalized bell function

$$\mu_{Ai}(x) = \frac{1}{1 + [(x-c_i)^2/(a_i)^2]^{b_i}}$$

{a_i, b_i, c_i} set of premise parameters

Layer 2: Fixed Nodes

$o_{2,i} = \omega_i = \mu_{Ai}(x)\ \times \mu_{B\ i}(y)$, i = 1, 2

↓ T-Norm

Layer 3: Fixed Nodes

$$o_{3,i} = \bar{\omega}_i \text{ (médio)} = \frac{\omega_i}{\omega_1 + \omega_2}, \qquad i = 1,2$$

Layer 4: Adaptive Nodes

$$o_{4,i} = \frac{\omega_i}{\omega_1 + \omega_2} . f_i = \frac{\omega_i}{\omega_1 + \omega_2} . (p_i x + q_i y + r_i)\ , \qquad i = 1,2$$

{ p_i, qi, r_i} set of the consequent parameters

Layer 5: Fixed Node

$$o_{5,i} = \Sigma_i \frac{\omega_i}{\omega_1 + \omega_2} . f_i = \frac{\Sigma_i \omega_i f_i}{\Sigma_i \omega_i}$$

ANFIS Hybrid Learning

1 – Fix the premisse parameters

⇒ Output is a linear combination of the consequence parameters

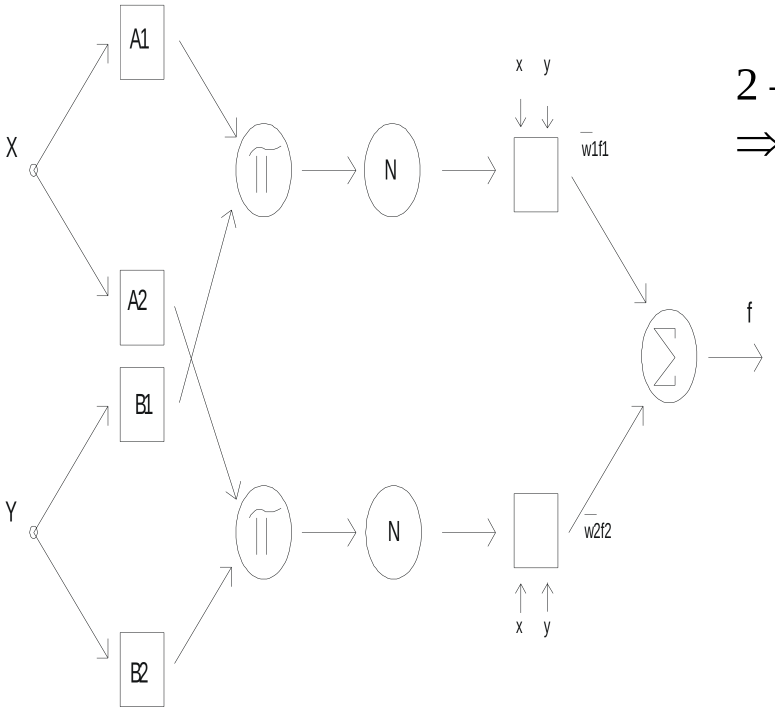
$$f = \frac{\omega_1}{\omega_1 + \omega_2} . f_1 + \frac{\omega_2}{\omega_1 + \omega_2} . f_2$$

$$= \frac{1}{\omega_1 + \omega_2} . (\omega_1 x p_1 + \omega_1 y q_1 + \omega_1 r_1 + \omega_2 x p_2 + \omega_2 y q_2 + \omega_2 r_2)$$

⇒Identify consequence paraters using Least Mean Squares method.

2 – Backpropagtion of the errror signals toadapt the premisse parameters

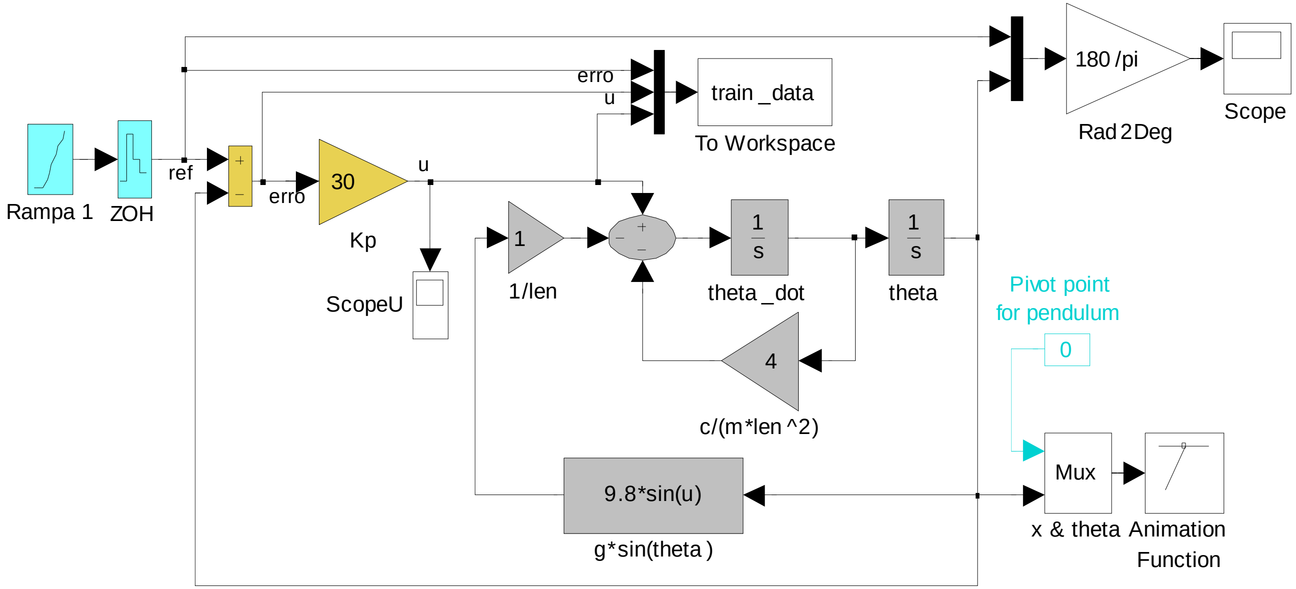
⇒ Gradient descent method.

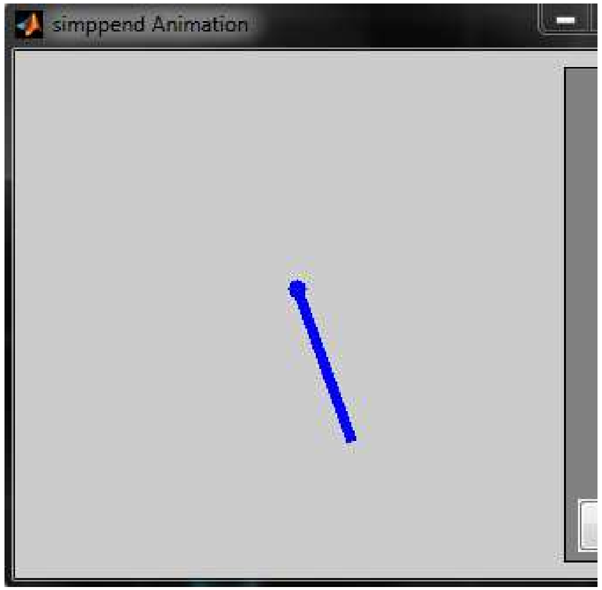


	Forward Step	Backward Step
Premisse Parameters	Fixed	Gradiente descent
Consequence Parameters	LMS Estimate	Fixed
Signals	Output nodes	Error Signals

ANFIS

Adaptive Neuro Fuzzy Inference System



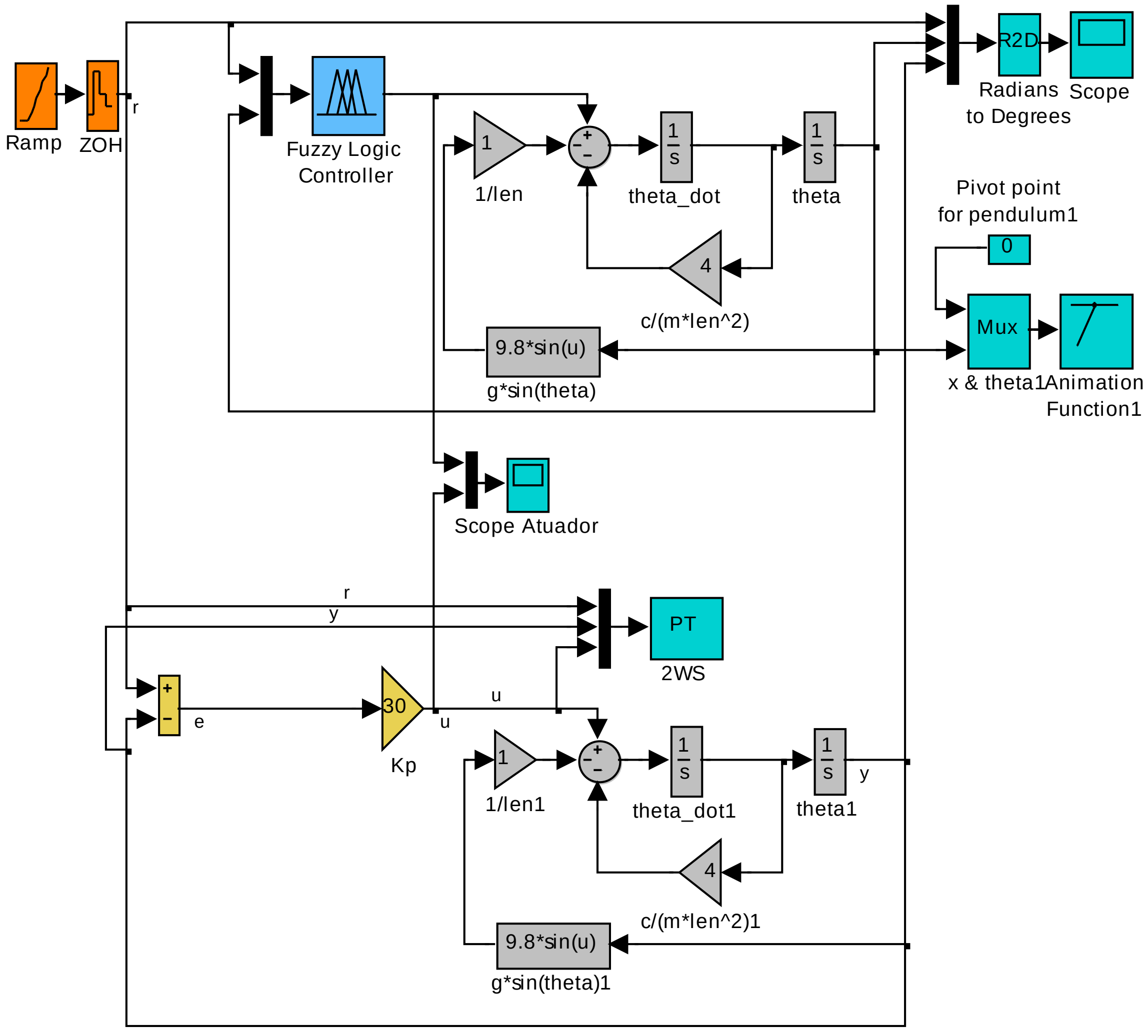


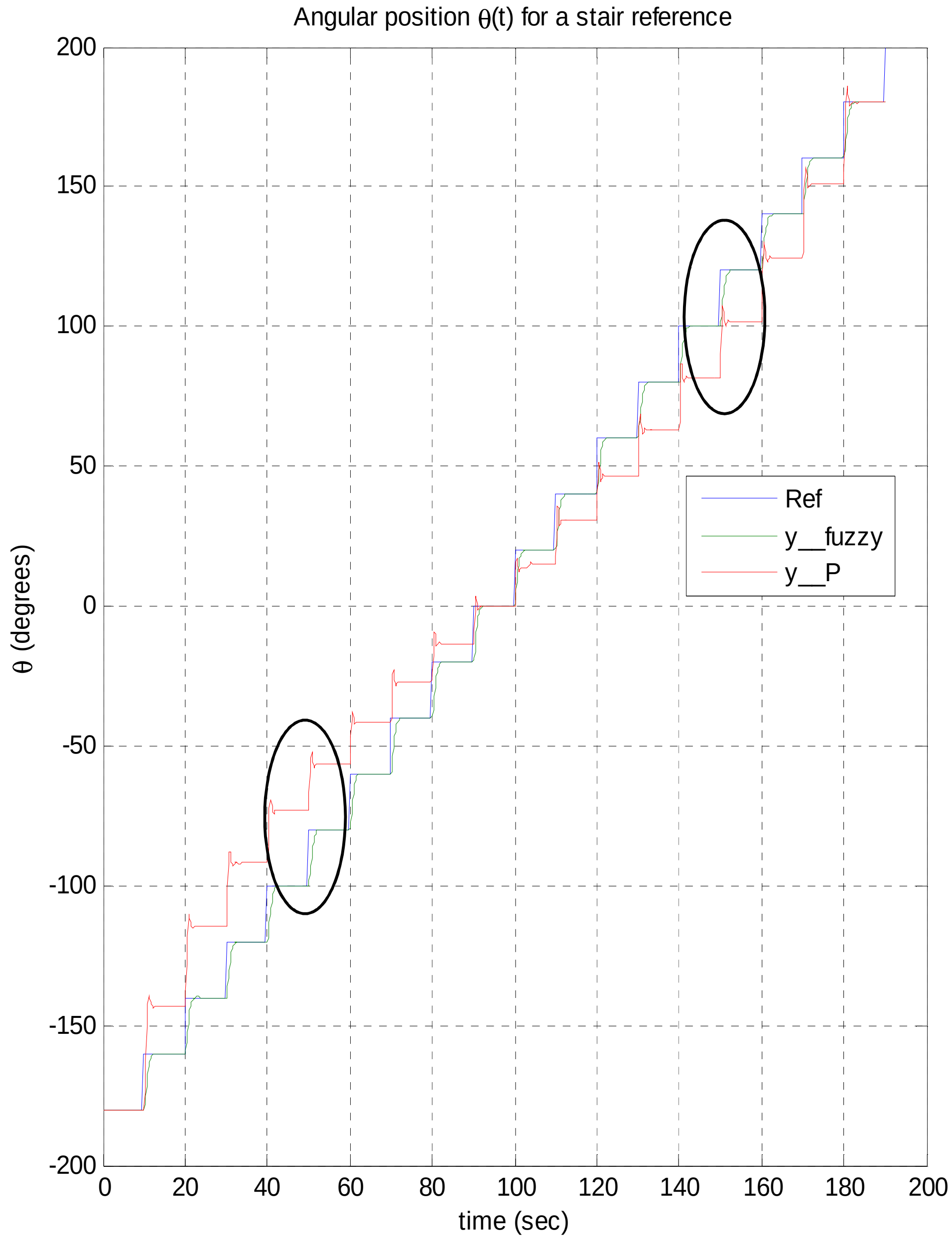
$$\ddot{\theta} = -\frac{c}{ml^2}\dot{\theta} - \frac{g\sin(\theta)}{l} + u$$

$$\ddot{\theta} + 4\dot{\theta} + 9.8\sin(\theta) = u$$

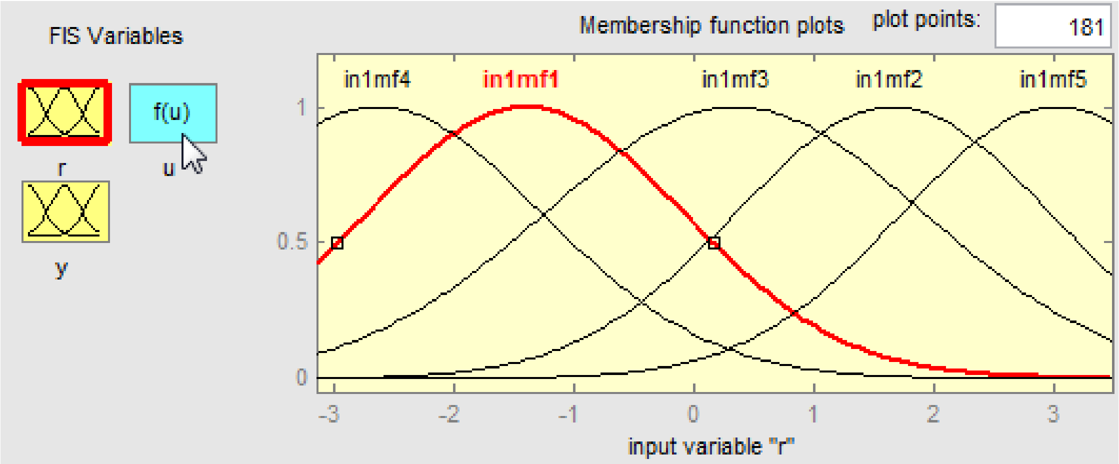
$$Control\ \ law \qquad u = 30(r - \theta)$$

ANFIS x P-Controller

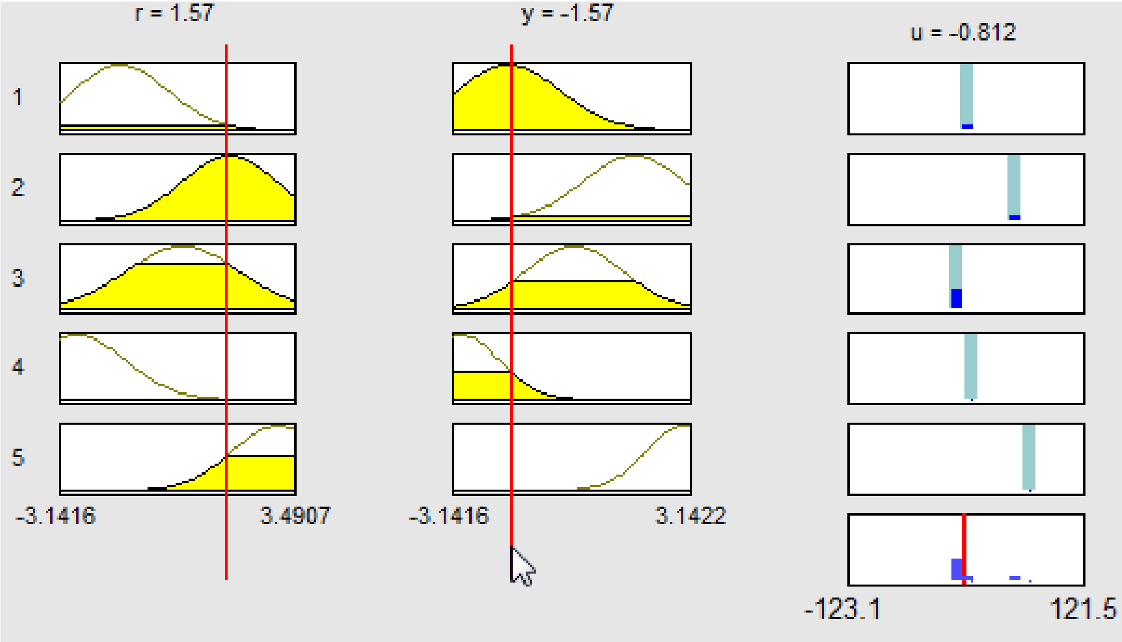




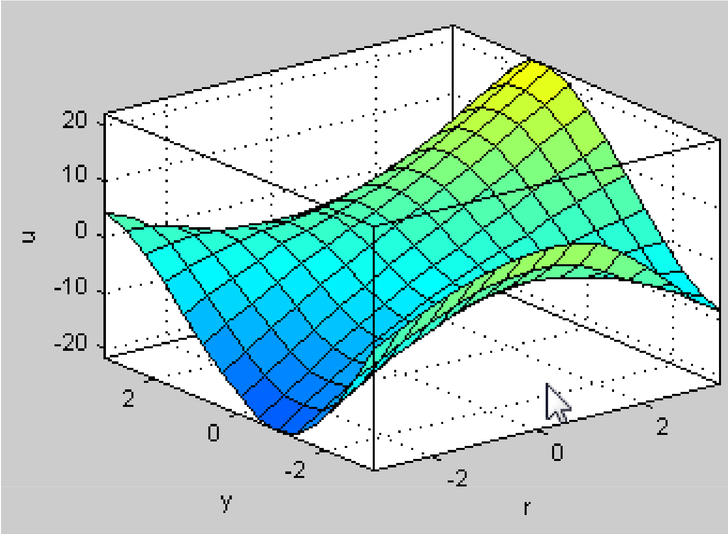
ANFIS Controller



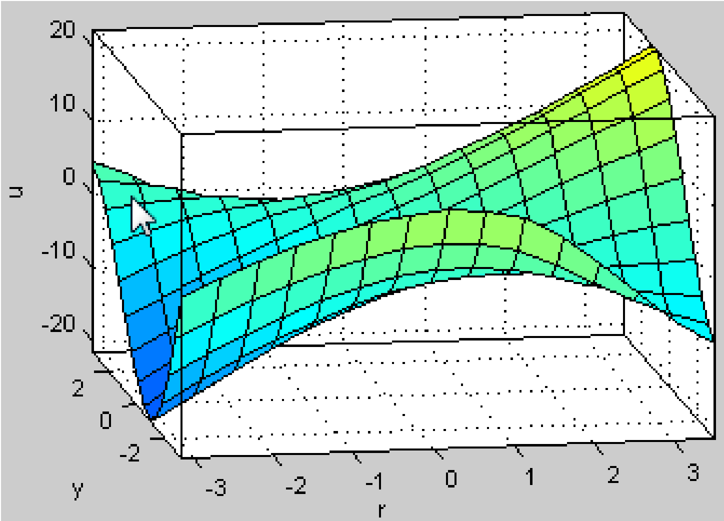
Membership functions of r (reference)



Rule Viewer

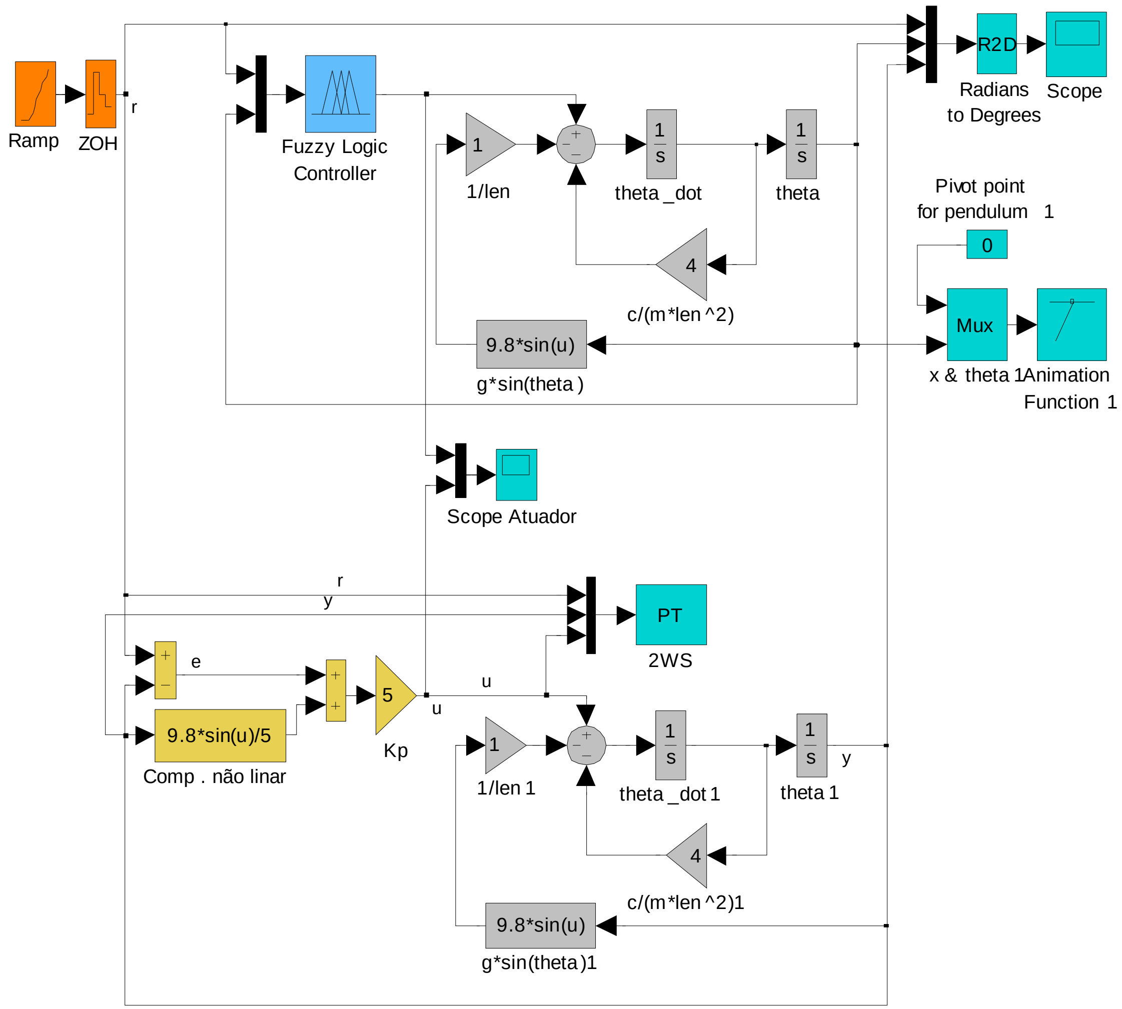


View of the ***non-linear*** control surface



Another view of the ***non-linear*** control surface

ANFIS – Who is the Expert?



Expert Knowlege

Linear behavior if the control signal, u, can cancel the non-linear dynamics of the process.

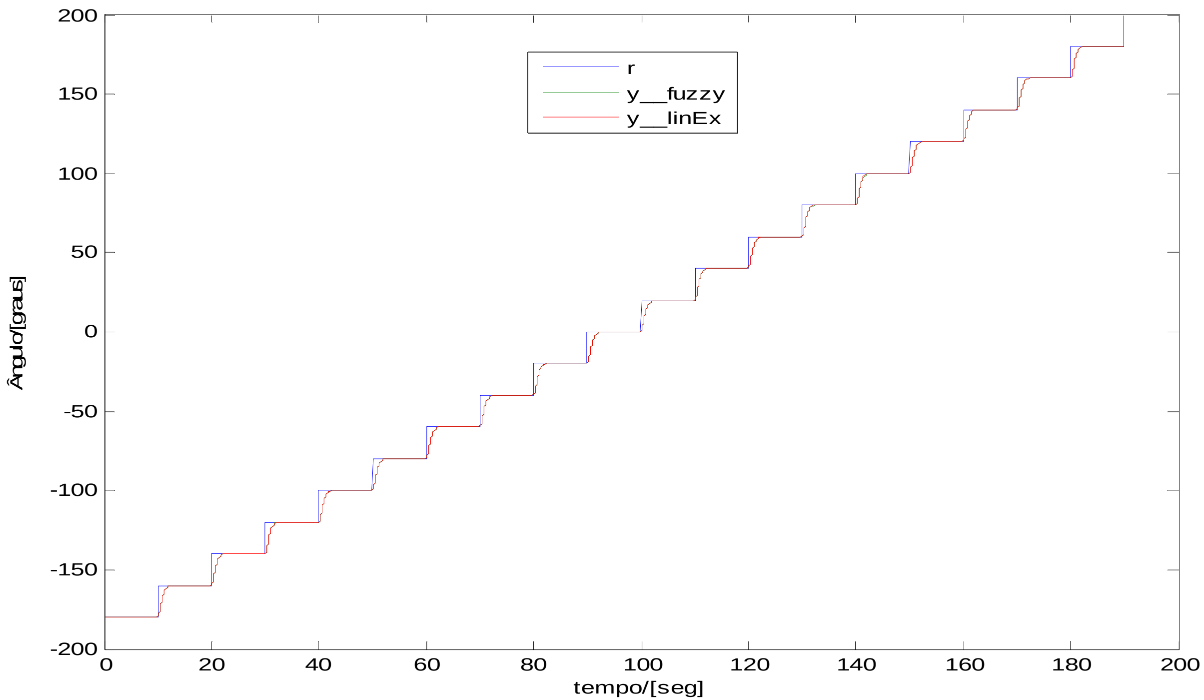
→ “Exact Linearization” (E.L.)
(not only operating point)

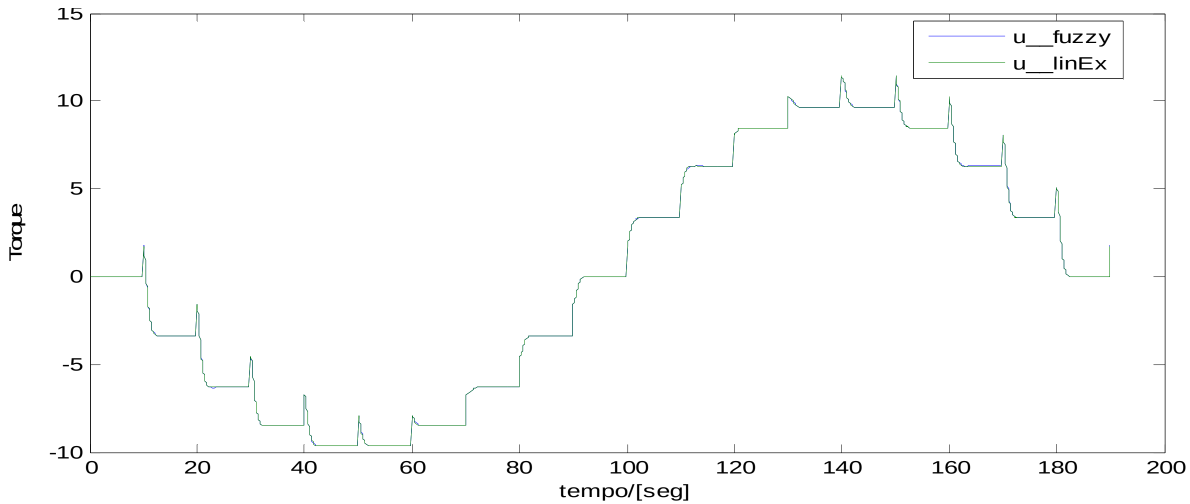
$$\ddot{\theta} + 4\dot{\theta} + 9.8\sin(\theta) = u$$

$$P-Control\ law \quad u = K_p(r-\theta)$$

$$E.L.-Control\ law \quad u = K_p(r-\theta-9.8\sin(\theta))$$

ANFIS – Controller





Rules Trained by an ANN!

-You can explain and add new rules (*fuzzy*)

-You can train with real data (ANN)

-Drawback *more parameters...*