A PREDICTIVE ALGORITHM WITH FINE INTERPOLATION FOR VISION GUIDED ROBOTS

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Abstract: A predictive algorithm with fine interpolation for the optimization of the path tracking of vision guided robotic manipulators is presented. The look-ahead trajectory information extracted from the image sensor is processed in a way that it can be used in the sampling rate of the joint control loop. The predictive control law is obtained from the short-term future trajectory and the minimization of differences between the state-variables of each linearized robot joint and the corresponding state-variables of an internal path generator model. Simulations and experimental results for a vision-guided hydraulic robot are presented.

Keywords: Predictive Control, Robot Vision, Path Tracking

1. INTRODUCTION

In the flexible automation robots equipped with CCD-cameras are increasingly being used due to the availability of effective visual servoing techniques (Nelson *et al*, 1993). An important field where they are used is, e.g., arc-welding (Bauchspiess and Alfaro, 1997). A great flexibility is obtained by eliminating the tedious programming of the robot. The trajectory can be on-line captured by a feature tracker. In welding, for example, the junction of two metal parts define a clear feature that is easily detected in a CCD-image. The autonomously seam tracking can thus greatly simplify the welding of complex parts.

A video camera as trajectory sensor is typically very slow when compared with the rate in which the drivers are actuated. Thus it is necessary to have a fine interpolation that generates reference values for each joint driver. A shortcut of the use of sensor-guided robots is that due to their mechanical inertia they can react only relatively slowly to changes in the trajectory information captured by the sensor system. In this paper it will be shown how the look ahead information can be used to improve substantially the tracking precision. The proposed algorithm virtually eliminates the tracking error by considering the dynamic model of the robot and the captured future trajectory information. Incorporating an internal trajectory generator model leads thus to the novel **F**ollowing **M**odel **P**redictive servo-controller algorithm (**FMP**).

2. PREDICTIVE CONTROL

Predictive control gathered wide acceptance in the control community due to excellent performance dealing with complex, time-varying, non-linear systems and also in industry, in sight of many reported successful industrial implementations

(Keyser, 1991). A great spectrum of predictive algorithms have been proposed (Soeterboek, 1992). MAC (Model Algorithmic Control), DMC (Dynamic Matrix Control), EPSAC (Extended Prediction Self-Adaptive Control), PFC (Predictive Functional Control), GPC (Generalized Predictive Control), UPC (Unified Predictive Control) are some of the most known schemes. They differ essentially in the adopted Cost Function.

In the GPC (Clarke *et al.*, 1987), perhaps the most successful predictive algorithm, the cost function is:

$$J = \sum_{j=N_1}^{N_2} \left[y(t+j) - w(t+j) \right]^2 + r \sum_{j=1}^{N_u} \Delta u^2 (t+j-1)$$

where r is a weighting factor, N_1 and N_2 are prediction horizons and N_u is the control horizon.

The proposed FMP (Following Model Predictive) is an extension of previous algorithms by incorporating the basic idea of ideal following (Wurmthaler, 1994): minimize the error between the state variables of the controlled system and the state variables of an internal trajectory generator.

3. FOLLOWING MODEL PREDICTIVE PATH TRACKING

Consider the controllable and observable scalar n-th order discrete time linear system described by:

$$\begin{aligned} &\mathbf{x}(k+1) = \mathbf{A}_k \mathbf{x}(k) + \mathbf{b}_k u_s(k); \qquad \mathbf{x}(0) = \mathbf{x}_0 \\ &y(k) = \mathbf{c}_k^T \mathbf{x}(k), \end{aligned} \tag{1}$$

where $\mathbf{x}(k)$ is the nx1 state vector at time $t=kT$ (T is the sampling period), $u_s(k)$ is the input and $y(k)$ is the system output.

Problem formulation: Given a third order plant described by (1). The reference trajectory and its derivatives $w(t), \dot{w}(t), \ddot{w}(t)$ are known from $t = (k+1)T$ until the horizon $t = (k+m)T$. The current state of the plant is $\mathbf{x}(k)$. The position, velocity, acceleration, correspond to the states of the system in the controllable canonical form. The problem consists on the calculation of the control sequence $u_s(k)$, $u_s(k+1)$, ... $u_s(k+m-1)$, such that the following *predictive path tracking* ***Optimization Criteria*** (or ***Cost Function***) will be minimized:

$$J = \sum_{i=0}^{2} \varepsilon_i^T \mathbf{Q}_i \varepsilon_i + \beta^T \mathbf{R} \beta \tag{2}$$

The weighting matrices $\mathbf{Q}_i$ and $\mathbf{R}$ are symmetric positive defined. The vector error terms ε_i are:

Position Error: $\varepsilon_0(k) \mathrel{\hat{=}} \mathbf{y}(k) - \mathbf{w}(k)$

$$\mathbf{y}(k) \mathrel{\hat{=}} \begin{bmatrix} \hat{y}(k+1) \\ \hat{y}(k+2) \\ \vdots \\ \hat{y}(k+m) \end{bmatrix}, \mathbf{w}(k) \mathrel{\hat{=}} \begin{bmatrix} w(k+1) \\ w(k+2) \\ \vdots \\ w(k+m) \end{bmatrix},$$

Velocity Error: $\varepsilon_1(k) \mathrel{\hat{=}} \dot{\mathbf{y}}(k) - \dot{\mathbf{w}}(k)$

$$\dot{\mathbf{y}}(k) \mathrel{\hat{=}} \begin{bmatrix} \hat{\dot{y}}(k+1) \\ \hat{\dot{y}}(k+2) \\ \vdots \\ \hat{\dot{y}}(k+m) \end{bmatrix}, \dot{\mathbf{w}}(k) \mathrel{\hat{=}} \begin{bmatrix} \dot{w}(k+1) \\ \dot{w}(k+2) \\ \vdots \\ \dot{w}(k+m) \end{bmatrix}, \ldots$$

Acceleration Error: $\varepsilon_2(k) \mathrel{\hat{=}} \ddot{\mathbf{y}}(k) - \ddot{\mathbf{w}}(k)$

$$\ddot{\mathbf{y}}(k) := \begin{bmatrix} \hat{\ddot{y}}(k+1) \\ \hat{\ddot{y}}(k+2) \\ \vdots \\ \hat{\ddot{y}}(k+m) \end{bmatrix}, \ \ddot{\mathbf{w}}(k) := \begin{bmatrix} \ddot{w}(k+1) \\ \ddot{w}(k+2) \\ \vdots \\ \ddot{w}(k+m) \end{bmatrix},$$

Energy: $\beta(k) \mathrel{\hat{=}} \mathbf{u}_s(k) - \mathbf{w}(k)$

$$\mathbf{u}_s(k) \mathrel{\hat{=}} \begin{bmatrix} u_s(k) \\ u_s(k+1) \\ \vdots \\ u_s(k+m-1) \end{bmatrix}.$$

The searched control sequence is the vector $\mathbf{u}_s$ (k). Each vector error term in (2) embraces a prediction horizon of m samples. The design matrices $\mathbf{Q}_i$ and $\mathbf{R}$ weight the corresponding error. For diagonal $\mathbf{Q}_i$ and $\mathbf{R}$ the cost function J reduces to a weighted sum of quadratic errors that penalizes reference deviations in each state space variable (i.e., position, velocity, and acceleration) over the known m future sampling times. The choice of the elements in $\mathbf{Q}_i$ and $\mathbf{R}$ establishes a particular dynamic behavior of a ***FMP*** servo-controlled system.

The last vector term $\beta(k)$, the control variable displacement, stands in the cost function for energy limitation. It constrains the control signal $u_s(k)$ in the vicinity of the actual set point $w(k)$. It can be assumed, without loss of generality, that the gain of the linearized scalar system (1) is unitary (in steady state $u_s(k) = w(k)$).

The calculation of the optimal control sequence using a receding horizon leads to (Bauchspiess, 1995):

$$u_s(k) = \mathbf{m}_w^T \mathbf{w}(k) + \mathbf{m}_{\dot{w}}^T \dot{\mathbf{w}}(k) + \mathbf{m}_{\ddot{w}}^T \ddot{\mathbf{w}}(k) - \mathbf{m}_x^T \mathbf{x}(k)$$

where the vectors $\mathbf{m}_w^T, \mathbf{m}_{\dot{w}}^T, \mathbf{m}_{\ddot{w}}^T$ can be interpreted as digital filters of the reference signal and their velocity and acceleration. $\mathbf{m}_x^T$ correspond to a linear state feedback.

4. VISUAL GUIDED ROBOTS

For visual guided robots the reference trajectory is generated on-line using features extracted from the CCD-image.

This sampling of the reference trajectory in global coordinates, is an inherent error source that reduces the tracking performance. Sharp edges, that are described by two converging lines will be rounded. A significant reduction of this error can be obtained if the exact analytic trajectory is used in the joint coordinates level. If the trajectory is given in joint coordinates, we eliminate the cinematic couplings of the trajectory error, that is typical when using a global trajectory description.

To obtain this improvement in the sensor guided trajectory programming, we propose to transform each measured sensor value in joint coordinates and to evaluate the analytical fitting function directly in this coordinate system.

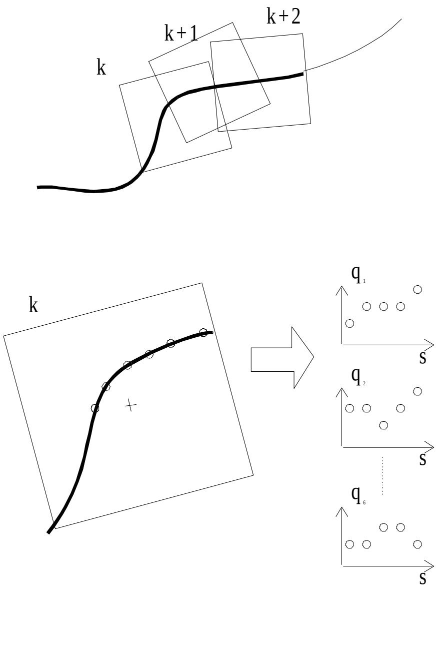

Figure 1 - CCD-Guided trajectory acquisition showing moving Vision Field and feature extraction.

The model (eq. 1), with n=3, will be used for robotic manipulators that are linearized and decoupled by an underlying non-linear multivariable joint controller. The resulting integrator chain that is obtained, is then transformed in a P-T_3 system by means of linear state feedback. Finally the discrete model (1) is obtained using a Step Invariant Transformation. That model will be here, therefore, denominated ***predictor model***, because it allows the prediction of the dynamic behavior of each robot joint for a given reference trajectory. Consequently the servocontroller will be designed for such SISO systems.

5. FINE INTEPOLATION USING POLYNOMIAL COEFFICIENTS

The use of cubic splines is well accepted in robotics to describe reference trajectories []. In this section the use of this analytical describing function in conjunction with the predictive servo control will be elucidated. From the measured sensor values transformed in joint coordinates fitting polynomials can be calculated for each joint. A predefined amount of values are aggregated using the *least squares criteria*. The trajectory of each joint is so a sequence of cubic polynomials segments. Each segment is described by:

$$w(t) = a_0 + a_1(t-t_i) + a_2(t-t_i)^2 + a_3(t-t_i)^3 .$$

The validity segment of the fitting polynomial comprises sampling point $t_i = k\cdot T$ to $t_p = (k+p)T$, with $k \le p - m$ (figure **2**). The reference velocity and acceleration are given by:

$$\begin{aligned} \dot{w}(t) &= a_1 + 2a_2(t-t_i) + 3a_3(t-t_i)^2 \\ \ddot{w}(t) &= 2a_2 + 6a_3(t-t_i). \end{aligned} \qquad (3)$$

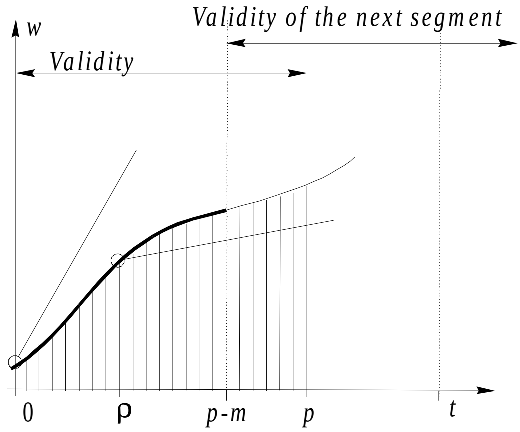

Figure 2 Incorporation of the time-axis shifting in the polynomial coefficients.

By sampling the analytic trajectory in the fine-interpolation sampling rate T we obtain the following reference vectors:

$$\mathbf{w}(k):=\begin{bmatrix} w(k+1)\\ w(k+2)\\ \vdots\\ w(k+p)\end{bmatrix}=\begin{bmatrix}1 & T & T^2 & T^3\\ 1 & 2T & 4T^2 & 8T^3\\ \vdots & \vdots & \vdots & \vdots\\ 1 & pT & (pT)^2 & (pT)^3\end{bmatrix}\begin{bmatrix}a_0\\ a_1\\ a_2\\ a_3\end{bmatrix}$$

$$\dot{\mathbf{w}}(k):=\begin{bmatrix} \dot w(k+1)\\ \dot w(k+2)\\ \vdots\\ \dot w(k+p)\end{bmatrix}=\begin{bmatrix}0 & 1 & 2T & 3T^2\\ 0 & 1 & 4T & 12T^2\\ \vdots & \vdots & \vdots & \vdots\\ 0 & 1 & 2pT & 3(pT)^2\end{bmatrix}\begin{bmatrix}a_0\\ a_1\\ a_2\\ a_3\end{bmatrix}, \text{ and}$$

$$\ddot{\mathbf{w}}(k):=\begin{bmatrix} \ddot w(k+1)\\ \ddot w(k+2)\\ \vdots\\ \ddot w(k+p)\end{bmatrix}=\begin{bmatrix}0 & 0 & 2 & 6T\\ 0 & 0 & 2 & 12T\\ \vdots & \vdots & \vdots & \vdots\\ 0 & 0 & 2 & 6pT\end{bmatrix}\begin{bmatrix}a_0\\ a_1\\ a_2\\ a_3\end{bmatrix}.$$

In the predictive control law (eq.) the digital pre-filter of the reference signal can be given explicitly in dependence of the fitting polynomial parameters. The predictive control law will be considered at $t = (k+\rho)T$:

$$u_s(k+\rho)=\mathbf{m}_{w1}^T\mathbf{w}(k+\rho)+\mathbf{m}_{\dot w1}^T\dot{\mathbf{w}}(k+\rho)+\mathbf{m}_{\ddot w1}^T\ddot{\mathbf{w}}(k+\rho)-\mathbf{m}_{x1}^T\mathbf{x}(k+\rho)$$

where ρ represents any sampling instant in the validity segment of the fitting polynomial. So, in general, as can easily be seen, for $\tau = (\rho+1)T$

$$\begin{bmatrix}1 & \tau & \tau^2 & \tau^3\\ 1 & 2\tau & 4\tau^2 & 8\tau^3\\ \vdots & \vdots & \vdots & \vdots\\ 1 & p\tau & (p\tau)^2 & (p\tau)^3\end{bmatrix}=$$

$$\begin{bmatrix}1 & T & T^2 & T^3\\ 1 & 2T & 4T^2 & 8T^3\\ \vdots & \vdots & \vdots & \vdots\\ 1 & pT & (pT)^2 & (pT)^3\end{bmatrix}\begin{bmatrix}1 & T & T^2 & T^3\\ 0 & 1 & 2T & 3T^2\\ 0 & 0 & 1 & 3T\\ 0 & 0 & 0 & 1\end{bmatrix}^{\rho}.$$

The matrix $P(\rho)=\begin{bmatrix}1 & T & T^2 & T^3\\ 0 & 1 & 2T & 3T^2\\ 0 & 0 & 1 & 3T\\ 0 & 0 & 0 & 1\end{bmatrix}^{\rho}$ (4)

describes a "polynomial-coefficient shift operator". So polynomial functions whose coefficients are referenced originally to the beginning of the segment can easily be shifted. The "polynomial-coefficient shift operator" can, of course, also be applied to the derivated polynomials. In the realization of the algorithm the matrix $P(\rho)$ can be very effectively calculated in a recursive manner, because from step to step only the six values, that are dependent on T, must be updated. After substitution of the reference vectors by the results of the polynomial evaluation we obtain

$u_s(k+\rho)=$

$$\mathbf{m}_{w1}^T\begin{bmatrix}1 & T & T^2 & T^3\\ 1 & 2T & 4T^2 & 8T^3\\ \vdots & \vdots & \vdots & \vdots\\ 1 & pT & (pT)^2 & (pT)^3\end{bmatrix}\begin{bmatrix}1 & T & T^2 & T^3\\ 0 & 1 & 2T & 3T^2\\ 0 & 0 & 1 & 3T\\ 0 & 0 & 0 & 1\end{bmatrix}^{\rho}\begin{bmatrix}a_0\\ a_1\\ a_2\\ a_3\end{bmatrix}+$$

$$\mathbf{m}_{\dot w1}^T\begin{bmatrix}0 & 1 & 2T & 3T^2\\ 0 & 1 & 4T & 12T^2\\ \vdots & \vdots & \vdots & \vdots\\ 0 & 1 & 2pT & 3(pT)^2\end{bmatrix}\begin{bmatrix}1 & T & T^2 & T^3\\ 0 & 1 & 2T & 3T^2\\ 0 & 0 & 1 & 3T\\ 0 & 0 & 0 & 1\end{bmatrix}^{\rho}\begin{bmatrix}a_0\\ a_1\\ a_2\\ a_3\end{bmatrix}+$$

$$\mathbf{m}_{\ddot w1}^T\begin{bmatrix}0 & 0 & 2 & 6T\\ 0 & 0 & 2 & 12T\\ \vdots & \vdots & \vdots & \vdots\\ 0 & 0 & 2 & 6pT\end{bmatrix}\begin{bmatrix}1 & T & T^2 & T^3\\ 0 & 1 & 2T & 3T^2\\ 0 & 0 & 1 & 3T\\ 0 & 0 & 0 & 1\end{bmatrix}^{\rho}\begin{bmatrix}a_0\\ a_1\\ a_2\\ a_3\end{bmatrix}-\mathbf{m}_{x1}^T\mathbf{x}(k+\rho),$$

resulting the predictive control law:

$$u(k+\rho)=\begin{bmatrix}b_0 & b_1 & b_2 & b_3\end{bmatrix}P(\rho)\begin{bmatrix}a_0\\ a_1\\ a_2\\ a_3\end{bmatrix}-\mathbf{m}_{x1}^T\mathbf{x}(k+\rho)$$

for $\rho = 1,2,\ldots p$ (5)

with $b_0=\sum_{\mu=1}^{m} m_{w1}(\mu)$, $b_1=\sum_{\mu=1}^{m} m_{w1}(\mu)\mu T+\sum_{\mu=1}^{m} m_{\dot w1}(\mu)$,

$b_2=\sum_{\mu=1}^{m} m_{w1}(\mu)(\mu T)^2+\sum_{\mu=1}^{m} m_{\dot w1}(\mu)2\mu T+2\sum_{\mu=1}^{m} m_{\ddot w1}(\mu)$ and

$b_3=\sum_{\mu=1}^{m} m_{w1}(\mu)(\mu T)^3+\sum_{\mu=1}^{m} m_{\dot w1}(\mu)3(\mu T)^2+\sum_{\mu=1}^{m} m_{\ddot w1}(\mu)6\mu T.$

The result obtained in equation (5) show a direct way to adequate the sampling rate of the vision system and that used in the control loop so that the tracking error is minimized. The procedure for the optimized path tracking can thus be summarized as follows:

1) Scan image for features that represent trajectory points,
2) Transformed trajectory points from camera to joint coordinates,
3) Interpolate joint points with a spline that describes the trajectory for the current frame (coefficients a_0, a_1, a_2, a_3)
4) Generate the control signal in the joint control loop that minimizes the trajectory error using equation (5).

6. SIMULATION RESULTS

The simulation of the ***FMP*** servo-control for a linearized robot joint described by

$$F(s) = \frac{13824}{s^3 + 72s^2 + 1728s + 13824},$$

and a typical trajectory is shown in figure **3**. The reference signal w and the output of the plant y are almost identical, the control signal antecede (predict) the plant lag dynamics, as seen in fig. 2-a. The small residual error can be seen in fig. 2-b. As expected, in the vicinity of reference velocity discontinuities occurs a greater error.

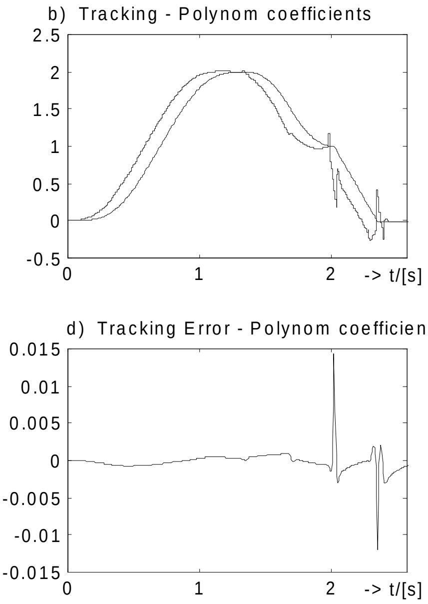

Figure 3 Predictive path-tracking using Polynom Coefficients, Design parameters: q_1=2e-3, q_2=1e-8, r=1.5e-3, m=5, p=15, T=6ms.

For smooth trajectories (splines), the tracking error can be virtually eliminated by using a control signal that exactly anticipates the lag of each robot joint. Hence, in the proposed algorithm for vision guided robots the trajectory information extracted from the image is approximated by splines.

7. EXPERIMENTAL RESULTS

A hydraulic robot with two degrees of freedom equipped with a very fast CCD-camera was used to validate the proposed approach. The tracking trajectory was printed on paper fixed on a plate parallel to the working area of the robot, such that different trajectories could easily be tested. After finding a border the control scheme guide the robot along this border with a pre-established TCP velocity.

The axis control of the robot was implemented on a DSP32C, 32 bit floating point digital signal processor, with 0.5 ms sampling rate. An exact linearization (computed torque) plus a state-space controller established the dynamics of each axis. Another DSP32C with a 16bit fixed-point DSP16 were used to process the image and implement the predictive servo-control.

Separation of Path-Tracking and Disturbance Rejection Performance: Relative good decoupling and linearization of each robot joint can be obtained by using the computed torque control law (in most practical cases only the three first axis are controlled in this way) . After that a state-space controller is used to establish the closed-loop dynamics for each joint. This “regulating-dynamics” is the dynamics with which disturbances are rejected. Path-tracking (servo) dynamics can, however, be much faster. This can be obtained by separating the regulating and the servo dynamics. The proposed algorithm for vision guided robots leads to a feed-forward pre-filter, which is by construction not affected by disturbances. Thus, a very fast dynamics can be adopted for the servo-controller without peril for the stability of the system.

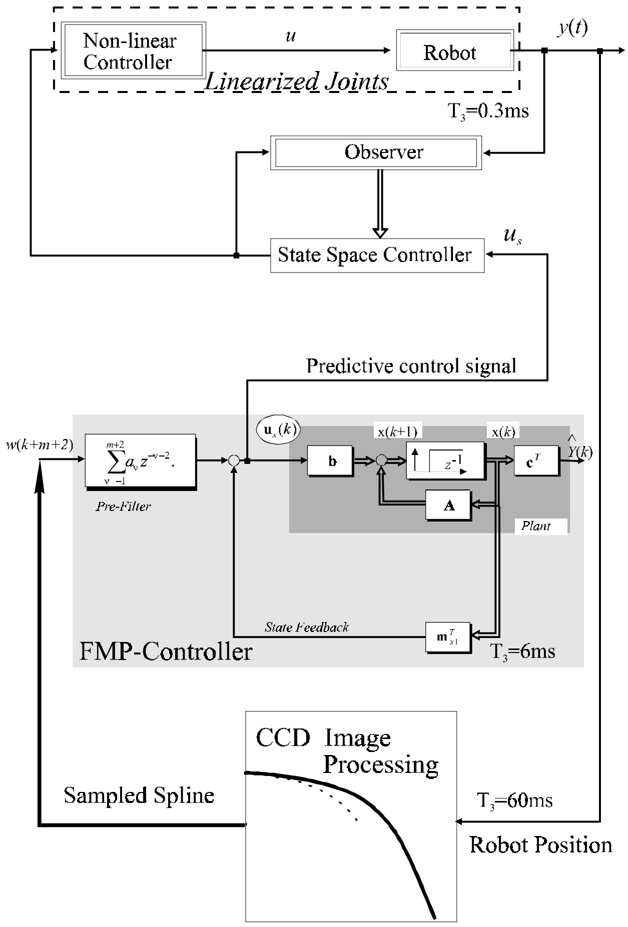

Figure 4 - *FMP* control of CCD guided robot.

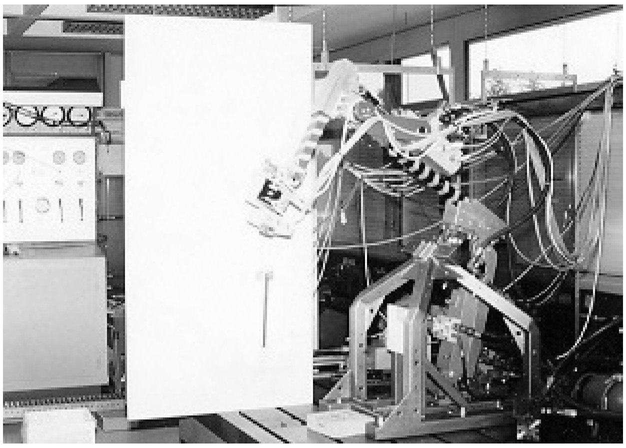

Figure 5-Hydraulic manipulator guided by a CCD-camera.

The implemented control scheme, **Figure 3**, shows the ***FMP*** controller as a feed-forward channel, that allows ***separation*** of path tracking and disturbance rejection characteristics. The slower nonlinear and state space dynamics of the robot is designed for disturbance rejection, while the reference tracking is enhanced by the fast predictive dynamics of the ***FMP*** controller. As a stable system by itself, the feed-forward ***FMP*** filter will not affect the stability of the robot joint control scheme.

From each image 10 border points in tracking direction are extracted. After a path length parameterization and a transformation in joint coordinates a spline fitting is employed. This analytical spline is then sampled to furnish new trajectory points to the ***FMP*** as suggested in Figure 4. For each image 200 points are generated for each robot joint.

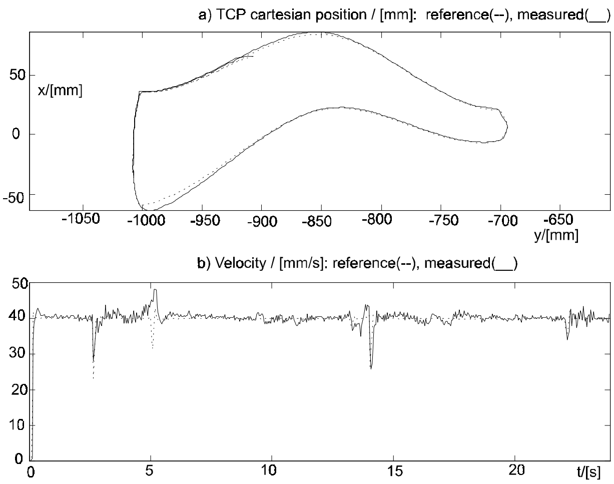

Figure 6 Sensor guided trajectory tracking for a typical trajectory.
Sampling period: T = 10 *ms*, Velocity: v_{ref} = 40 *mm/s*.

Figure **6** presents measures for a test trajectory for the predictive CCD-guided hydraulic robot. In this case the angular error is smaller than 0.3 degree.

8. CONCLUSION

A novel algorithm that minimize the tracking error of sensor guided robots was presented. An efficient predictive scheme that evaluates fitting splines (obtained from the vision sensor) in the control loop sampling rate was described. Simulation results for the polynomial approximation were discussed and experimental results for a hydraulic robot were presented.

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