
An introduction to models based on Laguerre, Kautz and other related orthonormal functions – part I: linear and uncertain models

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Abstract: This paper provides an overview of system identification using orthonormal basis function models, such as those based on Laguerre, Kautz, and generalised orthonormal basis functions. The paper is separated in two parts. In this first part, the mathematical foundations of these models as well as their advantages and limitations are discussed within the context of linear and robust system identification. The second part approaches the issues related with non-linear models. The discussions comprise a broad bibliographical survey of the subjects involving linear models within the orthonormal basis functions framework. Theoretical and practical issues regarding the identification of these models are presented and illustrated by means of a case study involving a polymerisation process.

Keywords: modelling; system identification; robust identification; orthonormal basis functions; OBF; linear systems.

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1 Introduction

The increasing complexity of production processes and the demand for high efficiency in industrial plants have imposed high performance goals on their associated control systems. Many strategies have been proposed to improve the performance of these systems. Among the most successful are the ones that use mathematical models of dynamic processes, such as model-based predictive controllers (MBPC) (Garcia et al., 1989; Soeterboek, 1992; Clarke, 1994; Camacho and Bordons, 1999; Henson, 1998; Allgower and Zheng, 2000; Balbis et al., 2006; Zhu, 2006). In MBPC, modelling plays an important role in the prediction of the dynamic behaviour of the process, since it allows the controller to make a decision in advance based on optimality criteria.

These MBPC calculate the control input by minimising a cost function over a future time horizon under certain process constraints. The closed-loop performance depends on the choice of an appropriate prediction model and on several tuning parameters. In most cases, a single linear model is adopted to describe the behaviour of the process, but this involves only an approximation; hence, great uncertainty in the value of the process parameters can result.

So, during the design stage, it is very important to consider the effect of these uncertainties on both the optimality and the stability of the closed-loop system.

Models of dynamic systems are usually constructed using the well-known auto-regressive with exogenous inputs – ARX – structure, in which the system output in a given discrete-time instant is represented in terms of past input and output (I/O) values (van den Bosch and van der Klauw, 1994; Sjöberg et al., 1995; Ljung, 1999; Nelles, 2001). Although this approach has several advantages, such as allowing parsimonious representations of unstable systems (infinite memory) (Aguirre et al., 2002), one of its main drawbacks is that the auto-regressive aspect generally increases the sensitivity regarding the choice of the model order. This characteristic generates a recursion of errors that can damage the quality of the prediction, especially for long-range prediction horizons.

In order to lessen the previously mentioned drawbacks, one approach of particular interest consists of using models without output feedback. The best known such models are the finite impulse response – FIR – models, in which the system output in a given discrete-time instant is represented only in terms of past samples of the input. The absence of

output recursion in such FIR models, however, has the drawback of usually requiring a large number of terms in the regression vector, especially when representing slow dynamics. The number of terms of the regression is proportional to the number of unknown model parameters, and this relation is exponential for most general-purpose non-linear models, as will be seen in subsequent sections. Models without output feedback that circumvent this drawback are the so-called orthonormal basis functions (OBF) models (Heuberger et al., 2005).

The Laguerre and Kautz basis functions (Broome, 1965; Wahlberg, 1991, 1994; Wahlberg and Mäkilä, 1996) are the most commonly used basis functions in the approximation of signals and systems and are, respectively, preferred for modelling systems with first and second-order dominant dynamics. To model systems with more complex dominant dynamics, basis functions constructed with more than one dynamic, such as the generalised orthonormal basis functions (GOBF) (Heuberger et al., 1995; Van den Hof et al., 1995) can be more appropriate. These functions have been widely used in the context of the identification and control of linear and non-linear systems (Schetzen, 1980; Dumont and Fu, 1993; Heuberger et al., 1995; Ninness and Gustafsson, 1997; Oliveira et al., 2000; Doyle et al., 2002; Heuberger et al., 2005), especially because the corresponding dynamic models have several desirable characteristics, such as the following (Dumont and Fu, 1993; Ninness and Gustafsson, 1995; Nelles, 2001; Heuberger et al., 2005):

- 1 there is no output recursion or feedback of prediction errors, which often leads to superior performance over long-range predictions and a natural decoupling between multiple outputs
- 2 prior knowledge of the relevant past terms of the system signal is not necessary, this represent an advantage particularly for non-linear systems where such knowledge is barely available
- 3 the representational capability of the models can be improved by simply increasing the number of functions in the orthonormal basis
- 4 the representation of a stable system is assuredly stable
- 5 these models are able to deal with time delays and are tolerant of unmodelled dynamics.

This paper presents an overview of the state-of-the-art in the identification of dynamic systems using OBF models. Issues related with non-linear OBF models are approached on the second part of this paper (Oliveira et al., forthcoming). For simplicity but without loss of generality, only the single-input single-output (SISO) case will be considered.

The outline of this paper is as follows. Section 2 discusses the problem of identifying linear dynamic systems using an OBF-based framework, including the robust identification of models with parametric uncertainties. Section 3 discusses several methodologies for the design of OBF that parameterise the corresponding models. Section 4 describes a case study involving an isothermic

polymerisation process. Finally, Section 5 addresses the conclusions.

2 Linear system identification

The idea behind dynamic linear models with an OBF framework is grounded in the completeness property of these orthonormal bases. This property ensures that any function of the Lebesgue space $\ell^2[0, \infty)$ can be approximated with arbitrary accuracy by a linear combination of functions of these bases. In other words, for any quadratically summable function $h(k): \mathbb{N} \rightarrow \mathbb{R}$ on

$[0, \infty)$, that is, $\sum_{k=0}^{\infty} h(k)^2 < \infty$, there exists an integer $n > 0$ such that for any $\epsilon > 0$ then

$$\sum_{k=0}^{\infty} \left(h(k) - \sum_{i=1}^n c_i \psi_i(k) \right)^2 < \epsilon, \quad (1)$$

where $\{\psi_1(k), \dots, \psi_n(k)\}$ is the set containing the first n orthonormal functions of the basis, and c_1, \dots, c_n are scalars. So, the representation $\hat{h}(k) = \sum_{i=1}^n c_i \psi_i(k)$ converges to $h(k)$ and it is exact for infinite n , that is:

$$h(k) = \sum_{i=1}^{\infty} c_i \psi_i(k). \quad (2)$$

For any integer $j > 0$, it follows from (2) that

$$\begin{aligned} \sum_{k=0}^{\infty} h(k) \psi_j(k) &= \sum_{k=0}^{\infty} \psi_j(k) \sum_{i=1}^{\infty} c_i \psi_i(k) \\ &= \sum_{i=1}^{\infty} c_i \sum_{k=0}^{\infty} \psi_j(k) \psi_i(k), \end{aligned} \quad (3)$$

and, using the orthonormality property of the functions

$$\sum_{k=0}^{\infty} \psi_i(k) \psi_j(k) = \begin{cases} 1 & \text{if } i = j \\ 0 & \text{if } i \neq j \end{cases}, \quad (4)$$

one then has from (3) that the coefficients of the series expansion are given by

$$c_j = \sum_{k=0}^{\infty} h(k) \psi_j(k). \quad (5)$$

The idea of the OBF framework in dynamic models is to represent the system's impulse response using a set of orthonormal functions. This representation is possible for bounded-input bounded-output (BIBO) stable dynamic systems whose impulse responses are absolutely (and therefore quadratically) summable. It is important to note that systems with integrators have impulse response with infinite energy and therefore fall outside the preceding requirement. Since, in general, the presence of integrators in a real system is known, one can model the variation of the system output instead of its absolute value, which

is equivalent to removing the integrator(s) from the identification loop.

The convolution equation describing a linear, causal, time-invariant dynamic system is given by $y(k) = \sum_{\tau=0}^k h(\tau)u(k-\tau)$, where $u(k)$ is the discrete-time input, $y(k)$ is the discrete-time output, and $h(\tau)$ is the impulse response of the system. The approximate expansion of $h(\tau)$ using n orthonormal functions is performed so that the convolution model is represented as:

$$\begin{aligned} \hat{y}(k) &= \sum_{\tau=0}^k \hat{h}(\tau)u(k-\tau), \\ &= \sum_{\tau=0}^k \sum_{i=1}^n c_i \psi_i(\tau)u(k-\tau), \\ &= \sum_{i=1}^n c_i \sum_{\tau=0}^k \psi_i(\tau)u(k-\tau), \\ &= \sum_{i=1}^n c_i l_i(k), \end{aligned} \quad (6)$$

where $l_i(k)$ is the convolution of the input signal $u(k)$ with the i^{th} orthonormal function $\psi_i(k)$. Since each function $\psi_i(k)$ of the orthonormal basis, from its Z -transform, can be associated with a transfer function representing a linear filter $\Psi_i(z)$, the term $l_i(k)$ is merely the result of filtering the input signal $u(k)$ by the function $\Psi_i(z)$. In other words, $l_i(k) = \Psi_i(q)u(k)$, where $\Psi_i(q)$ is the transfer function of the i^{th} orthonormal function represented in the shift operator q , where $qu(k) \triangleq u(k+1)$.

The OBF that are most commonly used in signal and system representations are presented below.

2.1 Orthonormal basis functions

The use of orthonormal filters to represent signals and systems has a long history, since the pioneering proposals of Takenaka (1925) and Wiener (1958). Discrete-time OBF can be generated by cascading different all-pass filters of order one or two, as follows (Ninness and Gustafsson, 1997; Heuberger et al., 2005):

$$\Psi_i(z) = \frac{z\sqrt{1-|\beta_i|^2}}{z-\beta_i} \prod_{j=1}^{i-1} \left(\frac{1-\bar{\beta}_j z}{z-\beta_j} \right), \quad i=1, 2, \dots, \quad (7)$$

where β_i are stable poles of the orthonormal basis ($\beta_i \in \mathbb{C} : |\beta_i| < 1$) and $\bar{\beta}_i$ denotes the complex conjugate of β_i . The functions in (7) are the so-called Takenaka-Malmquist functions (Heuberger et al., 2005). The corresponding realisations in the time-domain, $\psi_i(k)$, are given by the inverse Z -transform of (7) and satisfy the orthonormality property. The set $\{\psi_i\}$ is complete on $\ell^2[0, \infty)$ if and only if $\sum_{i=1}^{\infty} (1-|\beta_i|) = \infty$ (Ninness and Gustafsson, 1997; Heuberger et al., 2005), so any finite energy signal (including absolutely summable functions) can be

approximated with any prescribed accuracy by linearly combining a certain finite number of such functions. In general, functions $\psi_i(k)$ are complex-valued, although this is physically unrealistic in system identification problems. It is shown (Ninness and Gustafsson, 1997) that this drawback can be circumvented by constructing a modified basis of functions with real-valued impulse responses consisting of a linear combination of the complex-valued functions generated by (7).

When all the poles of (7) are real-valued and equal to each other, that is, $\beta_i = \bar{\beta}_i = p$, one obtains the Laguerre basis, which can be written in the z -domain as (Fu and Dumont, 1993; Oliveira e Silva, 1994; Belt and den Brinker, 1995; Tanguy et al., 1995):

$$\Psi_i(z) = \frac{z\sqrt{1-p^2}}{z-p} \left(\frac{1-pz}{z-p} \right)^{i-1}, \quad (8)$$

with $p \in \{\mathbb{R} : |p| < 1\}$ denoting the Laguerre pole. By setting $p = 0$, the Laguerre functions simplify to an ordinary pulse basis $\Psi_i(z) = z^{-(i-1)}$, which implies that the model in (6) reduces to the classical FIR model (Ljung, 1999).

Another important OBF realisation, which has also been shown to be a particular case of the unifying construction (7) (Ninness and Gustafsson, 1997), is obtained by cascading an all-pass filter with a pole at β and an all-pass filter with a pole at $\bar{\beta}$, in such a way that the pairs of conjugate poles are equal to each other for any value of i , that is, $\{\beta, \bar{\beta}, \beta, \bar{\beta}, \dots\}$. The result is the two-parameter Kautz functions, defined in the z -domain as (Wahlberg, 1994; Tanguy et al., 2002; Heuberger et al., 2005):

$$\begin{aligned} \Psi_{2i}(z) &= \frac{z\sqrt{(1-\alpha_2^2)(1-\alpha_1^2)}}{z^2 + \alpha_1(\alpha_2-1)z - \alpha_2} \\ &\quad \left[\frac{-\alpha_2 z^2 + \alpha_1(\alpha_2-1)z + 1}{z^2 + \alpha_1(\alpha_2-1)z - \alpha_2} \right]^{i-1}, \\ \Psi_{2i-1}(z) &= \frac{z(z-\alpha_1)\sqrt{1-\alpha_2^2}}{z^2 + \alpha_1(\alpha_2-1)z - \alpha_2} \\ &\quad \left[\frac{-\alpha_2 z^2 + \alpha_1(\alpha_2-1)z + 1}{z^2 + \alpha_1(\alpha_2-1)z - \alpha_2} \right]^{i-1}, \end{aligned} \quad (9)$$

where scalars α_1 and α_2 are real-valued parameters satisfying $|\alpha_1| < 1$ and $|\alpha_2| < 1$. These parameters are related to the pair of Kautz poles ($\beta, \bar{\beta}$) as

$$\alpha_1 = (\beta, \bar{\beta}) / (1 + \beta\bar{\beta}), \quad (10)$$

$$\alpha_2 = -\beta\bar{\beta}, \quad (11)$$

Laguerre and Kautz bases are preferred when modelling systems with first- and second-order dominant dynamics, respectively. Systems with more complex dominant dynamics are better represented using models based on GOBF because the mathematical description of such bases

involves multiple poles (modes). Although the term *generalised OBF* is originally due to the formula by Van den Hof et al. (1995) and Heuberger et al. (1995), this term will be used generically hereafter to refer to orthonormal bases of functions with multiple modes.

It is important to notice that the insertion of dynamics into the orthonormal filters in (7) allows the incorporation of prior knowledge about the dynamics of the system (Ninness and Gustafsson, 1995, 1997; Nelles, 2001). In fact, if the parameterisation of the basis is set close to the dominant modes of the system, then an accurate approximation can be obtained with fewer coefficients (see Section 3).

It is also worth noting that, as discussed previously, the OBF defined in (7) have complex-valued inverse Z-transforms when they are designed with complex-valued poles. This drawback can be circumvented by constructing a modified basis of functions with complementary pairs of real-valued impulse responses (Ninness and Gustafsson, 1997). Particularly, let us consider the basis functions in (7) parameterised by real-valued poles only, that is, $\beta_i = \bar{\beta}_i \triangleq p_i$ for $i = 1, 2, \dots$. In this case, one has:

$$\Psi_i(z) = \frac{z\sqrt{1-p_i^2}}{z-p_i} \prod_{j=1}^{i-1} \left(\frac{1-p_j z}{z-p_j} \right). \quad (12)$$

Then, let us initially consider an OBF-based model with the first $n-1$ basis functions $\{\Psi_1(z), \Psi_2(z), \dots, \Psi_{n-1}(z)\}$ parameterised by their respective $n-1$ real-valued poles, given by the set $\{p_1, p_2, \dots, p_{n-1}\}$, precisely as in (12). According to Ninness and Gustafsson (1997), if it is desired to include a complex-valued pole β_n into this set of poles, then two modified functions $\Psi'_n(z)$ and $\Psi''_n(z)$ with real-valued impulse responses must be constructed as a linear combination of $\Psi_n(z)$ and $\Psi_{n+1}(z)$ in (7). In this case, the new set of functions will be $\{\Psi_1(z), \dots, \Psi_{n-1}(z), \Psi'_n(z), \Psi''_n(z)\}$, with the corresponding set of poles $\{p_1, \dots, p_{n-1}, \beta_n, \bar{\beta}_n\}$. The pair of modified functions is given by Ninness and Gustafsson (1997):

$$\begin{aligned} \Psi'_n(z) &= \frac{z\sqrt{1-|\beta_n|^2}(\lambda'z + \gamma')}{z^2 - (\beta_n + \bar{\beta}_n)z + |\beta_n|^2} \prod_{j=1}^{n-1} \left(\frac{1-p_j z}{z-p_j} \right), \\ \Psi''_n(z) &= \frac{z\sqrt{1-|\beta_n|^2}(\lambda''z + \gamma'')}{z^2 - (\beta_n + \bar{\beta}_n)z + |\beta_n|^2} \prod_{j=1}^{n-1} \left(\frac{1-p_j z}{z-p_j} \right), \end{aligned} \quad (13)$$

where $\beta_n \in \mathbb{C}$ is the complex-valued pole included in the GOBF and $\lambda', \gamma', \lambda'', \gamma''$ are real-valued parameters that relate to β_n as follows:

$$\begin{bmatrix} \lambda' & \gamma' \end{bmatrix} \begin{bmatrix} 1+|\beta_n|^2 & \beta_n + \bar{\beta}_n \\ \beta_n + \bar{\beta}_n & 1+|\beta_n|^2 \end{bmatrix} \begin{bmatrix} \lambda' \\ \gamma' \end{bmatrix} = |1-\beta_n^2|^2, \quad (14)$$

$$\begin{bmatrix} \lambda'' \\ \gamma'' \end{bmatrix} = \frac{1}{\sqrt{1-\xi^2}} \begin{bmatrix} \xi & 1 \\ -1 & -\xi \end{bmatrix} \begin{bmatrix} \lambda' \\ \gamma' \end{bmatrix}, \quad (15)$$

$$\xi = \frac{\beta_n + \bar{\beta}_n}{1+|\beta_n|^2}, \quad (16)$$

There exist infinite solutions for the choice of the real-valued parameters $\lambda', \gamma', \lambda'', \gamma''$ satisfying (14) to (16). Any choice of such parameters satisfying these equations will assure the orthonormality of the basis (Ninness and Gustafsson, 1997; Ziaei and Wang, 2006).

If it is desired to include an additional pair of complex conjugate poles $\beta_{n+1}, \bar{\beta}_{n+1}$ into the GOBF, two additional functions $\Psi'_{n+1}(z)$ and $\Psi''_{n+1}(z)$ with real-valued impulse responses must be constructed as a linear combination of $\Psi_{n+2}(z)$ and $\Psi_{n+3}(z)$ in (7) (Ninness and Gustafsson, 1997). In this case, the new set of functions will be $\{\Psi_1(z), \dots, \Psi_{n-1}(z), \Psi'_n(z), \Psi''_n(z), \Psi'_{n+1}(z), \Psi''_{n+1}(z)\}$, with the corresponding set of poles $\{p_1, \dots, p_{n-1}, \beta_n, \bar{\beta}_n, \beta_{n+1}, \bar{\beta}_{n+1}\}$. After some algebraic manipulations, one obtains the two new functions given by:

$$\begin{aligned} \Psi'_{n+1}(z) &= \frac{z\sqrt{1-|\beta_{n+1}|^2}(\rho'z + \mu')}{z^2 - (\beta_{n+1} + \bar{\beta}_{n+1})z + |\beta_{n+1}|^2} F(z), \\ \Psi''_{n+1}(z) &= \frac{z\sqrt{1-|\beta_{n+1}|^2}(\rho''z + \mu'')}{z^2 - (\beta_{n+1} + \bar{\beta}_{n+1})z + |\beta_{n+1}|^2} F(z), \end{aligned} \quad (17)$$

with

$$F(z) = \left(\frac{1-\bar{\beta}_n z}{z-\beta_n} \right) \cdot \left(\frac{1-\beta_n z}{z-\bar{\beta}_n} \right) \prod_{j=1}^{n-1} \left(\frac{1-p_j z}{z-p_j} \right) \quad (18)$$

where $\rho', \mu', \rho'', \mu''$ are real-valued parameters that relate to β_{n+1} in the same way as $\lambda', \gamma', \lambda'', \gamma''$ relate to β_n . To include an arbitrary number of pairs of complex conjugate poles into the GOBF, the reasoning presented previously is repeated.

The OBF are recursive, which means that the i_{th} function can be written in terms of the $(i-1)^{\text{th}}$ one. It is then possible to describe the dynamics of the set of orthonormal functions using a state-space representation. In this case, the model in (6) can be represented as follows:

$$\mathbf{l}(k+1) = A\mathbf{l}(k) + \mathbf{b}u(k), \quad (19)$$

$$\hat{y}(k) = \mathcal{H}(\mathbf{l}(k)), \quad (20)$$

where the state vector $\mathbf{l}(k) = [l_1(k) \dots l_n(k)]^T$ is composed by the outputs of the orthonormal filters and \mathcal{H} is the static mapping given by the linear combination of these states, that is:

$$\mathcal{H}(\mathbf{l}(k)) = \sum_{i=1}^n c_i l_i(k). \quad (21)$$

The matrices A and \mathbf{b} in (19) depend solely upon the orthonormal basis. A description of these matrices is found in the literature for Laguerre and Kautz, for instance (Dumont and Fu, 1993).

Once the basis pole is chosen (see Section 3), the model becomes completely determined by the coefficients $c_{(i)}$ of the orthonormal series in (21). If the impulse response of the system is available, these coefficients can be computed analytically by using (5). Note that the impulse response of a BIBO-stable system necessarily vanish (or asymptotically tends to zero) with time, which allows the truncation of equation (5). Although this non-parametric approach is simple and mathematically well founded, it may not, however, be effective in practical problems since the impulse response of the system is usually not available. A more efficient approach involves considering the coefficients c_i as parameters to be estimated numerically using I/O data from the system, which can be performed in a simple way using, for example, a least-squares algorithm (Ljung, 1999).

Finally, it is worth remarking that, although no transport delay is explicitly represented in the previous model description, the orthonormal functions are able to represent dynamics with this characteristic (Mäkilä, 1990; Fu and Dumont, 1993). Nevertheless, any information available about the real delay of the system can be explicitly incorporated into the model, which allows reducing the number of functions and filters necessary for modelling the system with any prescribed accuracy. This feature can be obtained by replacing $u(k)$ with $u(k - \tau_d)$ in (19), where τ_d is the estimated delay.

2.2 Robust identification

In certain cases, a single linear model cannot adequately represent a complex system. The presence of external disturbances, for example, can rule out obtaining a single set of parameters that ensures the model will be a good representation of the system. To deal with such cases, the usual procedure consists of incorporating uncertainties associated with the model parameters. Models with parametric uncertainties are the basis of many control algorithms, the so-called robust algorithms. Robust controllers using OBF-based models are described in the literature (Oliveira et al., 2000; Araújo and Oliveira, 2009). In OBF-based models, such as those in equations (19) to (21), the problem of identifying models with uncertain parameters has been addressed in the linear case (Wahlberg and Ljung, 1992; Akçay and Ninness, 1998) and in the Wiener and Hammerstein cases (Figueroa et al., 2008; Biagiola and Figueroa, 2009).

Next, two approaches are presented for the robust estimation of the parameters in OBF-based models and their corresponding uncertainties (Oliveira, 1997; Oliveira et al., 1998, 2000). These approaches assume that the model uncertainty can be expressed as follows:

$$\hat{y}(k) = \sum_{i=1}^n c_i l_i(k) = \mathbf{c}^T \mathbf{I}(k). \quad (22)$$

Defining $c_i(\varepsilon_i)$ as the model uncertain parameters, one obtains:

$$\hat{y}(k, \varepsilon) = \sum_{i=1}^n c_i(\varepsilon_i) l_i(k) = \mathbf{c}(\varepsilon)^T \mathbf{I}(k). \quad (23)$$

where ε is a vector whose i^{th} element is ε_i , which, in turn, represents the uncertainty relative to the parameter c_i . The uncertain parameters $c_i(\varepsilon_i)$ characterise the process dynamics.

The first approach involves a set of I/O data measured from the system and the use of robust identification methods based on the unknown-but-bounded-error (UBBE) approach (Milanese and Belforte, 1982). The second approach considers prior knowledge of the uncertainty of the model associated with the impulse response of the process. An algorithm based on this latter approach for computing the uncertainty bounds is presented in the literature (Moreira, 2006).

In the context of robust control, the representation of model uncertainties is often approximated by geometric shapes within the space formed by the parameters, for example, polytopes, orthotopes, or ellipsoids (Walter and Piet-Lahanier, 1990; Favier and Arruda, 1996). The approaches discussed here represent the space of the parametric uncertainties as an orthotope, that is, the model's uncertainty is associated with the maximum and the minimum values for each parameter, i.e., $c_i(\varepsilon_i) \in [c_{i,\min}, c_{i,\max}]$. The robust identification problem with Laguerre models using ellipsoidal approximations has already been investigated (Wahlberg and Ljung, 1992).

The UBBE robust identification problem is formulated as follows. Let the model of a certain process be given by (22). For a given set of parameters c_i , the error between the model and process outputs is given by

$$e(k) = y(k) - \hat{y}(k), \quad (24)$$

where $y(k)$ is the process output and $\hat{y}(k)$ is the model output for the set of parameters c_i . Then, consider that $e(k)$ satisfies

$$e(k) \in \min[e_{\min}(k), e_{\max}(k)], \quad (25)$$

where $e_{\min}(k)$ and $e_{\max}(k)$ are, respectively, the lower and upper bounds of the model error at the time instant k . These bounds are assumed to be known or can be determined from experimental data. These bounds can even be arbitrarily chosen but this approach involves the following two risks:

- 1 if the bounds are underestimated, the problem of the robust identification of the parameters can have no solution
- 2 if the bounds are over-estimated, the solution can be very conservative.

So, given an initial set S^* defined so as to contain all the possible values for the parameters c_i , the robust identification problem consists of finding a subset of S^* , given by S , containing values of c_i that are consistent with equations (22), (24), and (25). The solution of this problem

is the identification of a set S having feasible values for the parameters c_i of model (22).

Let N be the number of input $u(k)$ and output $y(k)$ measurements available from the process. From the state-space representation of the orthonormal basis, the vector $\mathbf{l}(k)$ in (19) can be calculated for $k = 1, \dots, N$. Then, using a set of available I/O measurements and considering an exact representation of the polytope generated by S , one obtains the following:

$$S = \left\{ \mathbf{c} : y(k) - e_{\min}(k) \leq \mathbf{c}^T \mathbf{l}(k) \leq y(k) + e_{\max}(k), \right. \\ \left. k = 1, \dots, N \right\}, \quad (26)$$

and $\mathbf{c}(\varepsilon)$ in (23) represents all the vectors \mathbf{c} such that $\mathbf{c} \in S$.

Based on the definition of the set S in (26), the bounds of the uncertain parameters $\mathbf{c}(\varepsilon)$ can be calculated using different robust identification algorithms found in the literature (Milanese and Belforte, 1982; Mo and Norton, 1990; da Silva, 1995; Akçay and At, 2006). The computation of the polytope generated by S can become complicated as the number N of measurements increases. This drawback is circumvented in Milanese and Belforte (1982), where an algorithm for determining an approximation in the form of an orthotope O exterior to S , i.e., $S \subset O$, has been proposed. In this algorithm, $e(k)$ is assumed to be such that $|e(k)| \leq e_{\max}$ and every pair of bounds, $c_{i,\min}$ and $c_{i,\max}$, of the range of the uncertain parameter $c_i(\varepsilon_i)$ is associated with a linear programming problem. The constraints of this problem are the domain of $\mathbf{c}(\varepsilon)$, that is, S . Therefore, the computation of the orthotope O exterior to S requires solving $2n$ linear programming problems, each one with $2N$ constraints.

The second approach of robust identification (Oliveira, 1997) considers a set of M realisations of the impulse response that represents the uncertainty of the process, i.e., $h_m(k)$, $m = 1, \dots, M$. Assuming that the space formed by the uncertain parameters of the model is an orthotope, then each $c_i(\varepsilon_i)$ coefficient can be represented by means of a median value \bar{c}_i and an absolute value of the maximum deviation (ε_i) with respect to the median value. Mathematically, one has:

$$c_i(\varepsilon_i) = \bar{c}_i + \varepsilon_i \Delta c_i, \quad (27)$$

with $|\varepsilon_i| \leq 1$, that is, $c_i(\varepsilon_i) \in [c_{i,\min}, c_{i,\max}]$. Using equation (5), each impulse response of the process can be associated to a given model as:

$$c_{i,m} = \sum_{k=0}^{\infty} h_m(k) \psi_i(k), \quad (28)$$

Thus, from each element $c_{i,m}$ ($m = 1, \dots, M$), the bounds $[c_{i,\min}, c_{i,\max}]$ of each parameter i are given by:

$$c_{i,\min} = \min_{m=1, \dots, M} c_{i,m}, \\ c_{i,\max} = \max_{m=1, \dots, M} c_{i,m}. \quad (29)$$

It is worth remarking that the positive characteristics of the identification of OBF models cited previously are still valid in the context of robust identification presented in this section. One of these characteristics is that there is no need of predefining the order or delay of the process, which represents an advantage when compared to ARX or ARMAX models. Furthermore, compared to the FIR models, the reduction in the number of parameters to be estimated also reduces the complexity of the robust identification algorithm. Extensions of this latter methodology (Oliveira, 1997; Moreira, 2006) toward non-linear models have already been addressed (da Rosa, 2009).

3 Design of OBF

As discussed in Section 2, the OBF are complete on $\ell^2[0, \infty)$, which leads to two fundamental implications. The first one is that the number of functions in a truncated basis, n , represents a trade-off between the accuracy and parsimony of the model. The actual value for n required to provide an accurate approximation, however, also depends on the complexity of the specific problem at hand. Dynamic systems with multiple dominant modes, for example, typically require models with a larger number of functions. The second implication regarding the completeness of the basis is that the parameterisation of the set of functions is not critical. This condition means that for any stable basis pole β , there exists a corresponding number n of functions providing a certain capability of approximation. For a given number of functions, the underlying problem is how to select the poles that parameterise the basis functions so as to maximise the model's accuracy.

Often, a basis functions can be constructed from a given collection of rational orthonormal functions and the problem of selecting the best orthonormal rational basis from such a collection can be addressed (Bodin et al., 2000). When properly selected, the poles of an orthonormal basis can increase the speed of convergence of the series that approximates the model dynamics, thus allowing a reduction of the number of basis functions and, accordingly, an improvement of the estimator properties in identification problems (Van den Hof et al., 1995; Heuberger et al., 1995). In addition, the sensitivity of the estimators to measurement noise is affected by the poles. In this context, Ninness et al. (1999) presented accurate expressions for the estimate of the variance errors that highlight the role of orthonormal bases in system identification and the importance of appropriate procedures for pole selection.

The choice of the best Laguerre poles has been extensively addressed and is well established in the literature. The first study concerning this subject (Clowes, 1965) optimised the performance of models based on Laguerre functions in the case of linear continuous-time systems. Later (Masnadi-Shirazi and Ahmed, 1991), the problem of selecting Laguerre poles in discrete-time linear systems was investigated by minimising the error between

the impulse response of the system and the corresponding Laguerre model. This strategy's main drawback is that it requires finding the roots of high-order polynomials. In Fu and Dumont (1993), Tanguy et al. (1995), and den Brinker and Sarroukh (2004), different energy errors are minimised for obtaining analytical formulas for optimal Laguerre poles, whose solution is only valid for first-order Volterra models (linear models). In the context of non-linear systems, an analytical optimisation of Laguerre bases for the orthonormal series expansion of Volterra models was initially derived in the context of second-order models (Campello et al., 2001) and further extended to any order models (Campello et al., 2004; Kibangou et al., 2005; Campello et al., 2006). The list of works dealing with the Laguerre pole location also includes the derivation of optimality conditions for linear truncated Laguerre networks (Oliveira e Silva, 1994, 1995a). These conditions are of great theoretical interest but can, however, result in complicated computations in practical cases.

Particularly for Laguerre bases, whose functions are completely parameterised by a single real-valued pole (the Laguerre pole), the computation of optimal poles can be seen as a scalar optimisation problem. Despite the important theoretical results with regard to analytical approaches to the optimisation of the Laguerre poles, this optimisation can be carried out by performing a search within the feasibility interval $(-1, 1)$ (Oliveira et al., 2003). The main disadvantage of such an approach is that, for each evaluation of the objective function associated with a particular value of the pole, all the remaining model parameters – the static mapping \mathcal{H} in (20) – need to be re-estimated. Depending on the problem and the sort of linear and non-linear OBF model adopted, the consequence is a high computational cost.

As the Laguerre bases involve rational transfer functions with a single real-valued pole, they are preferred for representing well-damped dynamic systems. Systems with poorly damped dynamics, however, typically cannot be accurately approximated with a small number of Laguerre functions. In other words, such functions are not very well suited to approximate signals with strong oscillatory behaviour (Oliveira e Silva, 1995b; Tanguy et al., 2000; Heuberger et al., 2005). This drawback has led to an increasing interest in the two-parameter Kautz functions, introduced by Kautz (1954). These functions can better approximate systems with oscillatory behaviour because they are parameterised by a pair of resonant poles. Optimality conditions for the approximation of truncated linear models are derived (Oliveira e Silva, 1995b; den Brinker et al., 1996) by minimising the error between the impulse response of a given system and its Kautz approximation. In the context of pole location, a sub-optimal analytical choice of Kautz poles was proposed for discrete-time linear systems (Tanguy et al., 2002) and the corresponding non-linear counterpart was later addressed for Volterra models (da Rosa et al., 2007). More recently (da Rosa et al., 2008), an analytical solution for one of the parameters related to the Kautz poles was derived

when any-order Volterra kernels are decomposed into a set of independent orthonormal bases, each of which is parameterised by an individual pair of conjugate Kautz poles associated with the dominant dynamic of the kernel along a particular dimension. This is an extension of a previous method (da Rosa et al., 2007), where the solution involves a single Kautz basis for expanding a given kernel along all its dimensions.

In contrast to the analytical methods, a different approach to the pole location problem can be adopted that utilises numerical procedures for optimisation. In this context, numerical procedures for selecting OBF poles in an iterative manner have been proposed (Hacıoğlu and Williamson, 2001; Ngia, 2001; Favier et al., 2003; Kibangou et al., 2003, 2005; Patwardhan and Shah, 2005; da Rosa et al., 2009). Recursive algorithms for the estimation of the coefficients and poles of Kautz and Laguerre filters based on a separable non-linear least-squares method have been presented (Ngia, 2001). A general optimisation formulation that conceptually embodies both the optimality and convergence requirements has been suggested in Patwardhan and Shah (2005), but no strategy has been provided for determining the search directions to be followed by the optimisation algorithm. The main difficulty comes from the fact that the relations between the model output and the OBF parameters (poles) are governed by dynamic equations. The use of the gradient descent technique is proposed in the literature (Hacıoğlu and Williamson, 2001), but the gradient formulation and the corresponding search directions are roughly approximated by means of instantaneous (static) estimates of their partial derivatives. A method for the precise determination of search directions based on the analytical recursive computation of the derivatives of the output of the orthonormal basis filters with respect to their poles has been proposed (da Rosa et al., 2009). Such derivatives can then be used as part of an optimisation method to obtain exact search directions for the OBF poles that fully encompass the dynamic nature of these parameters. An application of this approach to the modelling of a real world magnetic levitation system with non-linear behaviour is presented in (Oliveira et al., forthcoming).

Regardless of the sort of model adopted, that is, the shape of the mapping \mathcal{H} in (20), a simple but effective heuristic for selecting the parameterisation of the orthonormal functions consists of using prior knowledge about the dominant dynamics of the system (Ninness and Gustafsson, 1995, 1997). Such a heuristic can often be selected by analysing the time or frequency response of the system (Zervos and Dumont, 1988; Wahlberg and Ljung, 1992).

4 Simulation example

In this section, the system identification using the techniques described in Section 2.2 is illustrated through a case study. It is related with the computational simulation of

a continuous stirred tank reactor (CSTR) polymerisation process.

The identification of a simulated CSTR polymerisation is considered, particularly an isothermal process that uses toluene as solvent (Doyle et al., 1995). The number average molecular weight (NAMW) of the resulting polymer, $y(t)$ [kg/kmol], is controlled by manipulating of the rate of flow of the substance initiator, $u(t)$ [m³/h]. A four-state model of this non-linear process is given by Maner et al. (1996) and Doyle et al. (2002):

$$\begin{cases} \dot{x}_1(t) = 10(6 - x_1(t)) - 2.4568x_1(t)\sqrt{x_2(t)} \\ \dot{x}_2(t) = 80u(t) - 10.1022x_2(t) \\ \dot{x}_3(t) = 0.002412x_1(t)\sqrt{x_2(t)} + 0.112191x_2(t) - 10x_3(t) \\ \dot{x}_4(t) = 245.978x_1(t)\sqrt{x_2(t)} - 10x_4(t) \\ y(t) = \frac{x_4(t)}{x_3(t)} \end{cases} \quad (30)$$

The differential equations in (30) are simulated using as initial conditions the nominal operation conditions, given by $x_1(0) = 5.506774$, $x_2(0) = 0.132906$, $x_3(0) = 0.0019752$, $x_4(0) = 49.3818$, $u(0) = 0.016783$, and $y(0) = 25000.5$ (Doyle et al., 1995; Maner et al., 1996). The process is simulated from $t = 0$ up to $t = 32$ hours, with the input u being a sequence of steps of the same duration and random amplitude uniformly distributed within the operational interval $[-0.6, 0.8]$. The data set is sampled using a sampling period of $T = 0.03$ hours and normalised in order to avoid numerical problems during the model estimation. This data set is then split into two parts: one half (16 hours) intended for estimation of the (N)OBF models of this process and the other half intended for validation of the resulting model. The first 1/2 hour of each set is used exclusively to recover the right Laguerre states. These states are unknown (and set equal to zero) at $t = 0$ because, in practice, the values of the input signal anterior to the data available for identification are usually unknown.

This section presents the identification of the CSTR polymerisation process using a linear OBF model with parametric uncertainties, as described in Section 2.2, and the two approaches described therein are adopted. Although the approach of obtaining a set of linear models via robust identification is directed primarily toward the representation of linear systems in the context of modelling non-linear systems, this approach can also be seen as the achievement of a set of linear approximations of the process at different set points.

In the second approach for robust identification presented in Section 2.2, one needs to get a set of the system's impulse responses. For the process of CSTR polymerisation described by the equations in (30), this set of impulse responses can be obtained by exciting the system with impulses of different amplitudes and later dividing the output signals by the amplitudes of the corresponding impulses. The goal is to recover the dynamic response of the process when it is excited by signals with different energies

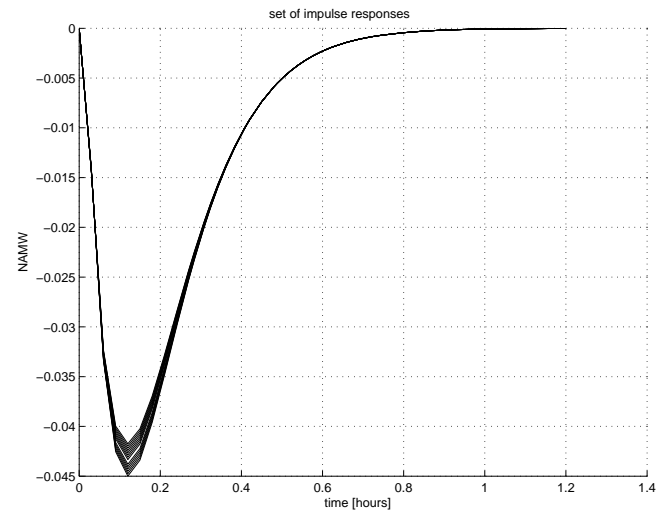
in such a way that the amplitude of the input signal has no influence on the result. Thus, if in addition to being deterministic the process is also linear, the set of impulse responses will be identical. Clearly, this is not the case in Figure 1, which displays the responses of the CSTR to impulses having amplitudes within the normalised range $[-0.6, 0.8]$, with sampling of 0.1.

As already discussed in Section 3, the method proposed by Fu and Dumont (1993) computes the optimal Laguerre pole from the impulse response of the (linear) system. The process described by the equations in (30) is non-linear and so the application of such method with six Laguerre functions generates a set of optimal poles (one pole for each impulse response) whose mean value is 0.56. Therefore, with six Laguerre functions parameterised at $p = 0.56$, the use of equations (28) and (29) leads to a linear model with uncertain parameters $c_i(\varepsilon_i)$ lying in the interval $[c_{i,\min}, c_{i,\max}]$ given by:

$$\mathbf{c}_{\min} = \begin{bmatrix} -0.0517 \\ -0.0839 \\ -0.0487 \\ -0.0223 \\ -0.0093 \\ -0.0036 \end{bmatrix}, \quad \mathbf{c}_{\max} = \begin{bmatrix} -0.0499 \\ -0.0783 \\ -0.0459 \\ -0.0218 \\ -0.0092 \\ -0.0036 \end{bmatrix}. \quad (31)$$

Next, an example of implementing of the first methodology for robust identification discussed in Section 2.2 (UBBE) is presented. In this case, a set of I/O estimation data is used with a maximum prediction error equal to $e_{\max} = 0.15$. For the sake of comparison, the pole value of the Laguerre pole ($p = 0.56$) and the number of Laguerre functions ($n = 6$) are the same as the ones adopted in the previous approach.

Figure 1 Set of normalised impulse responses of the CSTR.



So, using the state-space representation in (19), as well as the hypotheses (25) and (26), the method of robust identification proposed in Milanese and Belforte (1982) leads to a linear model with uncertain parameters $c_i(\varepsilon_i)$ lying in the interval $[c_{i,\min}, c_{i,\max}]$ given by:

$$\mathbf{c}_{\min} = \begin{bmatrix} -0.1514 \\ -0.2939 \\ -0.1972 \\ -0.2193 \\ -0.2478 \\ -0.1577 \end{bmatrix}, \quad \mathbf{c}_{\max} = \begin{bmatrix} 0.0994 \\ 0.0652 \\ 0.1018 \\ 0.1511 \\ 0.1806 \\ 0.1977 \end{bmatrix}. \quad (32)$$

From the preceding results, notice that the two strategies are capable of generating bounds of uncertainty for the parameters of the orthonormal model. When the two sets of results reported are compared, however, the method based on UBBE generates a larger orthotope (uncertainties more conservative than in the method based on the impulse response). Consequently, a more conservative control law will be generated by the robust control algorithm (e.g., see Oliveira et al., 2000). Another issue concerning this method is the selection of the maximum prediction error, which is usually hard to compute. Nevertheless, an important advantage is that the type of information required, i.e., the set of I/O data, is similar to that used by least squares algorithms for the estimation of linear systems. In this aspect, it is important to note that real world dynamic systems neither always react properly nor even admit operationally simulations involving the excitation via impulses. This property makes it difficult to obtain the information that is the basis for the computations of the method based on this sort of system.

5 Conclusions

An overview of the state-of-the-art in the areas of identification of dynamic systems using OBF models has been presented. This so-called OBF structure presents many advantages when compared to dynamic structures with traditional regressors, such as the absence of output recursion, the orthonormality of the elements of the regression vector, the ability to deal with time delays, tolerance of unmodelled dynamics and others.

The mathematical foundations of the OBF models have been discussed in the context of system identification based on linear models with uncertain parameters (robust identification). Moreover, a widespread bibliographical compilation about the identification of linear OBF models have been presented, including works comprising all the approaches mentioned previously. Practical aspects of modelling have also been presented and illustrated through a case study involving CSTR polymerisation. This work is followed by its part II, with issues regarding non-linear system identification using OBF models.

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