Online pathological tremor characterization using extended Kalman filtering

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Abstract—This paper describes different algorithms that perform online pathological tremor characterization in terms of acceleration. Two distinct parametric models are used, an Auto-Regressive (AR) model and an harmonic model. Both models are recursively estimated with Extended Kalman Filters (EKFs). Experimental data was obtained with low cost sensors and the results are compared in terms of spectrogram estimation and prediction performance.

I. INTRODUCTION

Pathological tremor is the most common movement disorder found in human pathology and it may be defined as an involuntary, approximately rhythmic and roughly sinusoidal movement [1]. It occurs mainly on the upper limb and its incidence increases with aging. Although tremor itself is not life-threatening, it may reduce considerably the patient's quality of life, especially because the ability to perform Activities of Daily Living (ADL), such as inserting the key in the keyhole, is seriously decreased.

Tremor is associated with several pathologies and diagnose and quantification by physicians is not straightforward. Although subjective methods are widely used, simple and quantitative methods are desired and may improve treatment effectiveness. In addition, tremor characterization methods are an important tool for the research conducted about the topic. For those purposes, methods for offline tremor spectral analysis have been presented, using both nonparametric and parametric approaches [2],[3].

As opposed to its offline version, online characterization of tremor provides information about the pathological movement during the acquisition of measurements. Its importance, however, does not lie only in the possibility to study and analyze tremors in real-time. If parametric methods are used, for example, such algorithms may be part of Human Machine Interfaces (HMIs) systems specially designed to filter tremorous movement or in active orthosis that directly compensate pathological tremor. In [4], for instance, an harmonic model was assumed and an adaptive filter, the Weighted Fourier Linear Combined (WFLC), was designed to model tremor. A similar approach was described in [5], although the adopted model was not harmonic, but rather a sum of sines and cosines of chosen frequencies.

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Antônio P. L. Bó (e-mail: bo0lirmm.fr) and Philippe Poignet are with the LIRMM UMR 5506 CNRS UM2, Montpellier, France. In this paper, an harmonic model and also an Auto-Regressive (AR) model are used to characterize pathological tremor. However, different from [4] and [5], Extended Kalman Filters (EKFs) are employed to recursively estimate the parameters of both models. The EKF presents the advantage of explicitly considering the measurement noise and concurrently performing estimation of the tremor state. This approach is similar to the two-stage KF used to estimate the AR models of intracranial pressure signals in [6].

Regarding experimental evaluation of the proposed algorithms, data was acquired with low cost accelerometers from patients with pathological tremor. Since the tremor exact model is unknown, the two methods are compared in terms of spectrogram estimation and RMS prediction error.

The paper is organized as follows: in the following section the problem is properly formulated and the framework shared by the two models is defined. Section III describes the models and the estimation procedure and the last two sections present the experimental results and the conclusions, respectively.

II. PROBLEM FORMULATION

Online pathological tremor characterization is treated, in this work, as the problem of estimating the pathological involuntary quasi-periodic and nonstationary tremor signal from the noisy measurements of a motion sensor (an accelerometer, in the present case). Concurrently, the parameters of a tremor model must be estimated. It is considered that frequency, amplitude and phase of the signal may slowly vary throughout time. This tremor model estimated at every instant may be used to provide online estimates of its spectrogram and to predict the tremor future state.

A set of considerations have been made at this stage of the research which are planned to be suppressed in future developments. First, a single-joint tremorous movement has been measured by a triaxial accelerometer mounted on the patient's hand. Hence, this paper does not address the problem of multi-articulation tremor. Also, it is considered that the movement measured by the sensor is due only to tremor motion. Further developments of the proposed algorithms will include the ability to track higher frequency components of the signal and then filter voluntary motion, which presents lower frequency components. Lastly, a simple model for the sensor is

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considered:

$$\tilde{s}_k = s_k + \nu_{s,k},\tag{1}$$

where ν_s is an additive white Gaussian noise, s is the tremor signal, \tilde{s} is the measured tremor signal corrupted with noise and k is a multiple of T, the sampling period.

III. MODELS AND ONLINE ESTIMATION

A. AR model

AR models are extensively used to model signals from which *a priori* knowledge is not available. In general, they can be used to model arbitrary time-series, if an increasing model order is considered. The AR model of a nonstationary signal is given by

$$s_k = \sum_{h=1}^{H} a_{h,k} s_{k-h},$$
 (2)

where $a_{h,k}$ are the parameters and H is the model order.

From the estimated model at an instant k, the corresponding Power Spectrum Density (PSD) and k-step ahead predictions may be computed. The instantaneous PSD have to be computed for all evaluated frequencies ω at every instant k. It may be given by [7]

$$P_{\omega,k} = \frac{\sigma_{\tilde{s}}T}{1 + \|\sum_{h=1}^{H} a_{h,k} e^{-ih\omega T}\|^2},$$
(3)

where $\sigma_{\tilde{s}}^2$ is an estimate of the signal noise variance. The *k*-step ahead predictions are computed with Eq. (2).

B. Harmonic model

Every periodic signal may be represented by an harmonic model, which is also often referred as Fourier series model. In the case of quasi-periodic signals, an approximation may be achieved with harmonic models. Since a nonstationary signal is considered, the following rectangular model is adopted:

$$s_{k} = \sum_{h=1}^{H} \left[a_{h,k} \sin\left(h \sum_{t=1}^{k} \omega_{t}\right) + b_{h,k} \cos\left(h \sum_{t=1}^{k} \omega_{t}\right) \right], \quad (4)$$

where ω_t is the fundamental frequency at instant k, $a_{h,k}$ and $b_{h,k}$ are the coefficients and H the number of harmonics, the model order.

For the harmonic model, the computation of the corresponding PSD and k-step ahead predictions are straightforward. The PSD is given by spikes on the fundamental frequency ω_k and its harmonics. The amplitude of each spike is given by

$$P_{h\omega_k,k} = \frac{\|b_h - ia_h\|^2}{4}.$$
 (5)

It is also a simple procedure the computation of the k-step ahead prediction, since for prediction purposes the system is considered stationary within the prediction horizon. Hence, it is enough to apply Eq. (4) with $\omega_t = \omega_k$, for $t \ge k$.

C. EKF

Since online pathological tremor estimation must be performed and tremor itself is considered a slowly timevarying signal, recursive estimation algorithms must be applied. Different filters were evaluated, like the WFLC, proposed by [4] for harmonic models, and the Recursive Least-Squares (RLS) for AR models, but finally the chosen algorithm was the EKF. The main reason for this choice is that the EKF explicitly considers both the model and the measurement uncertainties and also estimates recursively and concurrently both the noisecorrupted tremor signal measured from the motion sensor and the model parameters. In addition, it becomes easier to compare the two different models if a common framework is used for both of them. Also, initial evaluations have shown that the EKF presents better performance in terms of RMS prediction error when compared to the referred algorithms in the context of tremor characterization.

The KF is the optimal state estimator for linear systems that present additive Gaussian process and measurement noise. However, both estimation problems on this work are nonlinear. Hence, a modification of the KF for nonlinear systems, like the EKF, where Kalman equations are applied to the first-order linearization of the nonlinear system [8], must be used.

The filter states are the estimated tremor signal \hat{s}_k and the model parameters. For the states related to the tremor signal estimation, the models are given by Eqs. (2) and (4). Considering that the model parameters vary slowly, they are modeled as random walks. For the AR model the state vector may be given by

$$\begin{bmatrix} s_k & s_{k-1} & \cdots & s_{k-H} & a_{1,k} & \cdots & a_{H,k} \end{bmatrix}^T, \quad (6)$$

and for the harmonic model, in which the estimated tremor signal depends only on the model parameters, by

$$\begin{bmatrix} s_k & a_{1,k} & \cdots & a_{H,k} & b_{1,k} & \cdots & b_{H,k} & \omega_k \end{bmatrix}^T.$$
(7)

In the correction phase, the measurement from the motion sensor, \tilde{s}_k , is used to update the estimated tremor signal and the model parameters. Since the tremor signal is being estimated by the filter, the available information is a direct measurement of s_k .

The main parameters of the filter are the process covariance matrix, \mathbf{Q} , and the measurement variance, r. Variances related to the assumed model are lower for imprecise models, while variances related to parameters are proportional to the time-varying nature of those parameters. As to the variance r, it is considered equal to the variance $\sigma_{\tilde{s}}^2$.

The EKF is a recursive estimation algorithm and therefore an initial estimate of the model parameters is needed. For the case of the AR model, no *a priori* knowledge is considered about the tremor signal. Concerning the harmonic model, however, the initial estimate of the fundamental frequency is critical. Initial values distant from the actual fundamental frequency may not only slow down the convergence of the parameters, but may also decrease the quality of the final estimate.

IV. EXPERIMENTAL RESULTS

The acquisition system described in [9] was used to acquire data from patients diagnosed with different pathological tremors. The results presented in this paper, however, concern only data obtained from one patient diagnosed with Parkinson's Disease (PD) using a triaxial accelerometer (Analog Devices ADXL330) mounted on the patient's hand.

Two data sets were used. In a first experiment the patient was told to maintain a resting position. It can be seen on Fig. 1(a) that the pathological motion is almost stationary. In a second experiment the patient performed a common clinical test for tremor evaluation (drawing a spiral) resulting in a tremor signal with distinct features (Fig. 1(b)).

About the algorithms, the same filter parameters and initial conditions were used for both simulations whenever possible. It was not the case, naturally, for the variances related to parameters of different models, since they have distinct physical meanings. Also, in order to validate the robustness of the algorithm before different conditions, for different data sets the algorithm configuration was also kept unchanged.

Two main metrics were applied in order to evaluate the algorithms performance. First, the spectrogram estimated with the Short-Time Fourier Transform (STFT) was compared with the spectrograms estimated by the two different models, according to Eqs. (3) and (5). Also, the signal k-step ahead prediction and its RMS error was compared to the naive assumption in which the predicted state is equal to the present state. To compute RMS error, \tilde{s} is used instead of s, since the later is not available.

The experimental results for both data sets are illustrated on Fig. 1 and on Tab. I. Based on the smaller RMS errors obtained, a 4^{th} -order harmonic model and an AR model with H = 35 were chosen for comparison. It can be seen that both methods provided satisfactory and coherent spectrogram estimates. About the predicted tremor states, they presented small phase delay and the errors were considerably lower than the naive approach, as expected. A comparison between the AR model and the harmonic model may be conducted based on the prediction performance. Although in both data sets no great variation on frequency was observed, this preliminary analysis shows that regarding this performance index the AR model generally outperforms the harmonic model.

Some other remarks could be made to compare both models. AR models have the advantage that no *a priori* knowledge is needed about the signal. However, adaptation of harmonic models may be faster, specially for high variable tremors and when compared to high order AR

TABLE I EXPERIMENTAL RESULTS. VALUES ARE IN m/s^2 and %.

Data set/ Method		Estimation RMS error		1-step pred. RMS error		5-step pred. RMS error	
1^{st}	AR	0.014	2.7	0.103	19.9	0.139	29.6
	Harm.	0.020	3.9	0.132	25.6	0.157	30.4
	Naive	-	-	0.249	48.2	0.888	171.9
2^{nd}	AR	0.041	13.6	0.134	38.0	0.199	56.5
	Harm.	0.048	11.6	0.149	42.3	0.193	54.7
	Naive	-	-	0.172	48.8	0.465	131.9

models. Also, spectrogram and predictions computations are simpler with harmonic models.

V. CONCLUSIONS

This paper has presented and compared two different models used to perform online pathological tremor characterization. The method may be used not only to provide online tremor information to the physician, but also for applications of different natures, like active compensation of upper limb pathological tremor.

AR and harmonic models were briefly described, as well as the applied recursive estimation algorithm, the EKF. The performance of the proposed approach was evaluated with experimental data and both models presented satisfactory results in terms of spectrogram estimation and k-step ahead prediction errors. The analysis of the RMS prediction error based on the presented data shows that the AR model outperforms the harmonic model with respect to this performance index. However, the use of harmonic models may still be justified in some applications.

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Fig. 1. Results from the first set of data (a) and from the second set (b). The upper graphics shows the measured acceleration, \tilde{s} . Following, the estimated spectrograms with the STFT, with the AR model and Eq. (3) and with the harmonic model and Eq. (5). The bottom graphics shows the 5-step ahead prediction error of the naive prediction (gray), the AR model (red) and the harmonic model (green).