k_v-t SPARSE: Compressed sensing applied to flow quantitation MRI

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EE596 Project - Fall 2006 - University of Southern California

Abstract

Magnetic resonance (MR) based flow quantitation is important for the evaluation of many cardiovascular conditions, including valvular abnormalities, congenital defects, and coronary artery disease. Recently, a new approach to undersampling of MR data has peen proposed, using a compressed sensing framework. In this work, we apply this technique to Fourier velocity encoding (FVE) datasets. A different random subset of FVE samples is acquired in each temporal frame, and sparsity in velocity-frequency space is explored. We evaluate acceleration factors from 1.5 to 4 in FVE imaging using the proposed method (k_v-t SPARSE). Although sparsity evaluation experiments suggested great potential for a compressed sensing approach to FVE imaging, the proposed method did not work as well as expected. k_v-t SPARSE was able to reduced background noise due to undersampling, but clinically important information was lost. Acceptable results with 1.5-fold acceleration are presented, and possible approaches to improving the performance are proposed.

1 Introduction

Accurate flow quantitation is important for the evaluation of many cardiovascular conditions, including valvular abnormalities, congenital defects, and coronary artery disease. In cardiac magnetic resonance imaging (MRI), speed is of particular importance, as the acquisition time is typically limited to the duration of a breath-hold. Typical slice-selective MRI datasets are two-dimensional, i.e. an image (x, y). In cardiac MRI, the data is commonly time-resolved within the cardiac cycle, and datasets are three-dimensional

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(x, y, t). In flow imaging, an additional dimension - the velocity distribution - is acquired (x, y, v, t).To obtain such a dataset in a single breath-hold with clinically useful resolution, different fast imaging techniques have been proposed [1–3]. Resolution may be further improved by using undersampling techniques, in which only portions of the data is acquired. The most widely used undersampling techniques are partial Fourier [4], variable-density sampling [5, 6], temporal acceleration [7, 8] and parallel imaging [9, 10].

Recently, a new undersampling approach based on compressed sensing theory [11] has been proposed [12–14]. In this framework, the time required to image objects which are known to be sparse is some domain (e.g., x-f space in dynamic cardiac imaging, object domain in 3D angiography) is reduced by randomly undersampling the data. In this work, we apply this technique to Fourier velocity encoding (FVE) datasets.

In FVE [15] the velocity information is acquired in Fourier domain. Typically, 16 to 48 samples of the Fourier transform of the velocity distribution are obtained, and the acquired data (k_v) is inverse Fourier transformed to the velocity domain (v). This is repeated for each temporal frame (i.e., cardiac phase), and for each step of the spatial information acquisition process. After reconstruction, the time-velocity distribution (v-t space) in a region of interest of the object (e.g., aortic valve, common carotid artery) is displayed.

Our approach consists in acquiring a different random subset of these FVE samples in each temporal frame (i.e., undersampling in k_v-t space), and explore sparsity in v-f space (f denotes temporal frequency) using compressed sensing. We evaluate acceleration factors from 1.5 to 4 in FVE imaging using the proposed method (k_v-t SPARSE).

2 Theory

2.1 Fourier Velocity Encoding Datasets

The velocity distribution in a "voxel" or a region of interest of the image can be represented by a two-dimensional matrix, where different rows represent different velocities, different columns represent different temporal frames, and the numerical values represent the amount of blood flowing with a particular velocity in a particular time instant. The data is visualized in v-t space, where v denotes velocity in a particular direction (e.g., through-plane) and t denotes time within a cardiac cycle. Figure 1 shows typical aortic valve velocity distributions for a healthy subject and a patient with aortic stenosis.

However, MRI data is typically acquired in Fourier domain. In MR

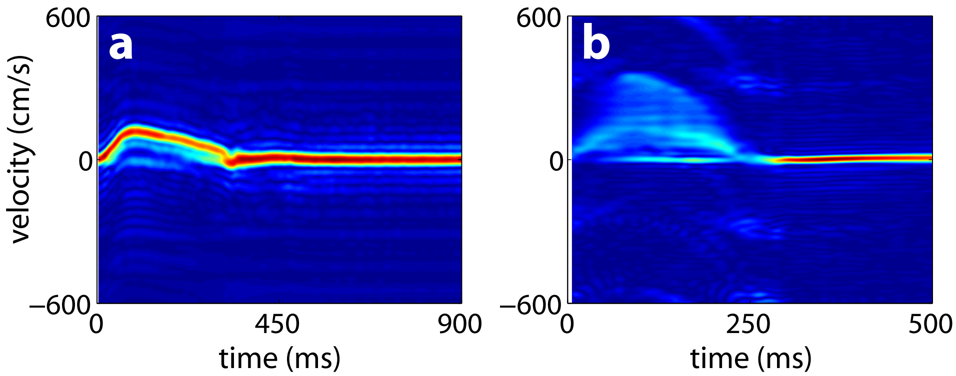

Figure 1: Aortic valve velocity distributions in a healthy subject (a) and a patient with aortic stenosis (b).

flow imaging, velocity information is obtained by acquiring samples of the Fourier transform ($S(k_v)$) of the velocity distribution ($s(v)$). Typically, 16-48 k_v samples are acquired in each phase of the cardiac cycle, thus the actual dataset that is acquired is in k_v-t space. An inverse Fourier transform along the velocity dimension is used to obtain the time-velocity distribution.

Additionally, a Fourier transform along the temporal dimension will convert the time-velocity distribution (v-t) to velocity-frequency space (v-f), where f denotes temporal frequency. Although velocity distributions in healthy subjects are considerably sparse in v-t domain, this is not necessarily true in patients (Figure 1). However, in v-f space, the data is likely to be sparse for both healthy subjects and patients [16]. Thus, this representation can be used in a compressed sensing framework to reduce scan time. Figure 2 illustrates typical FVE datasets acquired in k_v-t space, and their counterparts in v-t and v-f spaces.

2.2 Compressed Sensing

Conventional sampling theory defines the minimum sampling rate for a signal as at least twice its highest frequency component. According to this dogma, in order to acquire velocity distributions with 10 ms temporal resolution and 25 cm/s velocity resolution over a 600 cm/s velocity field-of-view, 24 FVE samples would have to be acquired in each temporal frame, and only one "view" (one velocity encode combined with one spatial readout) could be acquired during each heartbeat. This could imply in prohibitively long scan times.

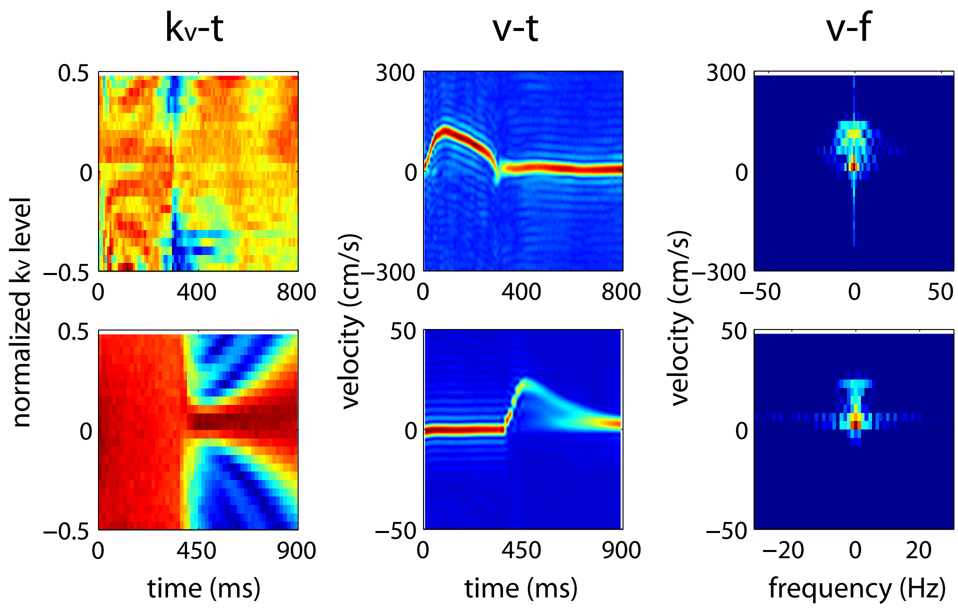

Figure 2: Typical FVE datasets acquired in k_v-t space (left) and their counterparts in v-t (center) and v-f spaces (right), from a healthy subject (top) and a flow phantom with a wide distribution of velocities (bottom). The distribution in the flow phantom shows characteristics that resembles distributions in patients.

However, velocity distributions are considerably sparse in v-f space (Figure 2, right). Compressed sensing theory states that signals that are known to be sparse in some known transform domain can be recovered from randomly under-sampled data [11,17]. If a signal s of length d can be expressed efficiently (few coefficients) in an orthonormal basis Ψ (the sparsity matrix), than we can write $s \approx \Psi\theta$, where θ is a vector with m non-zero entries ($m \ll d$). Instead of acquiring d samples of s, we take N random linear measurements of s, where $m \leq N \leq d$. This can be viewed as multiplication $y = \Phi s$, where Φ is a $N \times d$ measurement matrix. The signal s can be recovered from the undersampled data y using several algorithms, such as l_1 minimization [18], tree-based matching pursuit [19], and orthogonal matching pursuit (OMP) [20]. In this work, we use OMP to reconstruct the signals. The OMP algorithm is summarized in Table 1 [17].

In k_v-t SPARSE, we write $s(v,t) = \Psi\theta(v,f)$, and $y(k_v,t) = \Phi s(v,t)$. In other words, s is the time-velocity distribution, θ is its sparse representation in v-f space, and y is the acquired MR data (in k_v-t space). The

Initialize residual $r_0 = y$. **For** t=1,2,...,N **do** **A.** Find the column i_t of $\Phi\Psi$ such that $i_t = argmax_i \vert < r_{t-1}, (\Phi\Psi)_i > \vert$. **B.** Compute the new residual $r_t = y - P_t y$ where P_t is the orthogonal projector onto the span of the t columns chosen from $\Phi\Psi$. **Output** Columns $\{i_t\}$ and coefficients $\{\hat{\theta}_{i_t}\}$ s.t. $P_n y = \sum_{t=1}^{N} \hat{\theta}_{i_t} (\Phi\Psi)_{i_t}$.

Table 1: Orthogonal matching pursuit reconstruction algorithm

sparsity matrix Ψ is an inverse Fourier transform along the temporal frequency dimension, and the measurement matrix Φ is a Fourier transform along the velocity dimension, with N length-d rows corresponding the randomly picked k_v values in each temporal frame t. The distribution $s(v,t)$ is obtained from the randomly undersampled data $y(k_v, t)$ using the OMP algorithm described above, and inverse Fourier transforming the output (in v-f space) along the temporal frequency dimension to obtain s(v,t).

3 Methods

3.1 Sparsity Evaluation

In order to evaluate the actual sparsity in FVE data, we acquired fully sampled FVE datasets using spiral FVE [3]. The data was transformed to v-f space and the coefficients were sorted in descending order of magnitude values. From 1/3 to 15/16 of the lowest magnitude coefficients were discarded, and the data was reconstructed with only 2/3 to 1/16 of the originally acquired data. The goal of this experiment is to evaluate how sparse the data actually is in v-f space.

3.2 Choice of Sampling Grid

Next, we designed an experiment to choose a sampling grid (in k_v-t space) that would potentially provide better reconstruction results in v-t, by avoiding coefficient misdetection in v-f. For each dataset being analyzed, and for each acceleration factor being evaluated (1.5 to 4), a large set of 5000 different random sampling grids was obtained using a random number generator (uniform distribution). The point-spread function of these sampling grids was obtained by Fourier transforming into v-f space. The point-spread functions were normalized by dividing all coefficients by the absolute value of the largest coefficient in each set. Then, the second largest normalized coefficient in each set was taken, and the set with the smallest value was selected as the sampling grid of choice. The point-spread functions of the chosen sampling grids are expected to have low magnitude sidelobes in v-f space, reducing the risk of coefficient misdetection.

3.3 Data Acquisition and Reconstruction

Rather than modifying the acquisition system to acquire random velocity encode levels at each temporal frame, we acquired fully sampled datasets, and simply discarded specific samples according to the sampling grid selected from the previous experiment. We used spiral FVE [3] to acquire two datasets, measuring through-plane velocities in the aortic valve of a healthy subject, and in a pulsatile flow phantom, respectively. The data was reconstructed in Matlab (Mathworks, Inc., South Natick, MA) using the OMP algorithm described in Table 1. The reconstruction algorithm was written from scratch, with the exception of the basis orthogonalization step, which was performed using the built-in function `orth`. For performance evaluation, the reconstructed velocity distributions were qualitatively compared with distributions obtained from the fully sampled datasets, and also to the undersampled datasets reconstructed with direct reconstruction (inverse Fourier transform along k_v).

4 Results

The results from the sparsity evaluation experiment are show in Figure 3. These results show that FVE data can be accurately reconstructed from only 1/4 of the v-f space coefficients, and adequately reconstructed from only 1/8 of the coefficients. Artifacts become significant if only 6% or less of the coefficients are used in reconstruction, and considerable blurring is observed. As a rule of thumb, the acceleration factor used in compressed sensing should be 1/2 to 1/3 of the true sparseness [21]. Thus, k_v-t SPARSE is not expected to work well for acceleration factors higher than 4.

The sampling grids selected for the *in vivo* and phantom datasets are shown in Figures 4 and 5, respectively. The corresponding point-spread functions suggest that aliasing artifacts due to undersampling will be incoherent, as the point-spread functions consist of a single "spike" at the v-f space origin, with random noise in the background with a much lower magnitude.

Figure 6 shows the reconstruction results for the *in vivo* data at 1.5-fold acceleration. Although the k_v-t SPARSE result (bottom, right) clearly reduced the background noise when compared to the direct reconstruction (bottom, center), considerable distortion is observed when compared to the fully sampled data (bottom, left). The distortion becomes even more severe when 2-fold acceleration (Figure 7, bottom, right), or higher, is used.

Figure 8 shows the reconstruction results for the phantom data at 1.5-fold acceleration. As observed in the *in vivo* results, k_v-t SPARSE (bottom, right) was able to reduced the background noise, but the low amplitude information is completely distorted. This is problematic, as flow jets due to stenosis and/or regurgitation will have similar low amplitude components, and such distortion could affect the ability of determining peak-velocities. The results for 2-fold acceleration or higher show even more severe distortion (not shown).

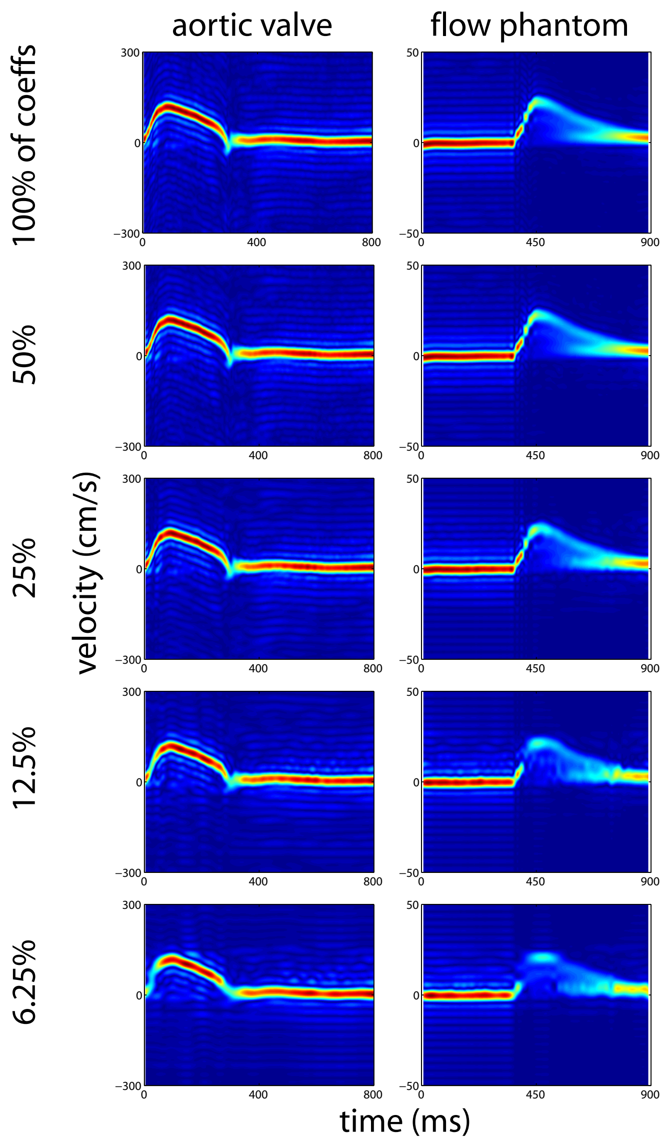

Figure 3: Sparsity evaluation in data obtained from a healthy volunter (left) and a pulsatile flow phantom (right). The data is resconstructed with 6.25% (bottom) to 100% (top) of the v-f space coefficients, which were sorted in descending order of magnitude values.

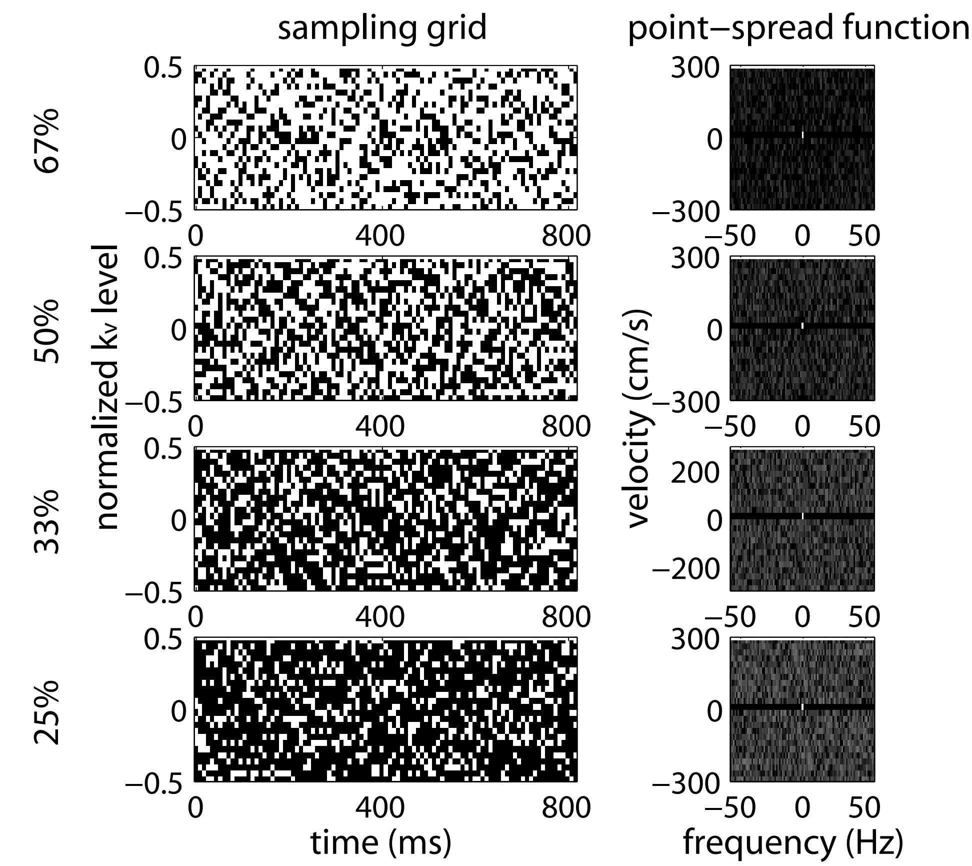

Figure 4: Sampling grids (left) and corresponding point-spread functions (right) used in the *in vivo* experiments, discarding from 33% (top) to 75% (bottom) of the acquired k_v-t samples. Black pixels in the sampling grids reflect discarded samples. Point-spread functions show in log scale with a 40 dB dynamic range.

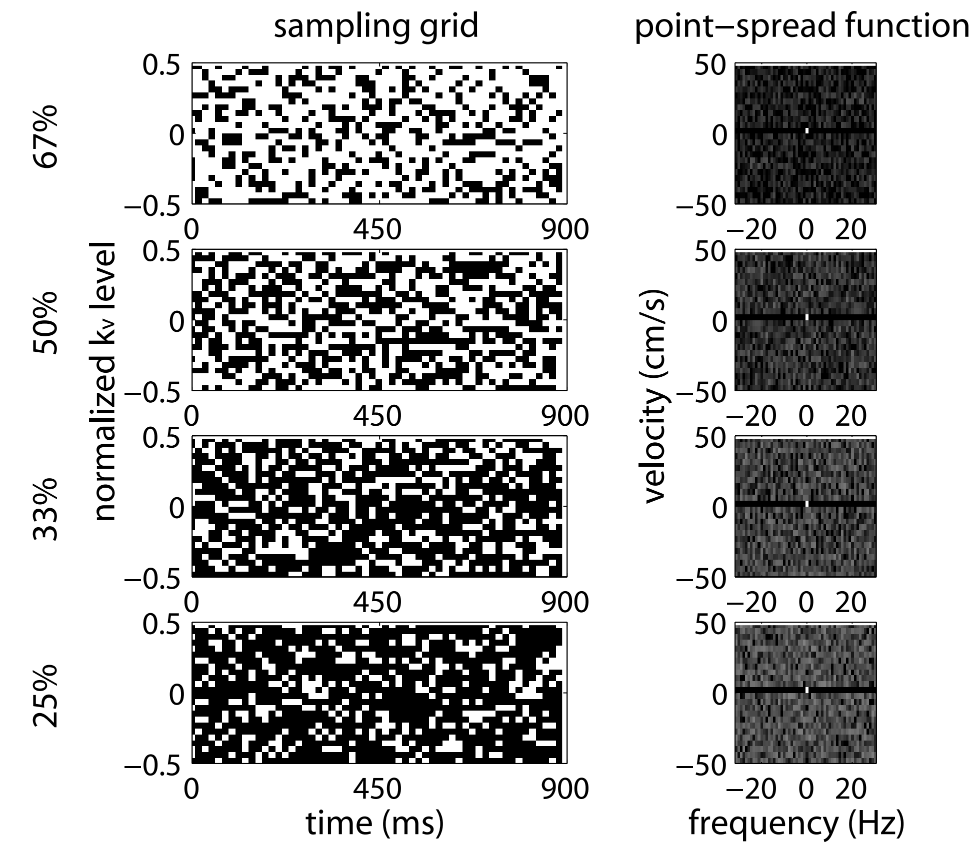

Figure 5: Sampling grids (left) and corresponding point-spread functions (right) used in the phantom experiments, discarding from 33% (top) to 75% (bottom) of the acquired k_v-t samples. Black pixels in the sampling grids reflect discarded samples. Point-spread functions show in log scale with a 40 dB dynamic range.

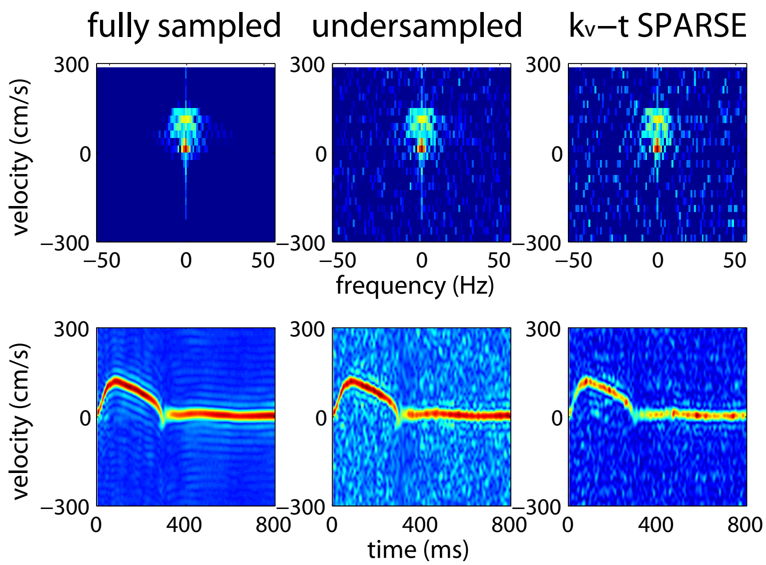

Figure 6: k_v-t SPARSE result for 1.5-fold acceleration in a healthy subject (right) compared to direct reconstruction (center) and fully sampled data (left), in v-f (top) and v-t spaces (bottom).

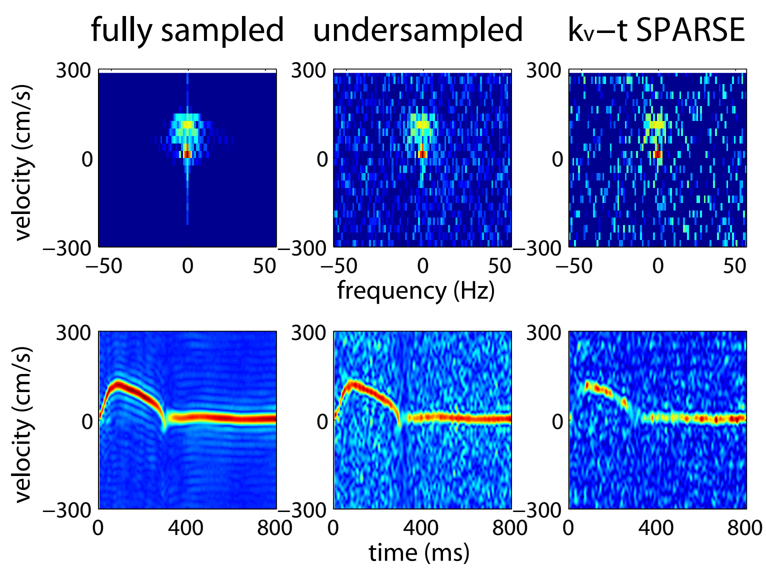

Figure 7: k_v-t SPARSE result for 2-fold acceleration in a healthy subject (right) compared to direct reconstruction (center) and fully sampled data (left), in v-f (top) and v-t spaces (bottom).

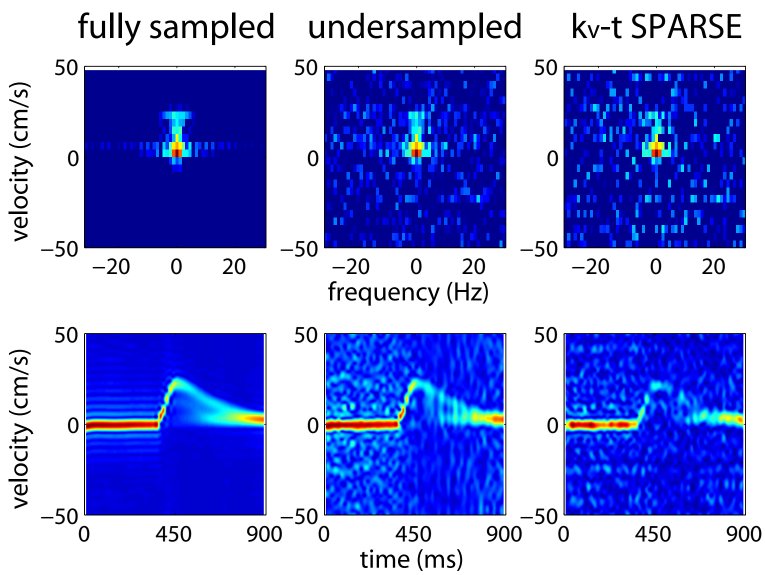

Figure 8: k_v-t SPARSE result for 1.5-fold acceleration in a flow phantom (right) compared to direct reconstruction (center) and fully sampled data (left), in v-f (top) and v-t spaces (bottom).

5 Discussion

Although the sparsity evaluation experiment suggested great potential for a compressed sensing approach to FVE imaging, the proposed method did not work as well as expected. k_v-t SPARSE is able to reduce background noise due to undersampling, but clinically important information may be blurred, distorted or even lost. Although FVE data is in fact sparse in v-f space, we usually observe a few very large coefficients at the v-f space origin, which may overshadow the numerous smaller amplitude coefficients - which are also clinically important - when convolved with the point-spread function of the sampling grid.

Acceptable results were observed at 1.5-fold acceleration in the healthy subject experiment. The results for the flow phantom experiment were not as good, probably due to the long repetition time (TR), which limited the number of temporal frames and reduced the temporal frequency bandwidth. With a shorter TR, such as the one used in the *in vivo* experiment, the number of temporal frames is increased, making the signal sparser in v-f space. Thus, we expect to achieve results comparable to that in Figure 6 when imaging patients.

The proposed sampling grid selection procedure is signal independent, and could easily be performed during pre-scan, which is performed right before signal acquisition. In this case, an appropriate sampling grid could be designed based on the current heart-rate and specified undersampling factor, velocity resolution and velocity field-of-view.

Several approaches could be applied to improve the performance of k_v-t SPARSE. One possible approach consists in finding a sparser representation for FVE data. A Fourier transform along time, along with a wavelet transform along the velocity dimension should provide better results [21]. Also, an alternate reconstruction approach could improve the method's efficiency. Orthogonal matching pursuit might not be the most adequate algorithm for this application, as significant background noise was observed in the reconstructed data. A non-linear approach such as l_1 minimization could potentially provide a sparser solution [18]. Finally, this technique could be directly combined to compressed sensing in spatial domain [13] to achieve much higher acceleration factors. For example, in spiral FVE, in each cardiac phase a random velocity encode associated with a random "perturbed" spiral readout [12] could be acquired, exploring sparsity in x, y, v-f space or in an alternate sparser representation (e.g., wavelet-based).

6 Conclusions

In this work, the performance of the compressed sensing framework in a flow quantitation application has been evaluated. Although sparsity evaluation experiments suggested great potential for a compressed sensing approach, the proposed method did not work as well as expected. k_v-t SPARSE was able to reduced background noise due to undersampling, but clinically important information was lost.

Acceptable results were observed at 1.5-fold acceleration. Better results, and/or higher acceleration factors could be achieved using a different reconstruction algorithm (e.g., l_1 minimization) or a sparser representation (e.g., wavelet-based). Also, k_v-t SPARSE may be directly combined to compressed sensing in spatial domain to achieve much higher acceleration factors.

7 Acknowledgments

The author thanks Michael Lustig for valuable discussion.

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