Does phase contrast MRI provide the mean velocity of the spins within a voxel?

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Introduction: Phase contrast (PC)[1] is the MRI gold standard for measuring blood flow. The underlying assumption with PC is that all spins within a voxel move at the same velocity. This assumption is broken if the spatial resolution is insufficient, if the voxel is partially occupied by static spins (e.g., vessel wall, plaque) or located at the flow's viscous sublayer, and/or if the flow is complex or turbulent (e.g., stenosis, aneurysm). PC measurements are considered unreliable in such conditions, due to partial volume effects[2]. We investigate the mathematical relationship between the velocity distribution of the spins within a voxel and the PC-measured velocity for that voxel.

Theory: The spatial-velocity spin distribution may be modeled as $\rho(\vec{r},v') = \rho(\vec{r})\delta(v' - v(\vec{r}))$, where $\rho(\vec{r})$ is a spin-density map, $v(\vec{r})$ is a velocity map, and $\delta(v')$ is the Dirac delta function[3,4]. A measurement with finite spatial resolution may be modeled as $\tilde{\rho}(\vec{r},v') = \varphi(\vec{r}) * \rho(\vec{r},v')$, where $\varphi(\vec{r})$ is the point-spread function associated with the k-space coverage. In PC, the velocities along each axis are measured from two finite-resolution images, $\tilde{\rho}_1(\vec{r})$ and $\tilde{\rho}_2(\vec{r})$, which are generally acquired independently using bipolar gradients with null zeroth moment, and different first moments, $M_{1,1}$ and $M_{1,2}$, respectively, in each acquisition[1]. Each image may be modeled as $\tilde{\rho}_i(\vec{r}) = \int \tilde{\rho}(\vec{r},v')e^{-j2\pi\kappa_i v'}\,dv'$, where $\kappa_i = (\gamma/2\pi)M_{1,i}$, and $(\gamma/2\pi) = 42.57$ MHz/T for ^{1}H spins[5]. Bipolar gradients are typically designed with first moments such that $\kappa_1 = (4v_{\text{enc}})^{-1}$ and $\kappa_2 = -\kappa_1$, where v_{enc} is the maximum velocity measurable without phase wrapping. The PC velocity map is then calculated from the phase difference between the two images as: $v_{\text{PC}}(\vec{r}) = (v_{\text{enc}}/\pi)\angle(\tilde{\rho}_2(\vec{r})/\tilde{\rho}_1(\vec{r}))$.

Methods: Our hypothesis is that the PC-measured velocity is equal to the mean spin velocity within a voxel, i.e., $v_{\text{PC}}(\vec{r}) \approx \bar{v}(\vec{r})$, where $\bar{v}(\vec{r}) = \int v'\tilde{\rho}(\vec{r},v')\,dv'$. Two-dimensional maps of through-plane velocities, $v(x,y)$, were obtained through computational fluid dynamics simulation of carotid flow[4,6]. A total of 31 maps were created, associated with different 1 mm "slices" along the z axis, covering 3 cm around the bifurcation. Signal intensities were assumed to be uniform throughout the images, i.e., $\rho(x,y) = 1$. The distributions $\tilde{\rho}(x,y,v')$ were calculated as shown above, but we replaced $\delta(v')$ with a symmetrical kernel $\psi(v')$ with FWHM = 1.5 cm/s, in order to allow for a discrete implementation. Grid spacing was 0.16 mm along each of the spatial dimensions, and 1 cm/s along the velocity dimension. We assumed 2DFT acquisitions; hence, $\varphi(x,y) = \text{sinc}(x/\Delta x)\text{sinc}(y/\Delta y)$ was used, with $\Delta x = \Delta y$ (spatial resolutions along x and y) varying from 0.25 to 8 mm. Finally, $v_{\text{PC}}(x,y)$ and $\bar{v}(x,y)$ were calculated (as shown above) and compared. One dimensional profiles $v(x) = v(x,0)$, and $\rho(x) = 1$, were also created for each slice, and $\tilde{\rho}(x,v')$ distributions were calculated, using $\varphi(x) = \text{sinc}(x/\Delta x)$. Grid spacing was 0.04 mm and 0.1 cm/s, and the FWHM of $\psi(v')$ was 0.15 cm/s. Lastly, $v_{\text{PC}}(x)$ and $\bar{v}(x)$ were calculated.

Results: Fig. 1 shows the signal-to-error ratio (SER) between $\bar{v}(x)$ and $v_{\text{PC}}(x)$, as a function of spatial resolution, for all 31 slices. The SER was greater than 30 dB for all resolution values. Fig. 2 shows a qualitative comparison between $\bar{v}(x)$ and $v_{\text{PC}}(x)$, for three slices near the center of the carotid bifurcation, and with $\Delta x = 2$ mm. Similarly, Fig. 3 compares $\bar{v}(x,y)$ and $v_{\text{PC}}(x,y)$, in a slice midway through the bifurcation (z = 0 mm). These representative results show that PC measurements very closely estimate the mean spin velocity within each voxel, even for voxels partially occupied by static spins, or at the viscous sublayer.

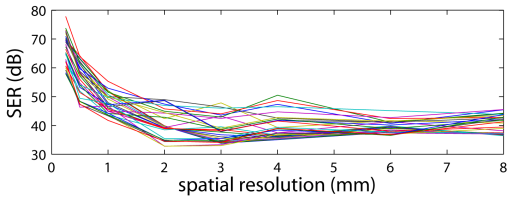

Fig. 1: Signal-to-error ratio between mean velocity and PC velocity, as a function of spatial resolution, for 31 slices, covering 3 cm around the bifurcation.

Discussion: We showed that phase contrast measurements may be accurately modeled as the mean velocity of all spins contained within each voxel. However, two important aspects must be considered in future studies. While we assumed signal intensities to be uniform throughout the images, in-flow enhancement and T_1 contrast must be considered. These could be easily incorporated into our model for $\rho(\vec{r},v')$, and will result in different weights being associated with spins moving with different velocities, or located in different tissues. More importantly, signal loss due to phase dispersion must be considered. Generally, the more disperse the velocity distribution within a voxel is, the lower $|\tilde{\rho}_1|$ and $|\tilde{\rho}_2|$ values are. Hence, such voxels are more susceptible to velocity estimation errors in the presence of noise. With this in mind, the proposed model could be used to better our understanding of partial volume effects in PC MRI velocimetry.

References: [1] O'Donnell M. *Med Phys* 12:59, 1985. [2] Tang C et al. *JMRI* 3:377, 1993. [3] Nishimura DG et al. *MRM* 33:549, 1995. [4] Carvalho JLA et al. *MRM* 63:1537, 2010. [5] Moran PR. *MRI* 1:197, 1982. [6] Ai L et al. *Am J Physiol Cell Physiol* 294:1576, 2008.

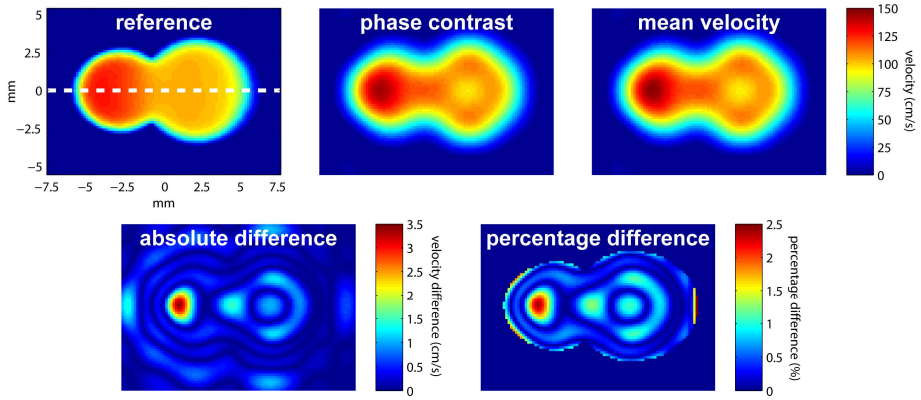

Fig. 3: Comparison between mean velocity and PC velocity maps, for a slice midway through the carotid bifurcation (z = 0 mm). Results correspond to "acquisitions" with 2 mm spatial resolution.

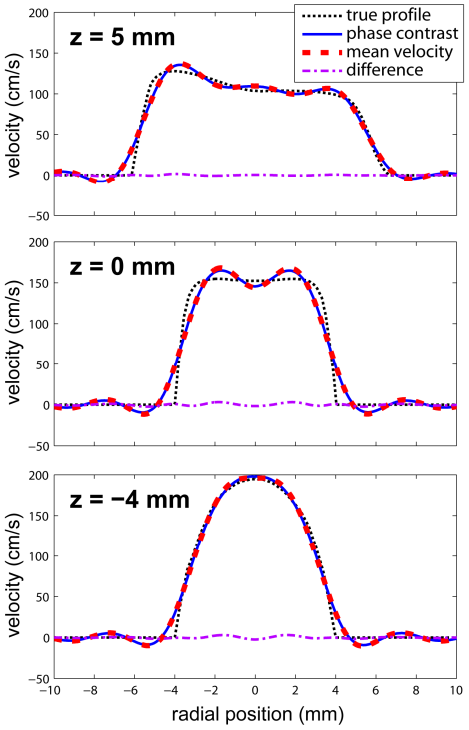

Fig. 2: Comparison between mean velocity and PC velocity profiles, for three slices near the carotid bifurcation (2 mm spatial resolution).