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Does phase contrast MRI provide the mean velocity of the spins within a voxel?

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PGEA — PÓS-GRADUAÇÃO EM ENGENHARIA DE SISTEMAS ELETRÔNICOS E DE AUTOMAÇÃO

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Introduction

- Phase contrast (PC) [1] is the MRI gold standard for measuring blood flow.
- Assumption: all spins within a voxel move at the same velocity.
- Broken if:
  - insufficient spatial resolution;
  - voxel partially occupied by static spins (e.g., vessel wall, plaque);
  - voxel located at viscous sublayer;
  - flow is complex or turbulent (e.g., stenosis, aneurysm).
- Consequence: errors due to partial volume effects [2].
- **Goal**: To investigate the mathematical relationship between the velocity distribution of the spins within a voxel and the PC-measured velocity for that voxel.

Spatial-velocity distribution

Spatial-velocity spin distribution [3,4]:

$$\rho(\vec{r}, v) = \rho(\vec{r})\delta(v - \nu(\vec{r})),$$

where:

- $\rho(\vec{r})$: spin-density map;
- $\nu(\vec{r})$: velocity map;
- $\delta(v)$: Dirac delta function.

Measurement with finite spatial resolution:

$$\tilde{\rho}(\vec{r}, v) = \varphi(\vec{r}) * \rho(\vec{r}, v),$$

where:

- $\varphi(\vec{r})$: spatial blurring kernel (point-spread function associated with k-space coverage).

Phase contrast

In PC, $\nu(\vec{r})$ is calculated from the phase difference between two finite-resolution images $\tilde{\rho}_1(\vec{r})$ and $\tilde{\rho}_2(\vec{r})$, as:

$$\nu_{\text{PC}}(\vec{r}) = \frac{V_{\text{enc}}}{\pi}\arg(\tilde{\rho}_2/\tilde{\rho}_1),$$

where V_{enc} is the maximum measurable velocity, and

$$\tilde{\rho}_i(\vec{r}) = \int \tilde{\rho}(\vec{r}, v)e^{-j2\pi\kappa_i v}dv,$$

where:

- $\kappa_i = \frac{\gamma}{2\pi}M_{1,i}$
- $M_{1,1}$ and $M_{1,2}$: first moment of the bipolar gradients used when acquiring $\tilde{\rho}_1(\vec{r})$ and $\tilde{\rho}_2(\vec{r})$, respectively [5].

Typically, $\kappa_1 = 1/(4V_{\text{enc}})$ and $\kappa_2 = -\kappa_1$.

Hypothesis

Does the PC-measured velocity in a voxel correspond to the mean velocity of the spins within that voxel?

$$\nu_{\text{PC}}(\vec{r}) \stackrel{?}{=} \bar{\nu}(\vec{r})$$

In time,

$$\bar{\nu}(\vec{r}) = \frac{\int v\tilde{\rho}(\vec{r}, v)dv}{\int \tilde{\rho}(\vec{r}, v)dv}.$$

Methods

- 2D maps of through-plane velocities, $\nu(x, y)$, were obtained through CFD of carotid flow [4,6].
  - 31 maps, one for each 1 mm "slice" along the z axis;
  - Total z-axis coverage: 3 cm around the bifurcation.
- Distributions $\tilde{\rho}(x, y, v)$ were derived from $\nu(x, y)$.
  - Signal intensities assumed spatially invariant: $\rho(x, y) = 1$.
  - $\delta(v)$ replaced w/ symmetrical kernel $\psi(v)$, FWHM $= 1.5$ cm/s.
  - Grid spacing: 0.16 mm along x and y; 1 cm/s along v.
  - Assumed 2DFT acquisitions:
    - Spatial blurring: $\varphi(x, y) = \text{sinc}(x/\Delta x)\,\text{sinc}(y/\Delta y)$
    - Spatial resolution: $\Delta x = \Delta y$ varying from 0.25 to 8 mm.
- $\nu_{\text{PC}}(x, y)$ and $\bar{\nu}(x, y)$ were calculated from $\tilde{\rho}(x, y, v)$, and compared.

- 1D profiles were also created for each "slice":
  - $\nu(x) = \nu(x, 0)$;
  - $\rho(x) = 1$.
- Distributions $\tilde{\rho}(x, v)$ were derived from $\nu(x)$:
  - Spatial blurring: $\varphi(x) = \text{sinc}(x/\Delta x)$;
  - FWHM of $\psi(v)$: 0.15 cm/s;
  - Grid spacing: 0.04 mm, and 0.1 cm/s.
- $\nu_{\text{PC}}(x)$ and $\bar{\nu}(x)$ were calculated from $\tilde{\rho}(x, v)$, and compared.

Results

PC measurements very closely estimate the mean spin velocity within each voxel, even for voxels partially occupied by static spins, or at the viscous sublayer.

- Fig. 1: signal-to-error ratio (SER) between $\bar{\nu}(x)$ and $\nu_{\text{PC}}(x)$ was > 30 dB for all values of Δx, for all 31 slices.
- Fig. 2: qualitative comparison between $\bar{\nu}(x)$ and $\nu_{\text{PC}}(x)$, for three slices around bifurcation ($\Delta x = 2$ mm).
- Fig. 3: quantitative comparison beetween $\bar{\nu}(x, y)$ and $\nu_{\text{PC}}(x, y)$, for slice at z = 5 mm ($\Delta x = \Delta y = 2$ mm).

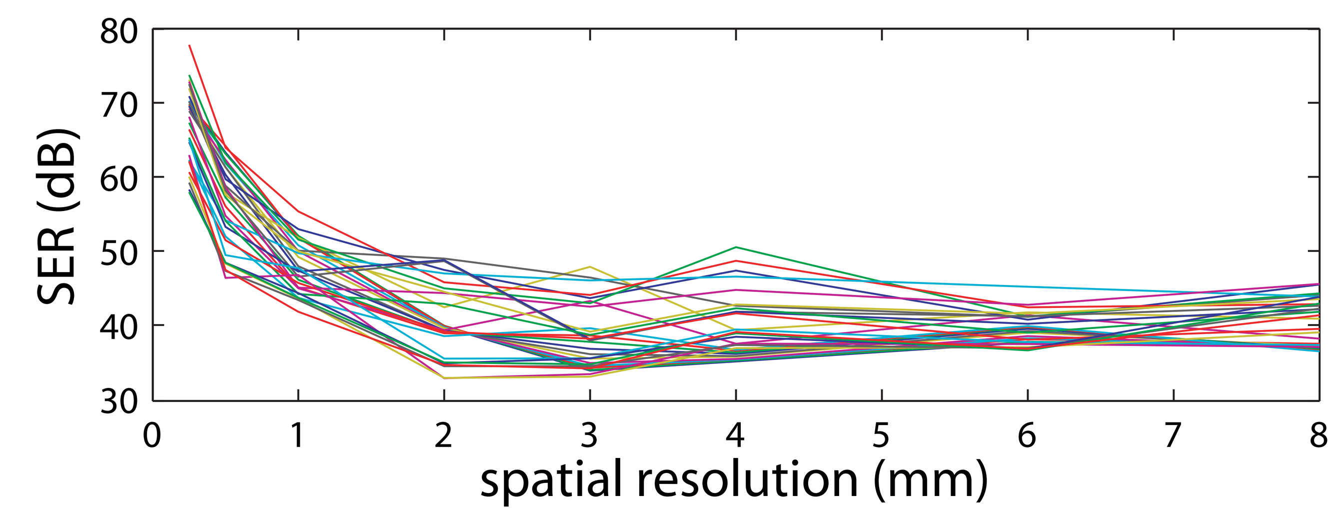

Fig. 1: Signal-to-error ratio between mean velocity and PC velocity, as a function of spatial resolution, for 31 slices.

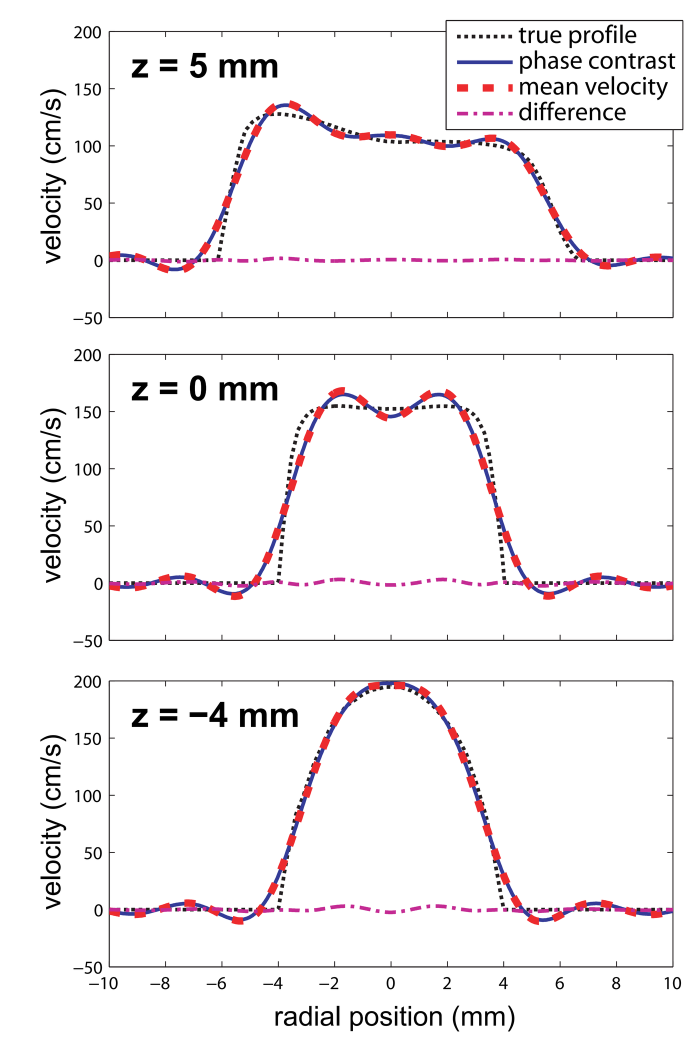

Fig. 2: Comparison between mean velocity and PC velocity profiles, for three slices near the carotid bifurcation (resolution: 2 mm).

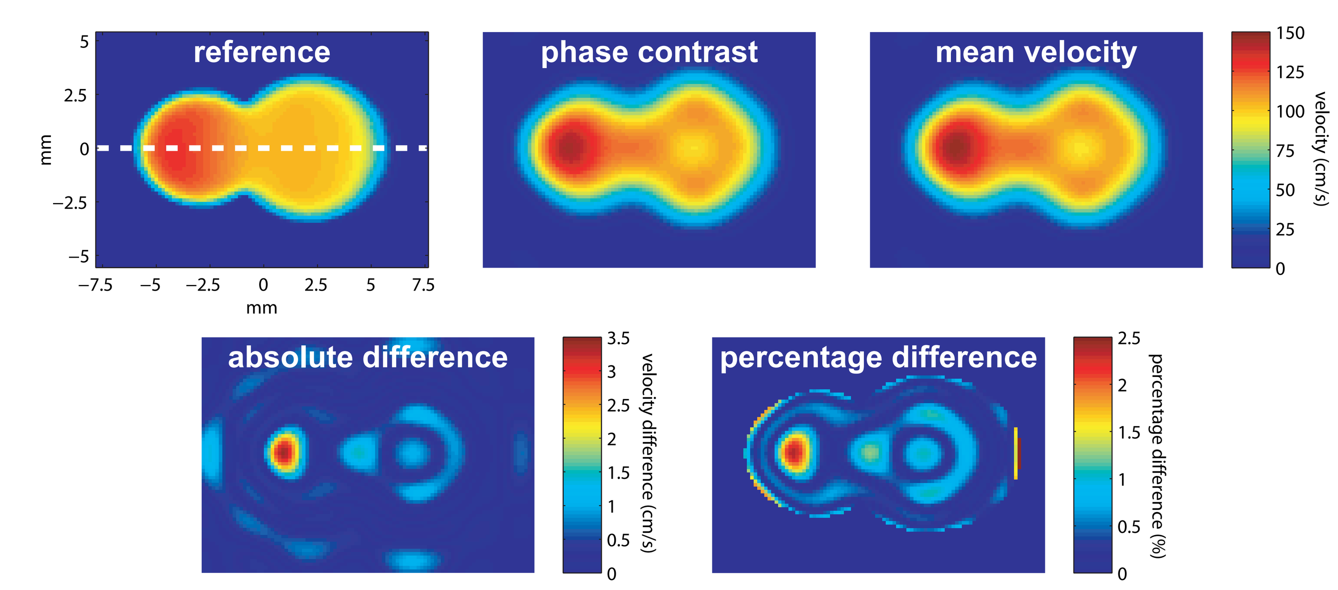

Fig. 3: Comparison between mean velocity and PC velocity maps, for a slice at z = 5 mm (resolution: 2 mm).

References

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