Finite-Element Computational Fluid Dynamics Simulations Constrained by Phase-Contrast MRI Data

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Introduction

- Hybrid PC-MRI/CFD solvers [1-4] can be used to:
  - reduce noise in PC-MRI data;
  - enforce PC-MRI data to satisfy fluid physics equations;
  - generate CFD-MRI data that is closer to PC-MRI than pure CFD.
- On the literature, hybrid solvers' fluid mechanics equations are solved using:
  - finite volume method and SIMPLER algorithm [1,3-4];
  - commercial software and synthetic data [2].
- **Goal**: implement a free hybrid PC-MRI/CFD solver based on Finite Element Method (FEM)*.
  - FEM is more flexible with complex geometries and boundary conditions [5];
  - FEM allows higher order approximations [5].

Numerical Procedure

- Numerical procedure is based on the solution of the PDE-constrained optimization

$$\min_{\mathbf{u}} \frac{1}{2}\int_{\Omega} |\mathbf{u}-\mathbf{u}_{\mathrm{mri}}|^2 d\Omega,$$

subject to the steady incompressible 2D Navier-Stokes-continuity system of equations [5]

$$\rho\mathbf{u}\cdot\nabla\mathbf{u} = -\nabla p + \mu\Delta\mathbf{u} \ \text{ and } \ \nabla\cdot\mathbf{u} = 0.$$

- Equations were discretized using the method of weighted residuals, which yelds a linear system [5]

$$\mathbf{Jc} = -\mathbf{R},$$

$\mathbf{J}$ is the residuals' Jacobian matrix and $\mathbf{R}$ is the residuals vector.

- Simulated velocity field $\mathbf{c}$ is then obtained solving the quadratic minimization problem

$$\min_{\mathbf{c}} \frac{1}{2}|\mathbf{Jc}+\mathbf{R}|_2^2 + \frac{\lambda}{2}|\mathbf{Sc}-\mathbf{u}_{\mathrm{mri}}|_2^2,$$

where $\mathbf{S}$ is an adjustment matrix, since MRI measurements are acquired on a coarse grid.

Experiments

- 4D PC-MRI data acquired for a flow phantom (Fig.1)
  - 32-channel head coil; resolution: $0.5\times0.5\times1.0\ \mathrm{mm}^3$; FOV: $4.0\times3.5\times5.0\ \mathrm{cm}^3$; NEX: 10; Venc: $50\ \mathrm{cm/s}$; scan time: 5 hours.
- Two experiments were performed:
  - Combined solution with x and y PC-MRI velocity components constraining CFD for different λ;
  - Combined solution with x and y PC-MRI velocity components corrupted by 7.5cm/s Gaussian noise constraining CFD for different λ;
- CFD assumptions:
  - $\rho = 1100\ \mathrm{kg/m^3}$; $\mu = 0.005\ \mathrm{Pa\cdot s}$; voxel size $1.0\times0.5\ \mathrm{mm}^2$; elements Q_2P_{-1}; no-slip boundary condition.

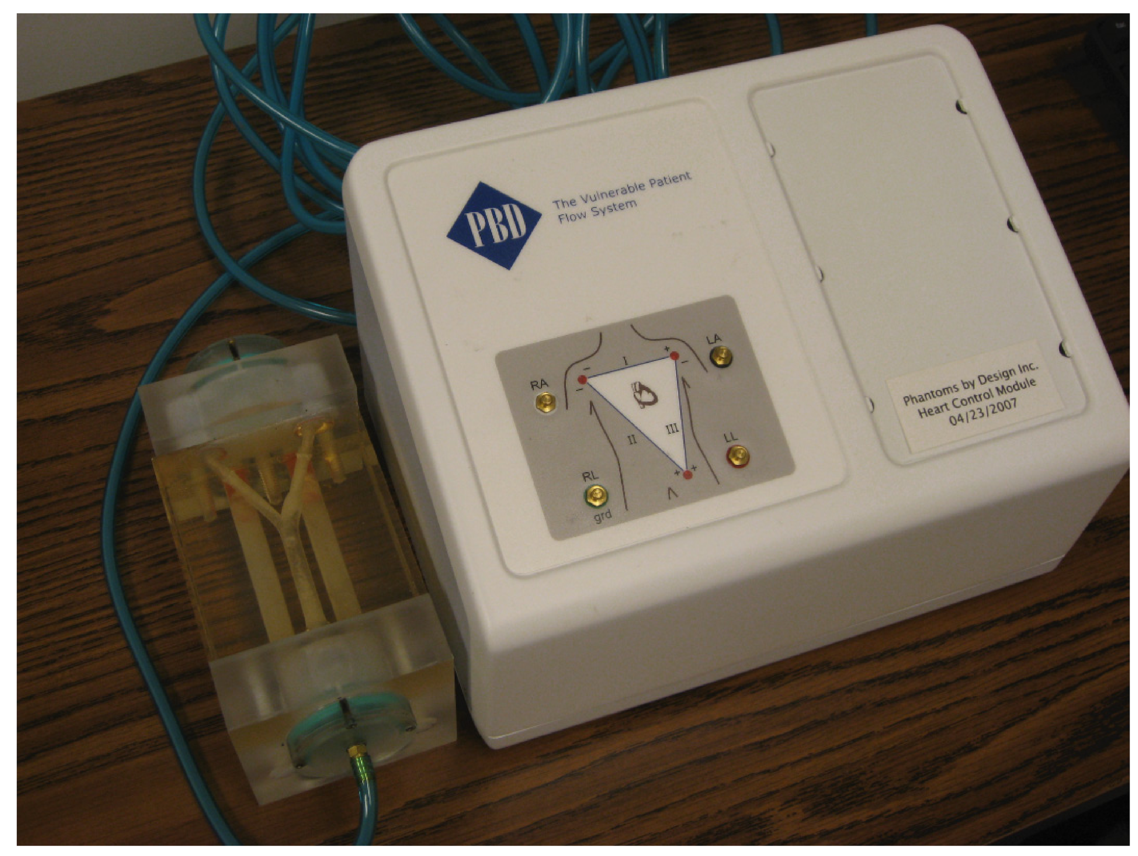

Figure 1: Pulsatile carotid flow phantom (Phantoms by Design, Inc., Bothell, WA).

Results and discussion

- Constrained solution (Fig.2(c)) is qualitatively closer to the measured PC-MRI data (Fig.2(a)) than pure CFD (Fig.2(b)).
- This behavior is quantitatively confirmed using the signal-to-error ratio (SER), considering measured PC-MRI as "ground truth":
  - pure CFD solution provided 4.53dB SER;
  - MRI-constrained CFD solution provided 8.17dB SER;
  - In Fig.1(b), the misbehavior of the solution near the bifurcation is caused by the difference between the Navier-Stokes equation's diffusive term ($\Delta^2\mathbf{u}$) and the convective term ($\mathbf{u}\cdot\nabla\mathbf{u}$), requiring a numerical stabilization term.
- On the denoising experiment, the constrained solution (Fig.3(c)) corrected the noisy PC flow (Fig.3(b)), leading to a solution that is very similar to the original PC-MRI data.
- This was quantitatively confirmed using the signal-to-noise ratio (SNR), considering PC-MRI as "ground truth":
  - noisy PC-MRI has 5.39dB SNR;
  - noisy MRI-constrained CFD solution provided 6.86dB SNR.

Conclusion and Future Works

- Hybrid solutions are closer to PC-MRI than pure CFD solution satisfying fluid equations.
- Works as a noise reduction technique.
- Convergence of hybrid solution is faster than pure CFD solution.
- FW: implementation of a stabilization term, in order to avoid numerical errors [6].
- FW: was proposed in [4] an undersampled PC-MRI reconstruction method with synthetic CFD data as *a priori* information. It is possible to adapt the method in [4], so the reconstruction can use N-S equations as a constrain for the l_1-minimization problem.

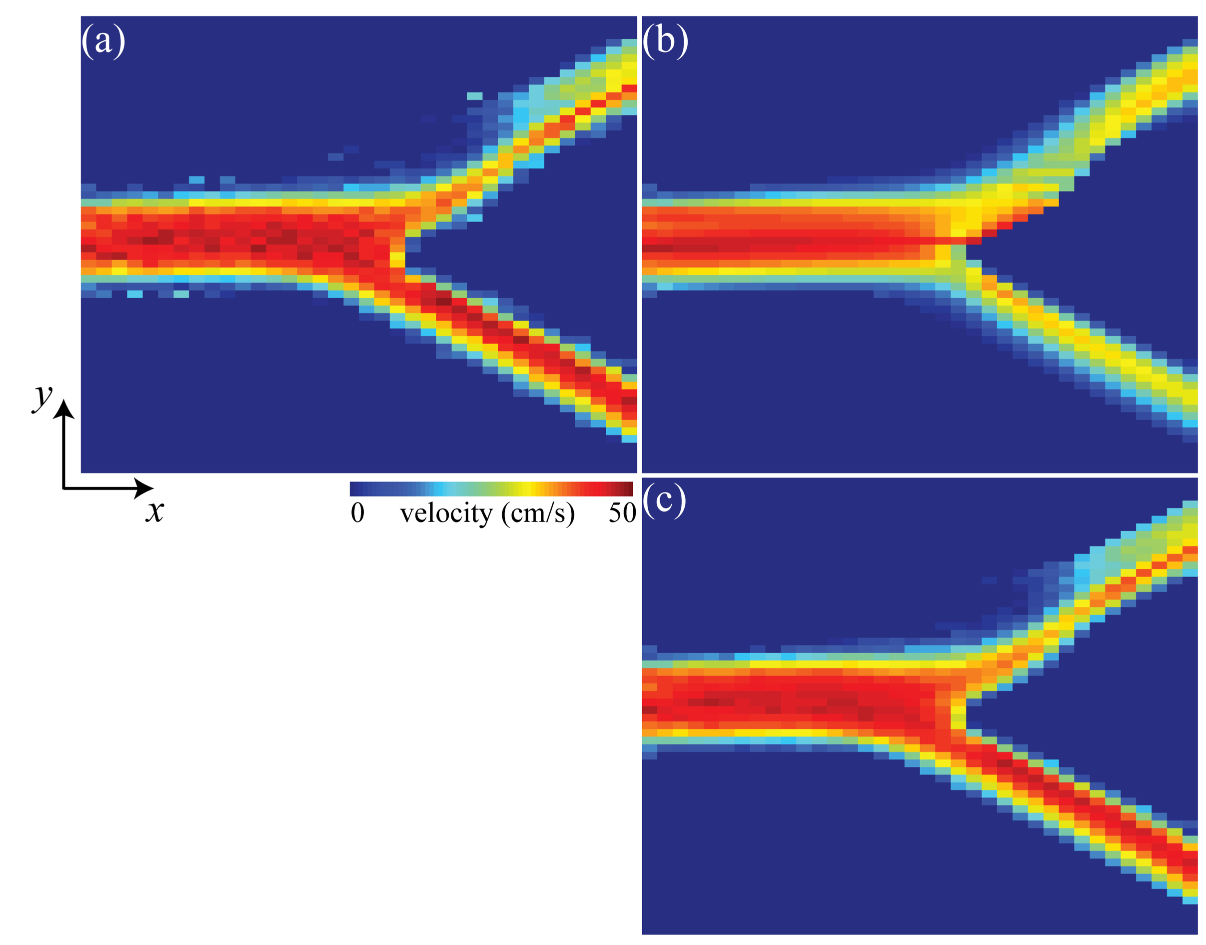

Figure 2: Velocity component on x direction: (a) measured PC-MRI data; (b) pure CFD solution ($\lambda = 0$); and (c) CFD solution constrained by PC-MRI ($\lambda = 10^{-2}$).

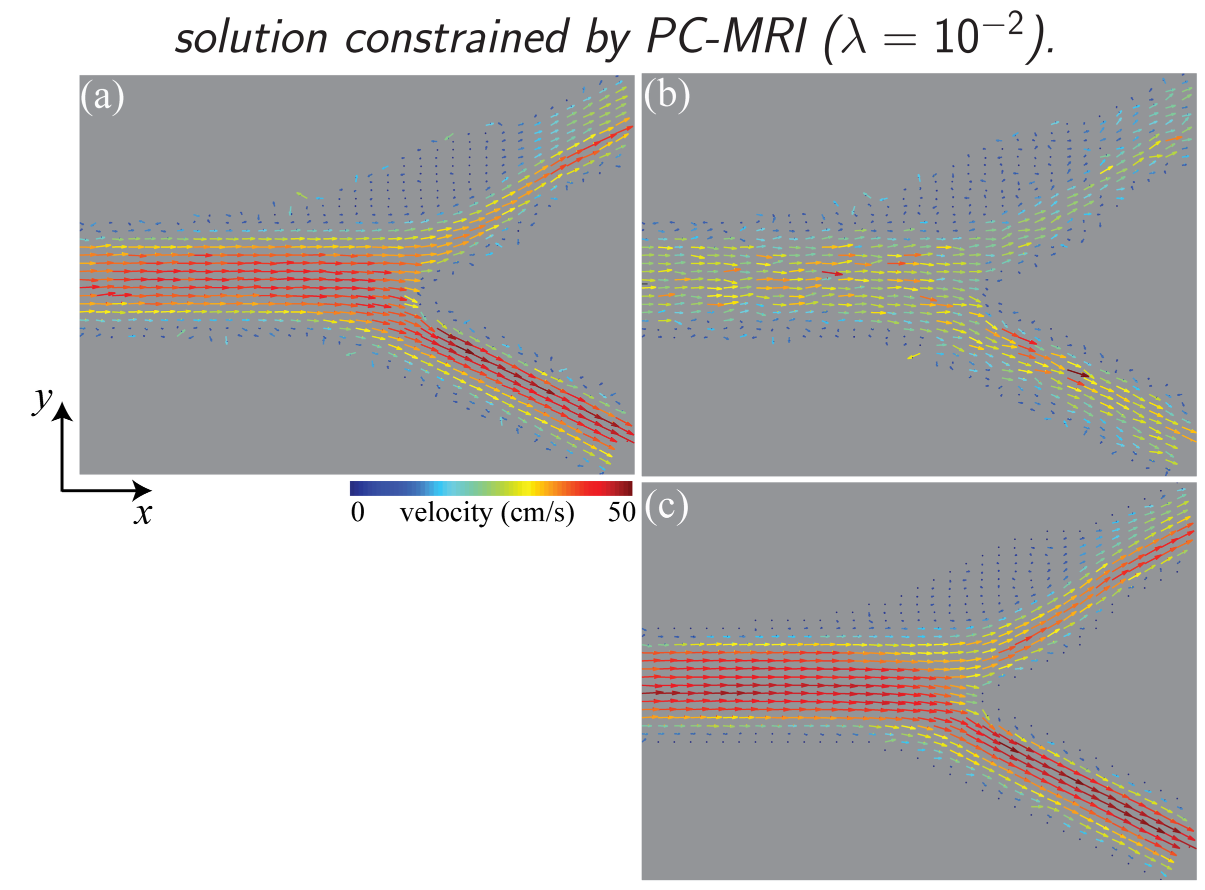

Figure 3: Vector field visualization: (a) measured PC-MRI data; (b) PC-MRI corrupted by Gaussian noise (standard deviation: 7.5cm/s); and (c) CFD solution constrained by noisy PC-MRI ($\lambda = 10^{-3}$).

References

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[*] MATLAB code avaliable at: http://bit.do/vrispoli

Support

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