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Estimating Highly Accurate Velocity Maps From FVE MRI Data Using a PDE-Constrained Optimization (#2930)

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Introduction

- Fourier velocity encoding (FVE) [1] provides considerably higher SNR than phase contrast (PC), and is robust to partial-volume effects.
- FVE data can be acquired fast with low spatial resolution.
- FVE provides the velocity distribution associated with a large voxel, but does not directly provides a velocity map.
- Previous methods for estimating velocity maps from FVE distributions were already proposed [2].
- **Goal**: proposed a better method for estimate high-spatial-resolution velocity maps from low-resolution FVE data based on the FVE signal model and a flow physics model

Methods

- FVE spatial-velocity distribution, $\hat{s}(x,y,v)$, model is given by [3]:

$$\hat{s}(x,y,w) = \left[m(x,y) \times \mathrm{sinc}\left(\frac{v - v_z(x,y)}{\Delta v}\right)\right] * \psi(x,y)$$

- Navier–Stokes equation gives the blood flow model [4]:

$$\rho\left(\frac{\partial \mathbf{v}}{\partial t} + \mathbf{v}\cdot\nabla\mathbf{v}\right) = -\nabla p + \mu\nabla^2\mathbf{v}.$$

- Velocity map can be estimated from a measured FVE dataset through the PDE-constrained optimization problem:

$$\min_{v_z} \sum_{k=1}^{K} \int_\Omega \left\{ f(\mathbf{x}, v_k) - \left[m(\mathbf{x}) \times \mathrm{sinc}\left(\frac{v_k - v_z}{\Delta v}\right)\right] * \psi(\mathbf{x}) \right\}^2 dA$$

subject to

$$\rho\mathbf{v}\cdot\nabla\mathbf{v} = -\nabla p + \mu\nabla^2\mathbf{v}.$$

- N-S equations were discretized using the method of weighted residuals yelding a linear system [5,6]

$$\mathbf{J}\mathbf{c} = -\mathbf{R},$$

J is the residuals' Jacobian matrix and **R** is the residuals vector.

- Numerically the PDE-contrained optimization problem can be written as:

$$\min_{\mathbf{v}_z} \sum_{k=1}^{K} \left\| \mathbf{f}_k - \left[\mathbf{m} \times \mathrm{sinc}\left(\frac{v_k - \mathbf{v}_z}{\Delta v}\right)\right] * \psi \right\|_{\ell_2}^2 + \lambda \left\| \mathbf{J}[\mathbf{v}_x; \mathbf{v}_z; \mathbf{p}] - \mathbf{r} \right\|_{\ell_2}^2.$$

Experiments

- One experiment was performed:
  1. Spiral FVE data was simulated from acquired PC data for $\Delta r = 1mm$ and $\Delta r = 2mm$;
  2. then the optimization was solved;
  3. resultant velocity map was compared with the acquired PC velocity map
- 4D PC-MRI data acquired for a flow phantom (Fig.1)
  - 32-channel head coil; resolution: $0.5 \times 0.5 \times 1.0\ \mathrm{mm}^3$; FOV: $4.0 \times 3.5 \times 5.0\ \mathrm{cm}^3$; NEX: 10; Venc: $50\ \mathrm{cm/s}$; scan time: 5 hours.
- CFD assumptions:
  - $\rho = 1100\ \mathrm{kg/m}^3$; $\mu = 0.005\ \mathrm{Pa\cdot s}$; voxel size $1.0 \times 0.5\ \mathrm{mm}^2$; elements Q_2P_{-1}; no-slip boundary condition.

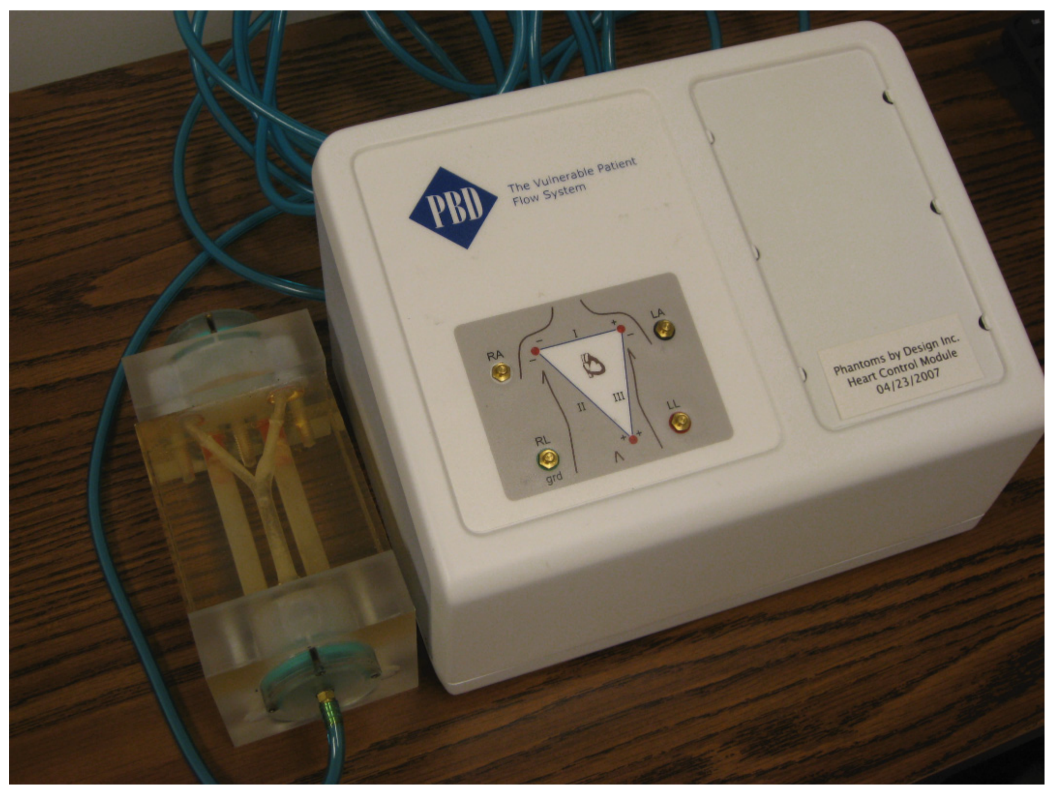

Figure 1: Pulsatile carotid flow phantom (Phantoms by Design, Inc., Bothell, WA).

Results and discussion

- The velocity maps estimated from simulated low spatial resolution FVE data (Figs.(2)-(3)b and Figs.(2)-(3)c) are very similar qualitatively to the reference map (Figs.(2)-(3)a).
- At first glance can be noted that the velocity map obtained using the previous method previously proposed[2] (Figs.(2)-(3)c) is more similar to the acquired PC-MRI velocity map (Figs.(2)-(3)a) than the proposed method (Figs.(2)-(3)b).
- However the error images show that the velocity map obtained using the technique proposed in this work (Figs.(2)-(3)b) was more accurate than the one obtained with the previous method[2] (Figs.(2)-(3)c) for both resolution.
- Quantitative comparison was also performed based on the signal-to-error ratio (SER) calculated (in decibels) as:

$$\mathrm{SER} = 10\log_{10}\left(\frac{\sum_{i,j}\left\|v_{\mathrm{pc}}(i,j)\right\|^2}{\sum_{i,j}\left\|v_e(i,j) - v_{\mathrm{pc}}(i,j)\right\|^2}\right),$$

where v_{pc} is the acquired phase contrast velocity map used as the ground-truth signal and v_e is the estimated velocity map.

- Measured SER, relative to the PC reference, was:
  - Proposed method 50.64 dB and 44.63 dB for $\Delta r = 1$ mm and $\Delta r = 2$ mm resolution, respectively.
  - Previously proposed[2] method 32.02 dB and 28.68 dB, for $\Delta r = 1$ mm and $\Delta r = 2$ mm resolution, respectively.

Conclusion

- Was proposed a novel method for estimating high-resolution velocity maps from low-resolution FVE measurements.
- This method is based on a PDE- constrained optimization that incorporates both the FVE signal model and the Navier-Stokes equation.
- Results showed that it is possible to obtain highly accurate velocity maps from the FVE distributions.
- These good results are important, meaning that FVE may potentially be a substitute of PC imaging, since it contains both a velocity distribution and also velocity map with considerably higher SNR and robustness to partial voluming.

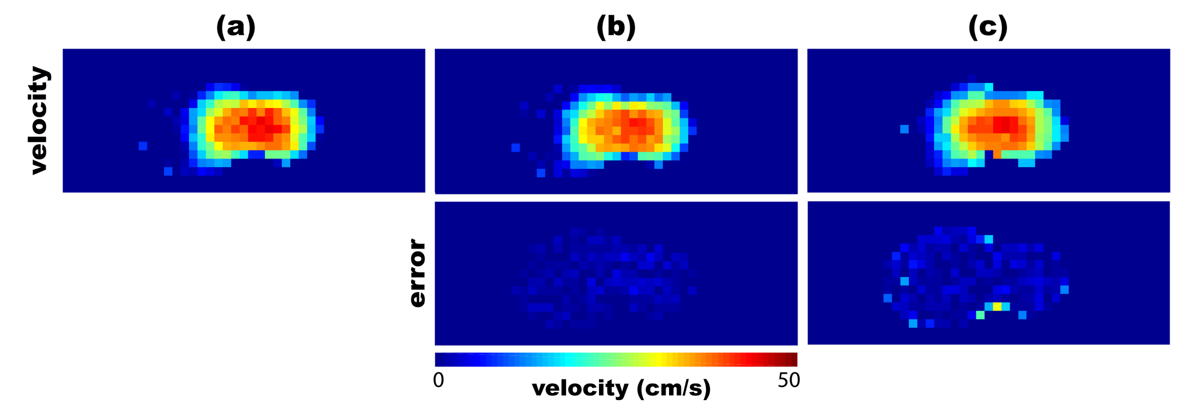

Figure 2: Validation experiment using a pulsatile carotid flow phantom: (a) reference phase contrast velocity map, measured at the phantom's bifurcation; (b) velocity map estimated from the simulated low-resolution spiral FVE data with $\Delta r = 1$ mm spatial resolution with the proposed method (and associated error percentages); and (c) velocity map estimated from the simulated low-resolution spiral FVE data with $\Delta r = 1$ mm spatial resolution with the method proposed in Ref.[2] (and associated error percentages).

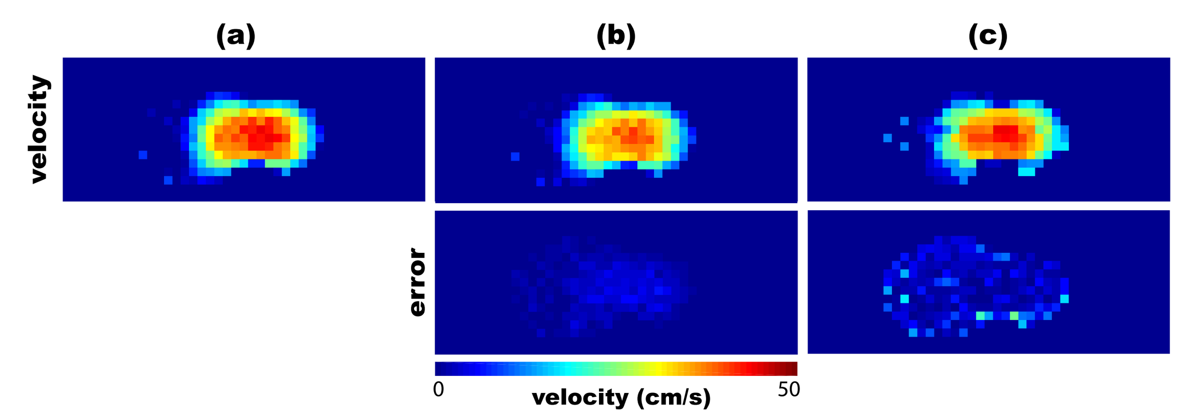

Figure 3: Validation experiment using a pulsatile carotid flow phantom: (a) reference phase contrast velocity map, measured at the phantom's bifurcation; (b) velocity map estimated from the simulated low-resolution spiral FVE data with $\Delta r = 2$ mm spatial resolution with the proposed method (and associated error percentages); and (c) velocity map estimated from the simulated low-resolution spiral FVE data with $\Delta r = 2$ mm spatial resolution with the method proposed in Ref.[2] (and associated error percentages).

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Support

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