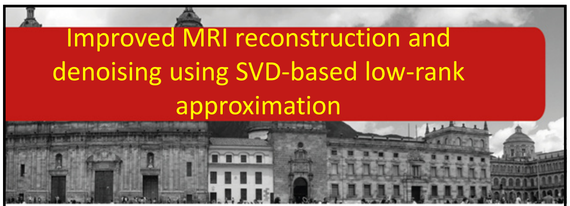

# Improved MRI reconstruction and denoising using SVD-based low-rank approximation

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## Sumary

- Motivation
- Magnetic Resonance Imaging (MRI)
- SVD
- Problem
- AIC
- Methods and Results
- Conclusion
- Bibliography



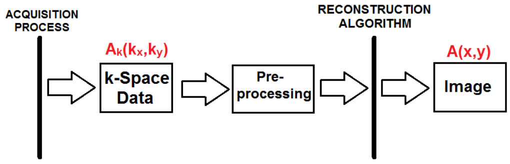
## Motivation

- Reconstruction of multi-dimensional magnetic resonance imaging (MRI) data
  - computationally demanding task
- Therefore, data compression can be applied:
  - to reduce reconstruction complexity and memory requirements,
  - and for denoising.



## Magnetic Resonance Imaging

- Magnetic Resonance Imaging acquisition and reconstruction process






## Magnetic Resonance Imaging

- The Fourier Transform of the image

$$A(k_x,k_y) = \int_x \int_y A(x,y)\, e^{-j2\pi(k_x x + k_y y)}\, dx\, dy$$

- The Fourier coordinates

$$k_x(t) = \frac{\gamma}{2\pi}\int_0^t G_x(\tau)\, d\tau$$

$$k_y(t) = \frac{\gamma}{2\pi}\int_0^t G_y(\tau)\, d\tau$$



## Singular Value Decomposition

$\boldsymbol{X} \in \mathbb{C}^{M\times N}$, rank$(\boldsymbol{X}) = r$

"Full SVD"

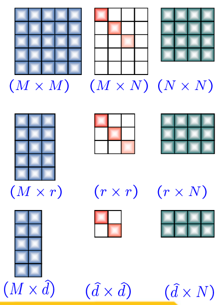
$\boldsymbol{X} = \boldsymbol{U} \cdot \boldsymbol{\Sigma} \cdot \boldsymbol{V}^H$

"Economy size SVD"

$\boldsymbol{X} = \boldsymbol{U}_s \cdot \boldsymbol{\Sigma}_s \cdot \boldsymbol{V}_s^H$

Low-rank approximation

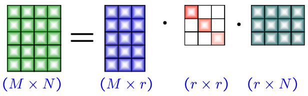
$\boldsymbol{X} \approx \boldsymbol{U}_s \cdot \boldsymbol{\Sigma}_s \cdot \boldsymbol{V}_s^H$



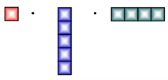
## Singular Value Decomposition

- Decomposition into a sum of rank one matrices:

$$\boldsymbol{X} = \boldsymbol{U} \cdot \boldsymbol{\Sigma} \cdot \boldsymbol{V}^H$$




$$= \sum_{n=1}^{r} \sigma_n \cdot \boldsymbol{u}_n \cdot \boldsymbol{v}_n^H$$





## Proposal

- Apply a low-rank approximation singular value decomposition to MRI data in order to:
  - remove noise components
    - verified through the Signal to Error Ratio
  - reduce the storage memory
    - with a high quality level



## Akaike Information Criterion

- Akaike Information Criterion
  - based on the difference between the maximum likelihood function and the free parameters.
- We shall find *m* that minimizes:

$$\mathrm{AIC}(m) = -\ln(\mathcal{L}(m)) + \kappa(m)$$

where

$\ln(\mathcal{L}(m))$ is the loglikelihood function.

$\kappa(m)$ are the free parameters.

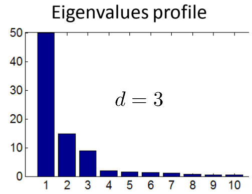


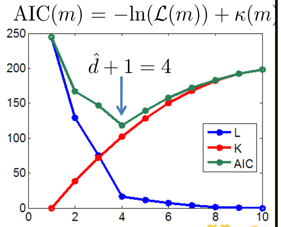
## Akaike Information Criterion

- The Loglikelihood as function of the eingevalues proposed by Wax and Kailath.

$$\mathrm{AIC}(m) = -\ln(\mathcal{L}(m)) + \kappa(m)$$

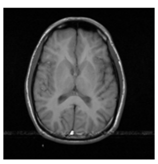
Eigenvalues profile



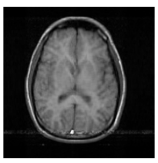






## Results

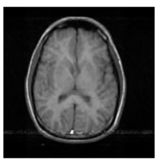


Original ground-truth image



Denoised in Image Domain via AIC/SVD

$\hat{d} = 30$



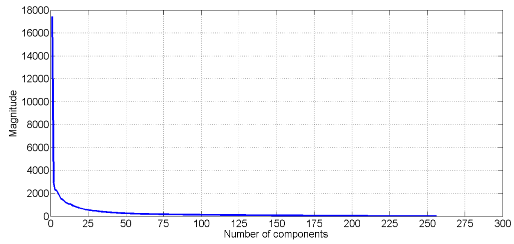
Denoised in Frequency Domain via AIC/SVD

$\hat{d} = 30$



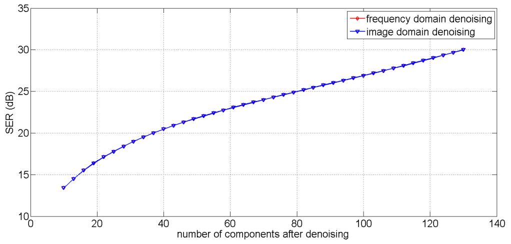
## Results

SVD profile in frequency domain



# Results

SER according to the number of components





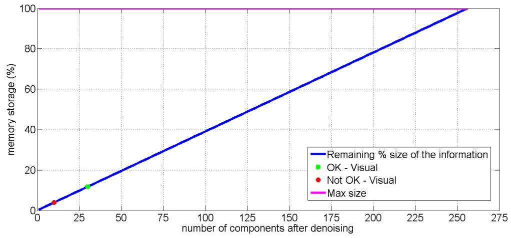

# Magnetic Resonance Imaging

| Number of Components | SER (dB) frequency domain | SER (dB) image domain | RMSE (dB) |
|---|---|---|---|
| 10 | 13.4 | 13.4 | −6.7 |
| 22 | 17.1 | 17.1 | −8.6 |
| 31 | 19.0 | 19.0 | −9.5 |
| 43 | 20.9 | 20.9 | −10.5 |
| 55 | 22.4 | 22.4 | −11.2 |
| 67 | 23.7 | 23.7 | −11.9 |
| 79 | 24.9 | 24.9 | −12.5 |
| 88 | 25.8 | 25.8 | −12.9 |
| 97 | 26.7 | 26.7 | −13.3 |
| 109 | 27.9 | 27.9 | −13.9 |
| 118 | 28.8 | 28.8 | −14.4 |
| 124 | 29.4 | 29.4 | −14.7 |
| 130 | 30.1 | 30.1 | −15.0 |



# Results

Memory storage according to the number of components






# Conclusion

- We propose a SVD-based low rank approximation for MRI data.
- Our approach:
  - can be applied for denoising;
  - can reduce significantly the memory storage;
  - can be applied in image and frequency domain.



# Future Works

- Extension for the low-rank approximation HOSVD
  - The MRI data is usually three dimensional
    - two dimensions related to image and one related to time



Thank you !

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