Image Processing

Spatial Transforms

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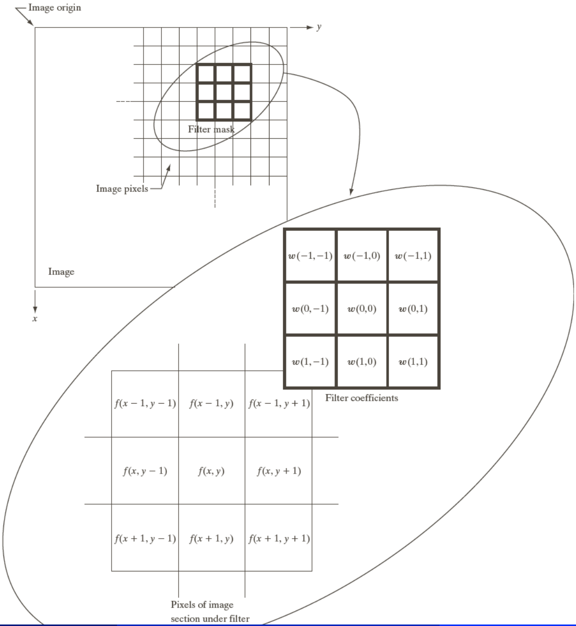
Class 04:

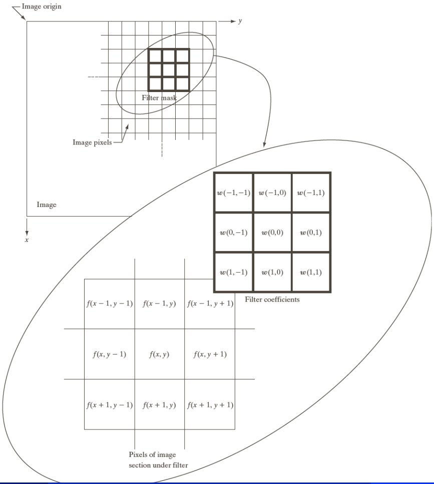
Sumário

- Spatial Filtering

Spatial Filtering



$$g(x,y) = \sum_{s=-a}^{a} \sum_{t=-b}^{b} w(s,t) f(x+s, y+t)$$

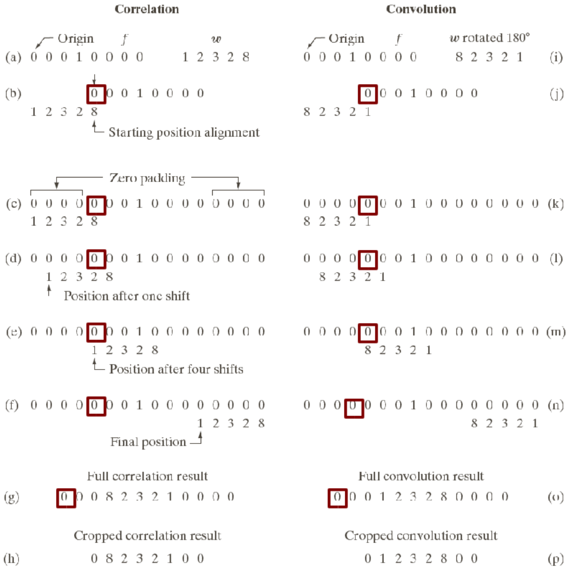


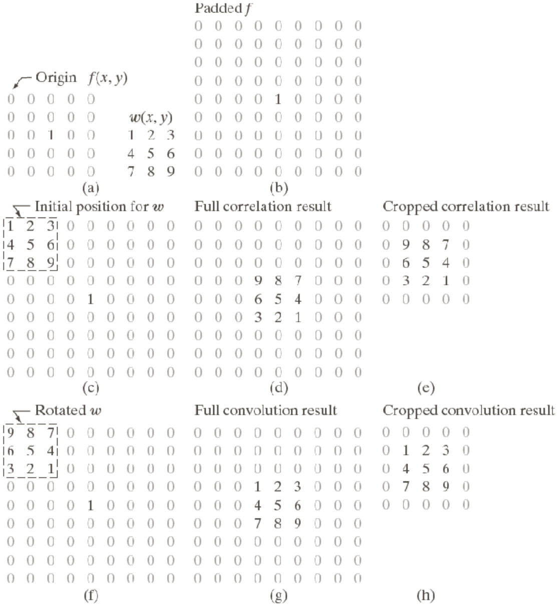
Spatial Filtering

$$R = w(-1,-1)f(x-1,y-1) + w(-1,0)f(x-1,y) + \dots$$
$$+ w(0,0)f(x,y) + \dots + w(1,0)f(x+1,y0) + w(1,1)f(x+1,y+1)$$

w_1	w_2	w_3
w_4	w_5	w_6
w_7	w_8	w_9

$$R = w_1 z_1 + w_2 z_2 + \dots w_m n z_m n$$
$$= \sum_{i=1}^{mn} w_i z_i$$


Correlation
Convolution
Origin f w
(a) 0 0 0 1 0 0 0 0 1 2 3 2 8
Origin f w rotated 180°
0 0 0 1 0 0 0 0 8 2 3 2 1 (i)
(b) 0 0 0 1 0 0 0 0
1 2 3 2 8
Starting position alignment
0 0 0 1 0 0 0 0 (j)
8 2 3 2 1
Zero padding
(c) 0 0 0 0 0 0 0 1 0 0 0 0 0 0 0 0
1 2 3 2 8
0 0 0 0 0 0 0 1 0 0 0 0 0 0 0 0 (k)
8 2 3 2 1
(d) 0 0 0 0 0 0 0 1 0 0 0 0 0 0 0 0
1 2 3 2 8
Position after one shift
0 0 0 0 0 0 0 1 0 0 0 0 0 0 0 0 (l)
8 2 3 2 1
(e) 0 0 0 0 0 0 0 1 0 0 0 0 0 0 0 0
1 2 3 2 8
Position after four shifts
0 0 0 0 0 0 0 1 0 0 0 0 0 0 0 0 (m)
8 2 3 2 1
(f) 0 0 0 0 0 0 0 1 0 0 0 0 0 0 0 0
1 2 3 2 8
Final position
0 0 0 0 0 0 0 1 0 0 0 0 0 0 0 0 (n)
8 2 3 2 1
Full correlation result
(g) 0 0 0 8 2 3 2 1 0 0 0 0
Full convolution result
0 0 0 1 2 3 2 8 0 0 0 0 (o)
Cropped correlation result
(h) 0 8 2 3 2 1 0 0
Cropped convolution result
0 1 2 3 2 8 0 0 (p)


Padded f
Origin f(x, y)
w(x, y)
(a)
(b)
Initial position for w
Full correlation result
Cropped correlation result
(c)
(d)
(e)
Rotated w
Full convolution result
Cropped convolution result
(f)
(g)
(h)

Spatial Filtering

$$R = w(-1,-1)f(x-1,y-1) + w(-1,0)f(x-1,y) + \dots$$
$$+ w(0,0)f(x,y) + \dots + w(1,0)f(x+1,y0) + w(1,1)f(x+1,y+1)$$

w_1	w_2	w_3
w_4	w_5	w_6
w_7	w_8	w_9

$$R = w_1 z_1 + w_2 z_2 + \dots w_m n z_m n$$
$$= \sum_{i=1}^{mn} w_i z_i$$

$$\frac{1}{9} \times \begin{array}{|c|c|c|} \hline 1 & 1 & 1 \\ \hline 1 & 1 & 1 \\ \hline 1 & 1 & 1 \\ \hline \end{array} \qquad \frac{1}{16} \times \begin{array}{|c|c|c|} \hline 1 & 2 & 1 \\ \hline 2 & 4 & 2 \\ \hline 1 & 2 & 1 \\ \hline \end{array}$$

$$R = \frac{1}{9}\sum_{i=1}^{mn} z_i$$

ou, genericamente:

$$R = \frac{\sum_{s=-a}^{a}\sum_{t=-b}^{b} w(s,t)f(x+s,y+t)}{\sum_{s=-a}^{a}\sum_{t=-b}^{b} w(s,t)}$$

Eliminating Details

Filters: $m = 3, 5, 9, 15,$ e $35.$

Eliminating Details

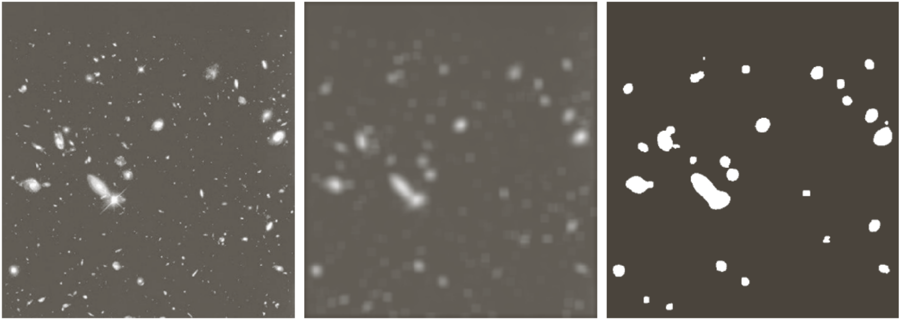
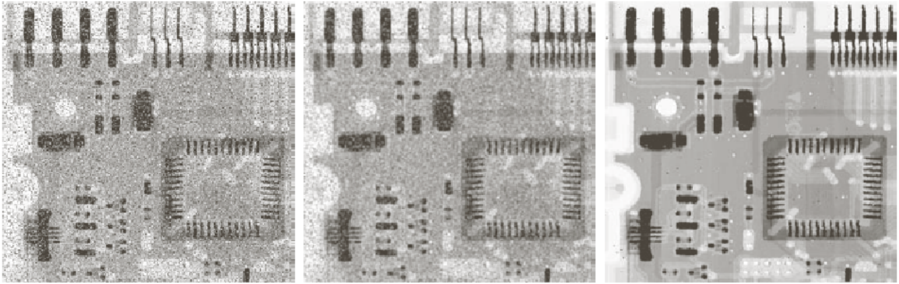


Image with 528 x 485 pixels. Filter with a 15 $\times$ 15 filter, followed by a thresholding operation.

Filters Based on Statistics

- Median:
  - Eliminates the pixels whose properties differ from the properties of the neighboring pixels;
  - Isolated Areas ($< m^2/2$ of the neighborhood) are eliminated;
  - Salt and paper
- Max, Min, percentile, etc.

Median



a b c

FIGURE 3.35 (a) X-ray image of circuit board corrupted by salt-and-pepper noise. (b) Noise reduction with a 3×3 averaging mask. (c) Noise reduction with a 3×3 median filter. (Original image courtesy of Mr. Joseph E. Pascente, Lixi, Inc.)

Sharpening Spatial Filters

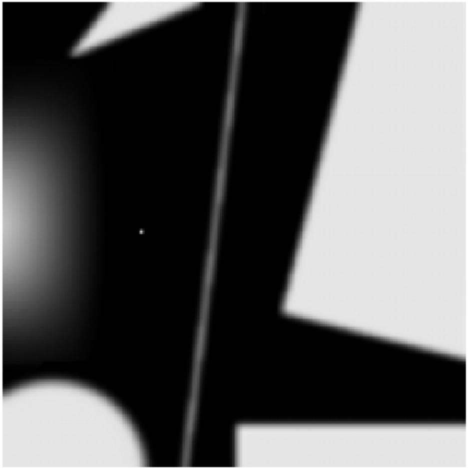
- 1st order derivative (discrete)

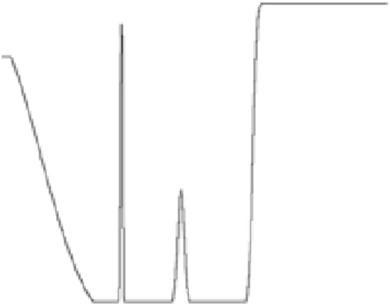
$$\frac{\partial f}{\partial x} = f(x+1) - f(x)$$

- 2nd Dorder derivative (discrete)

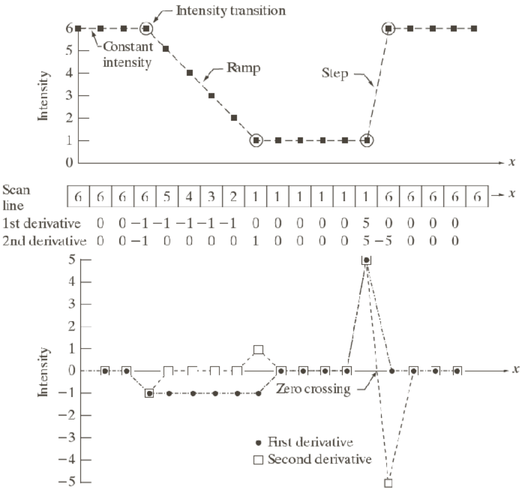
$$\frac{\partial^2 f}{\partial x^2} = f(x+1) + f(x-1) - 2f(x)$$

Sharpening Spatial Filters





Sharpening Spatial Filters



Laplacian Filters

$$\nabla^2 f = \frac{\partial^2 f}{\partial x^2} + \frac{\partial^2 f}{\partial y^2}$$

onde

$$\frac{\partial^2 f}{\partial x^2} = f(x+1, y) + f(x-1, y) - 2f(x, y)$$

e

$$\frac{\partial^2 f}{\partial y^2} = f(x, y+1) + f(x, y-1) - 2f(x, y)$$

Logo

$$\boxed{\nabla^2 f = [f(x+1, y) + f(x-1, y) + f(x, y+1) + f(x, y-1)] - 4f(x, y)}$$

Laplacian Filters

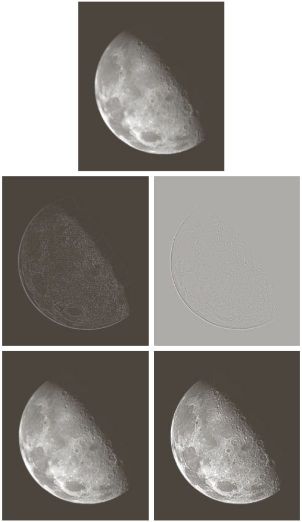
0	1	0
1	−4	1
0	1	0

1	1	1
1	−8	1
1	1	1

0	−1	0
−1	4	−1
0	−1	0

−1	−1	−1
−1	8	−1
−1	−1	−1

Laplacian Filters



(a) blurred image, (b) Laplacian (no scaling), (c) Laplacian (scaling), (d) Original image + (a), (d) Original image + (b)

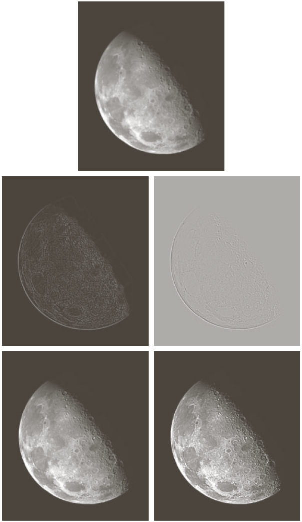
Steps:

1. $\bar{f}(x, y) = \text{conv}(h_{LP}(x, y), f(x, y)) = h_{LP}(x, y) * f(x, y)$

2. $g_{mask} = f(x, y) - \bar{f}(x, y)$

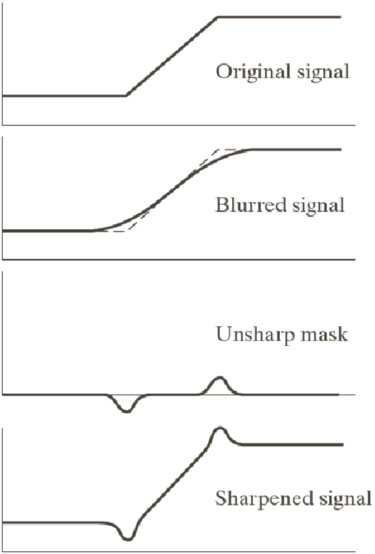
3. $g(x, y) = f(x, y) + k \cdot g_{mask}(x, y)$

Unsharp Masking



(d) Unsharp masking.

Unsharp Masking



PSteps:

$$\boxed{f_{hb}(x,y) = Af(x,y) - \bar{f}(x,y)}$$

$$f_{hb}(x,y) = (A-1)f(x,y) + f(x,y) - \bar{f}(x,y)$$

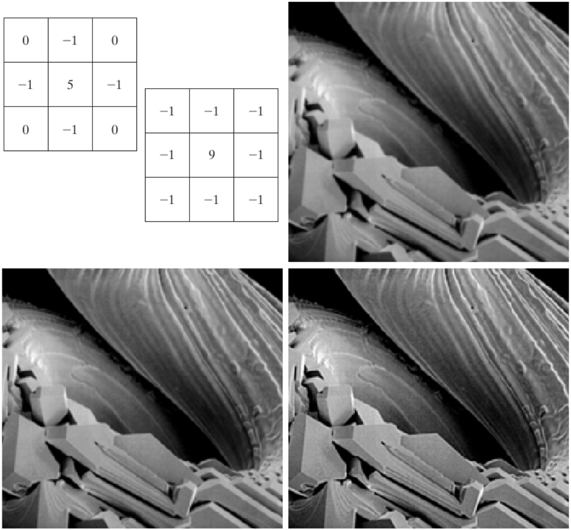
$$\boxed{f_{hb}(x,y) = (A-1)f(x,y) - f_s(x,y)}$$

0	−1	0
−1	$A+4$	−1
0	−1	0

−1	−1	−1
−1	$A+8$	−1
−1	−1	−1

High Boost

original, borrado com Gaussiano, unsharp mask, resultado do unsharp mask, resultado do high-boost



0	−1	0
−1	5	−1
0	−1	0

−1	−1	−1
−1	9	−1
−1	−1	−1

a b c
d e

FIGURE 3.41 (a) Composite Laplacian mask. (b) A second composite mask. (c) Scanning electron microscope image. (d) and (e) Results of filtering with the masks in (a) and (b), respectively. Note how much sharper (e) is than (d). (Original image courtesy of Mr. Michael Shaffer, Department of Geological Sciences, University of Oregon, Eugene.)

a b
c d

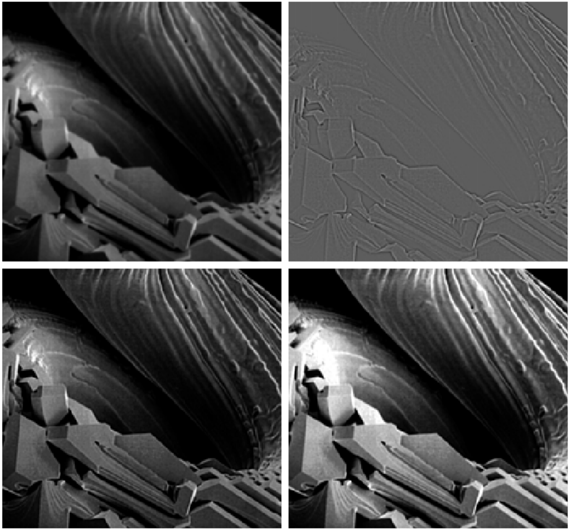


FIGURE 3.43
(a) Same as Fig. 3.41(c), but darker.
(a) Laplacian of (a) computed with the mask in Fig. 3.42(b) using $A = 0$.
(c) Laplacian enhanced image using the mask in Fig. 3.42(b) with $A = 1$. (d) Same as (c), but using $A = 1.7$.

1st Order Derivative

Passos:

$$\nabla f = \begin{vmatrix} G_x \\ G_y \end{vmatrix} = \begin{vmatrix} \frac{\partial f}{\partial x} \\ \\ \frac{\partial f}{\partial y} \end{vmatrix}$$

$$\begin{aligned} \nabla f &= \text{mag}(\nabla f) \\ &= \left[G_x^2 + G_y^2\right]^{1/2} \\ &= \left[\frac{\partial f}{\partial x}^2 + \frac{\partial f}{\partial y}^2\right]^{1/2} \end{aligned}$$

OR

$$\nabla f \approx |G_x| + |G_y|$$

Roberts & Sobel

z_1	z_2	z_3
z_4	z_5	z_6
z_7	z_8	z_9

−1	0
0	1

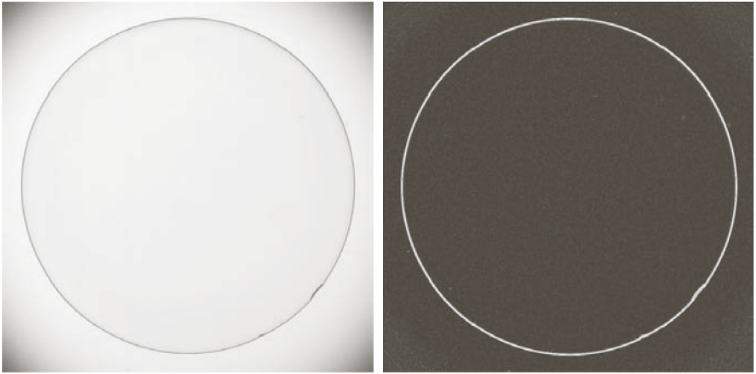
0	−1
1	0

−1	−2	−1
0	0	0
1	2	1

−1	0	1
−2	0	2
−1	0	1

a
b c
d e

FIGURE 3.41 A 3 × 3 region of an image (the *z*s are intensity values). (b)–(c) Roberts cross gradient operators. (d)–(e) Sobel operators. All the mask coefficients sum to zero, as expected of a derivative operator.

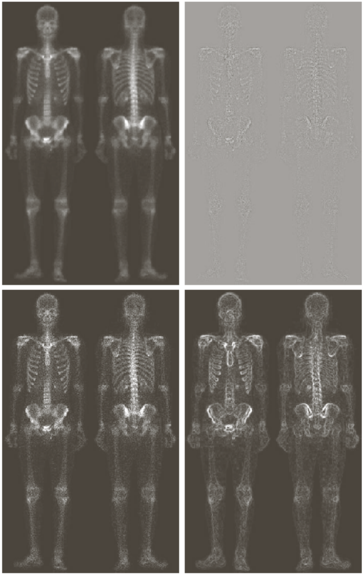


Original & Sobel

Roberts & Sobel

original

Laplaciano do original



a b
c d

FIGURE 3.43 (a) Image of whole body bone scan. (b) Laplacian of (a). (c) Sharpened image obtained by adding (a) and (b). (d) Sobel gradient of (a).

Original + Laplaciano

Sobel do original