

Controlle Digital

Parte II

Prof. Adolfo Bauchspiess
ENE/FT/UnB

... 7.3 - Projeto LGR digital



$$\text{Antena : } G(s) = \frac{1}{s(10s + 1)} \xrightarrow{T=1s} G(z) = 0.048374 \frac{(z + 0.9672)}{(z - 1)(z - 0.9048)}$$

Especificação $\rightarrow$ Plano z

$$M_p \leq 16\% \rightarrow \zeta = 0.5$$

$$\sigma \geq \frac{4}{10} \rightarrow \sigma = 0.4 \quad r = e^{-0.4T} = 0.67$$

$$\omega_n = \frac{1.8}{t_r} = 1 \rightarrow t_r = 1.8$$

$$N(0 : 10) \text{ em } z\text{grid : } \omega_n = \frac{N\pi}{10T}$$

$$\rightarrow N = \frac{10T\omega_n}{\pi} = 10/\pi = 3.183 \quad (0.318\pi/T)$$

$$e_{ss} \leq \frac{0.01}{0.01} = 1 \rightarrow K_v = \frac{1}{e_{ss}} \geq 1$$

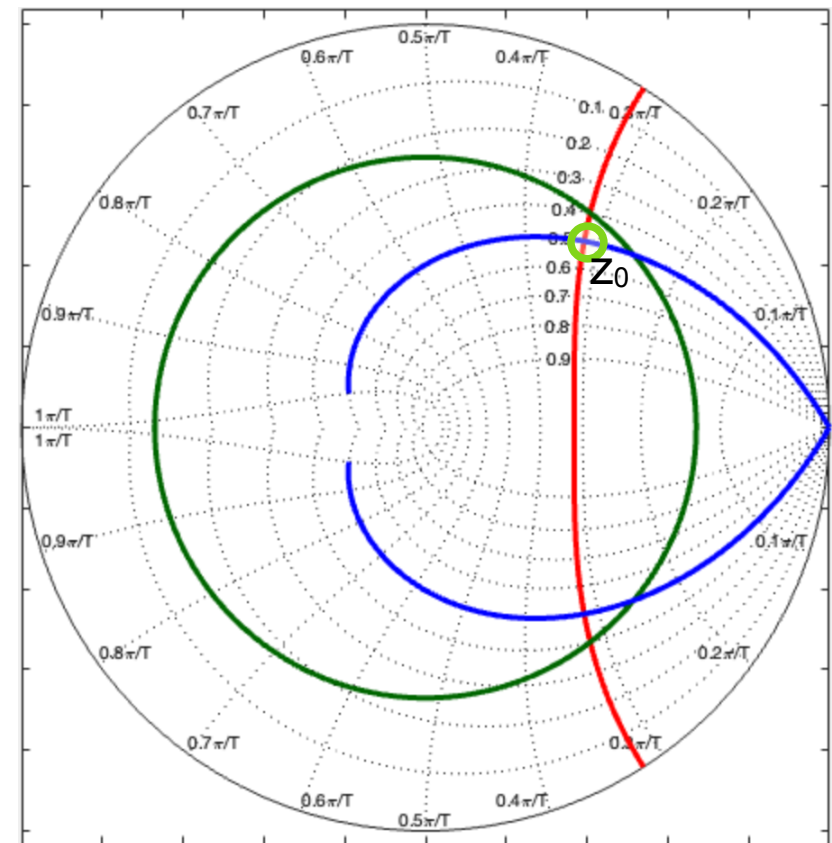
$$K_v = \lim_{z \rightarrow 1} \frac{(z-1)D(z)G(z)}{Tz} \geq 1$$

$$\lim_{z \rightarrow 1} \frac{(z-1)G(z)}{1.1} = 0.048374 * 1.9672 / (1 - 0.9048) = 0.9996$$

$$\Rightarrow |D(z)|_{z=1} \geq 1$$

CDig2-ENE/UnB

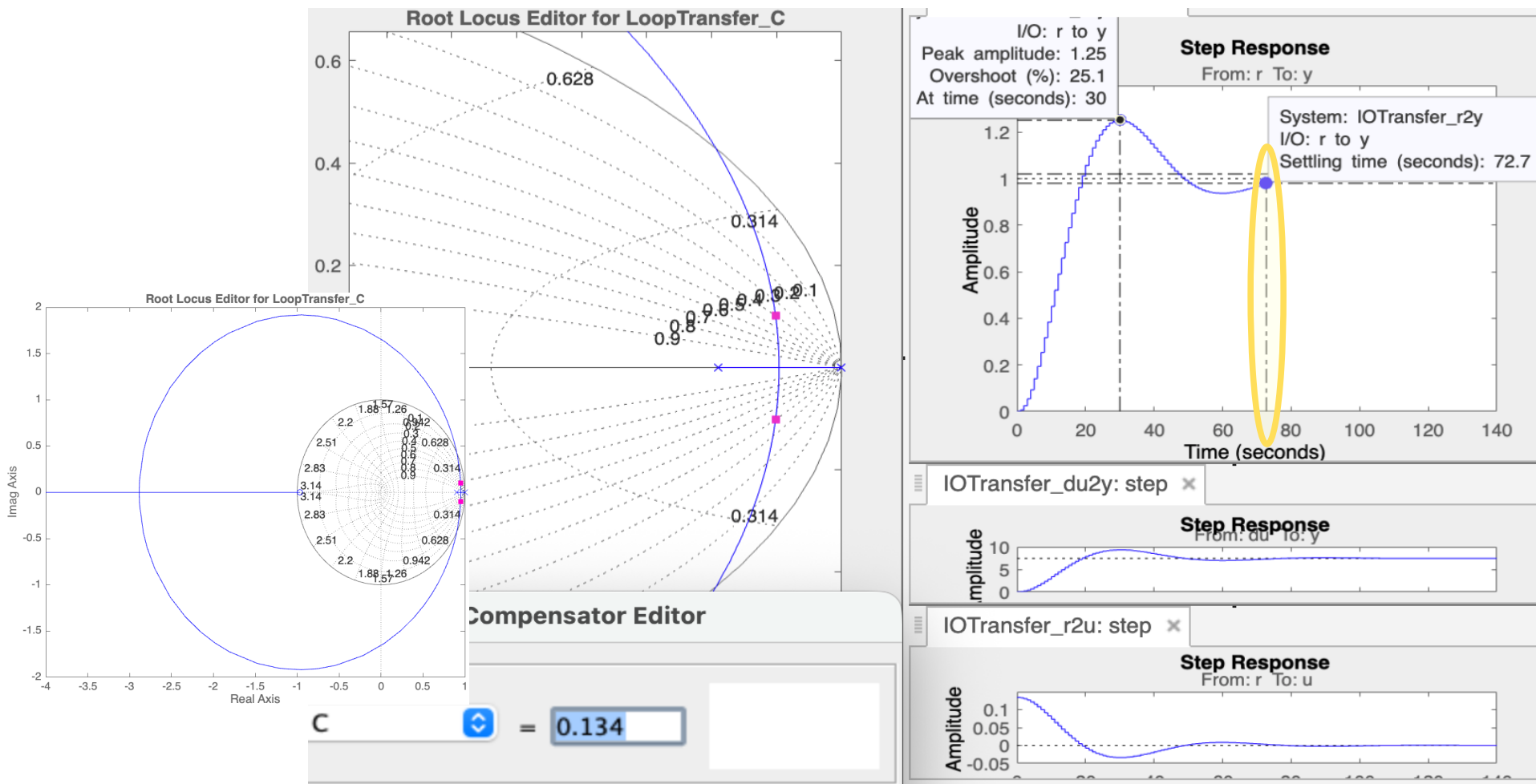
$e_{ss} 2\% \gg \text{spec2_z}(0.5, 1.8, 10, 1)$



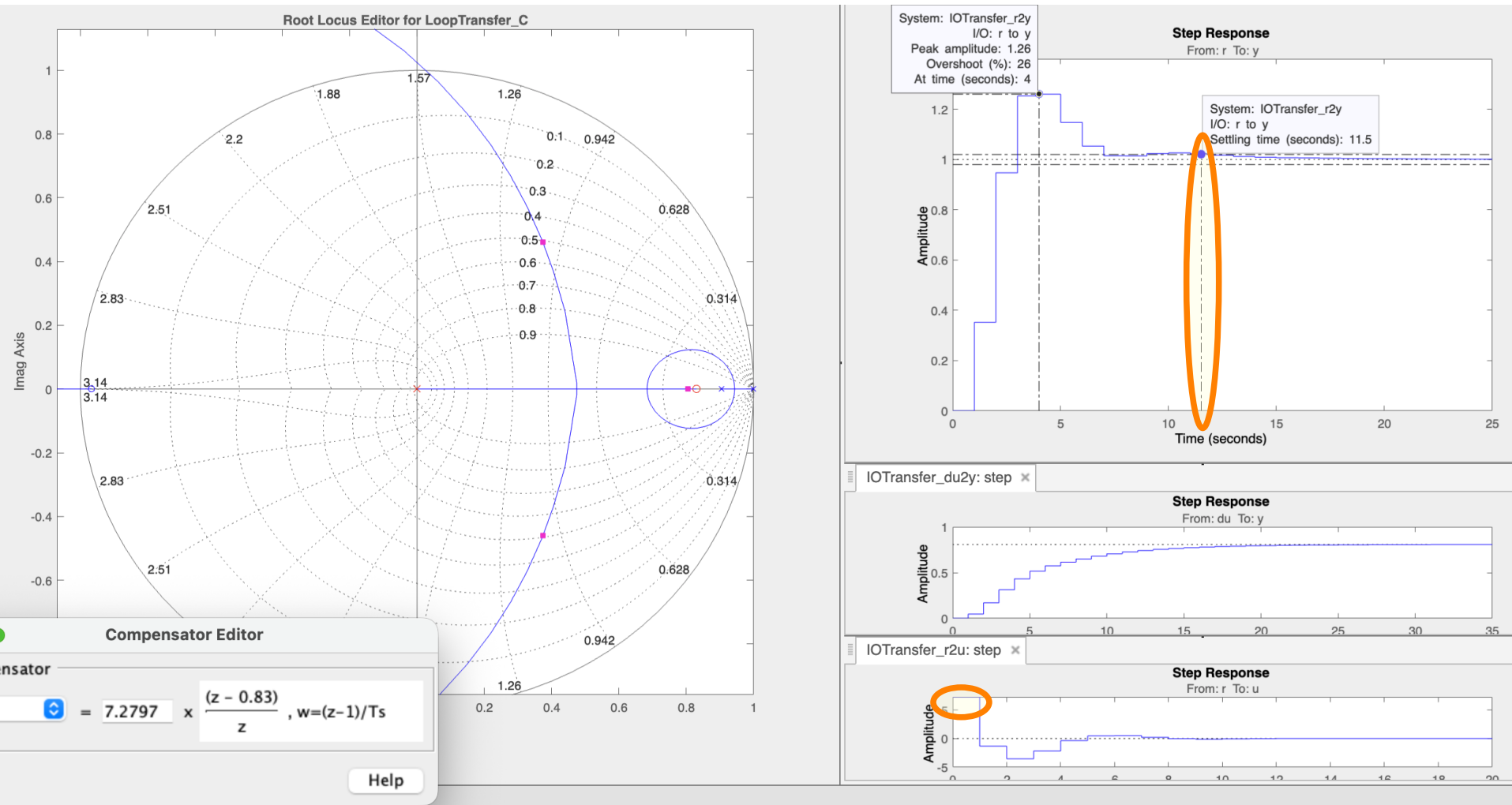
... 7.3 - Projeto LGR digital - P



$$\text{Antena : } G(s) = \frac{1}{s(10s + 1)} \xrightarrow{T=1s} G(z) = 0.048374 \frac{(z + 0.9672)}{(z - 1)(z - 0.9048)}$$

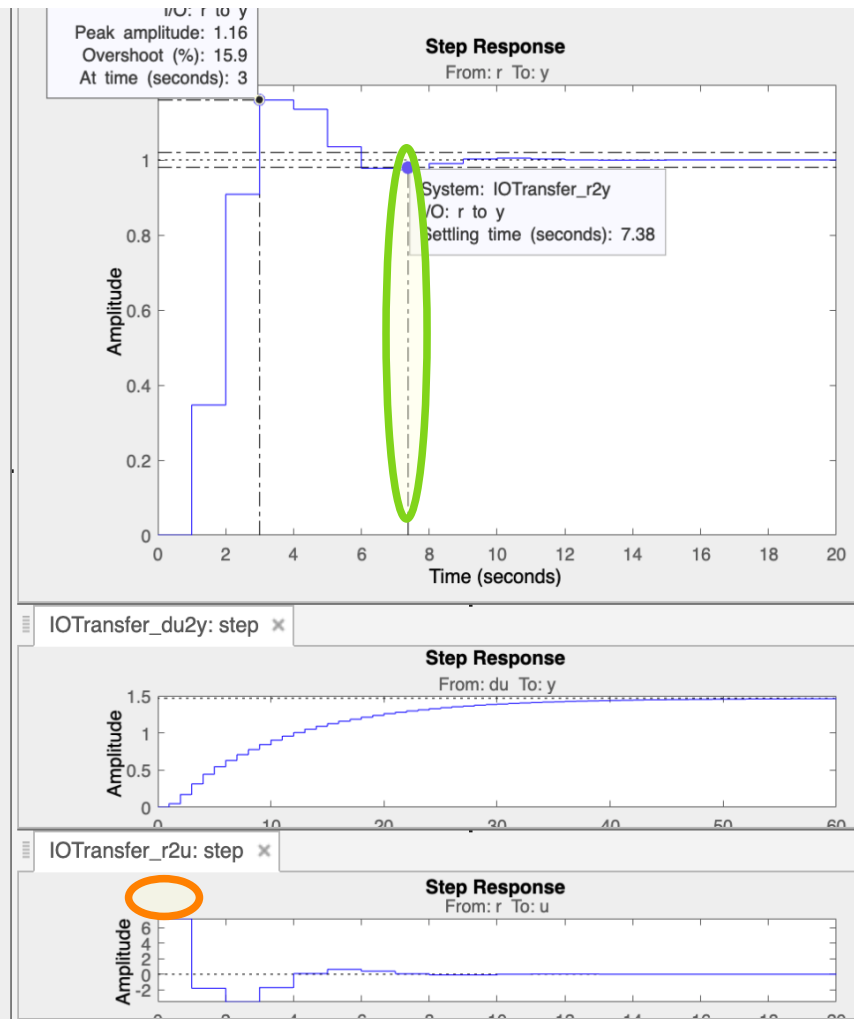
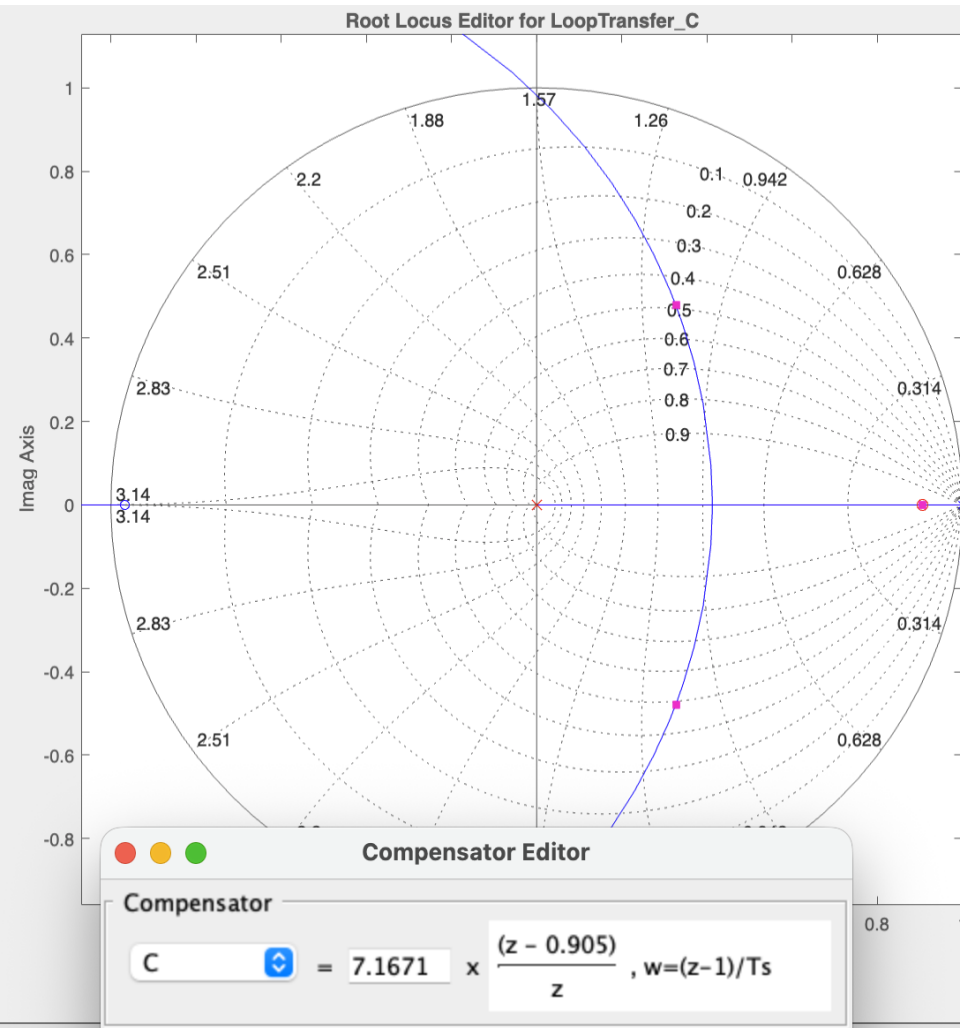


... 7.3 - Projeto LGR digital – Avanço



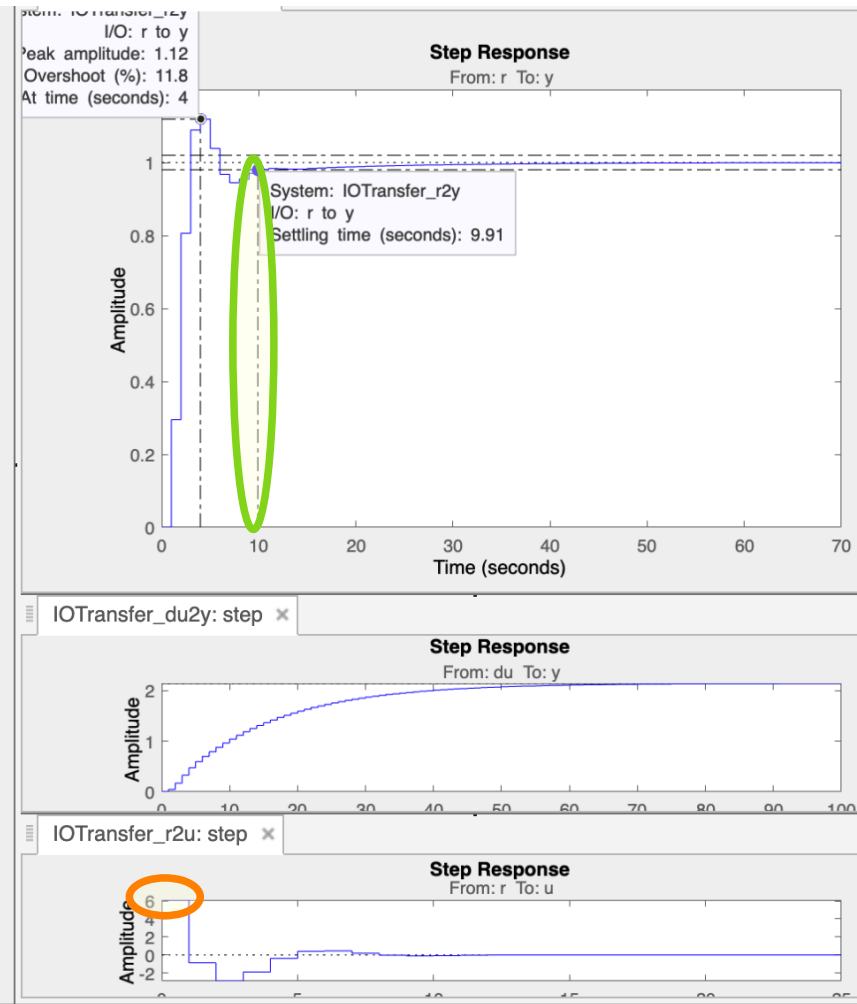
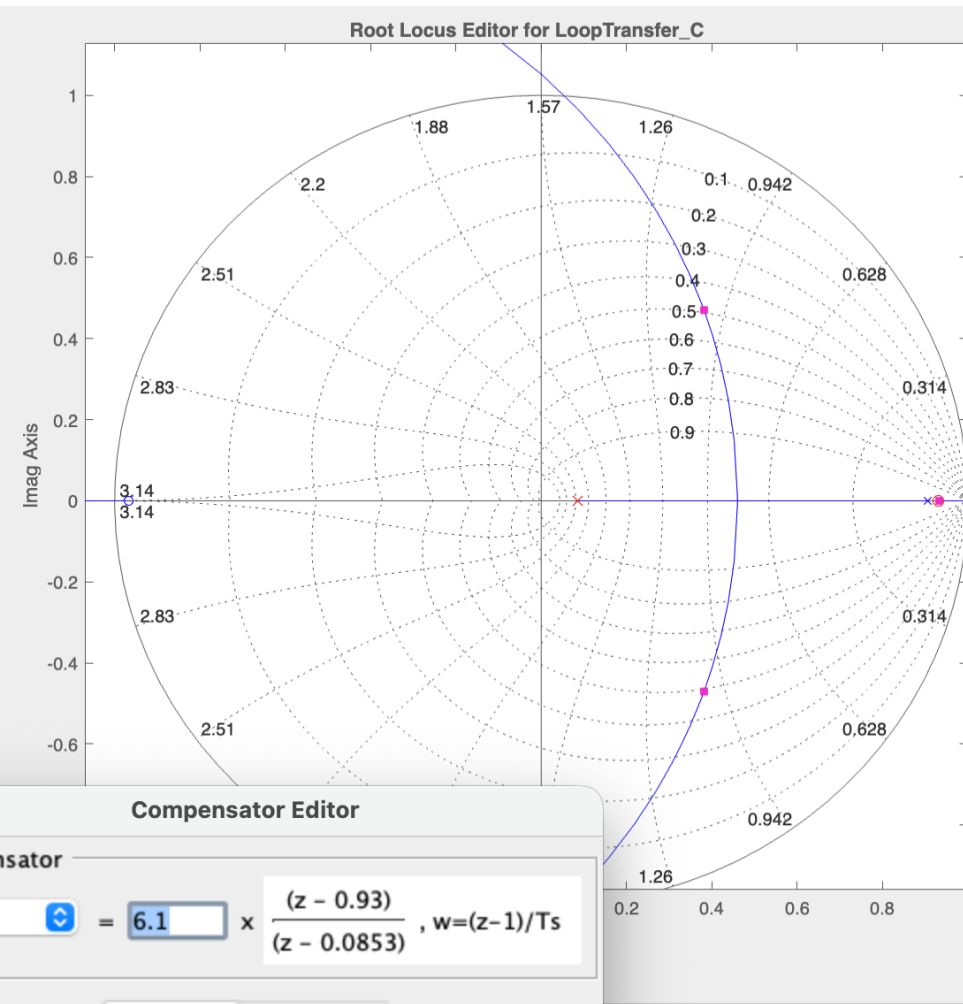
$$D(z)_{z=1} = 7.2797 \cdot (1 - 0.83) = 1.2375 \text{ OK!}$$

... 7.3 - Projeto LGR digital –Avanço 2



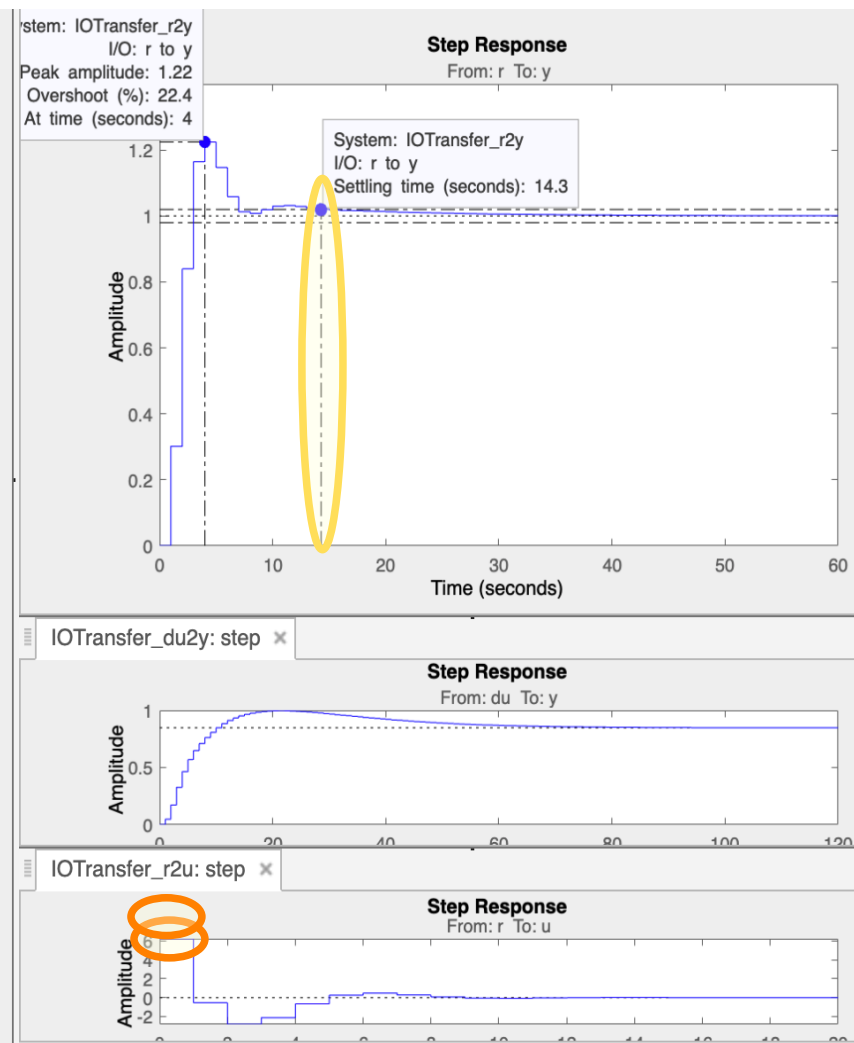
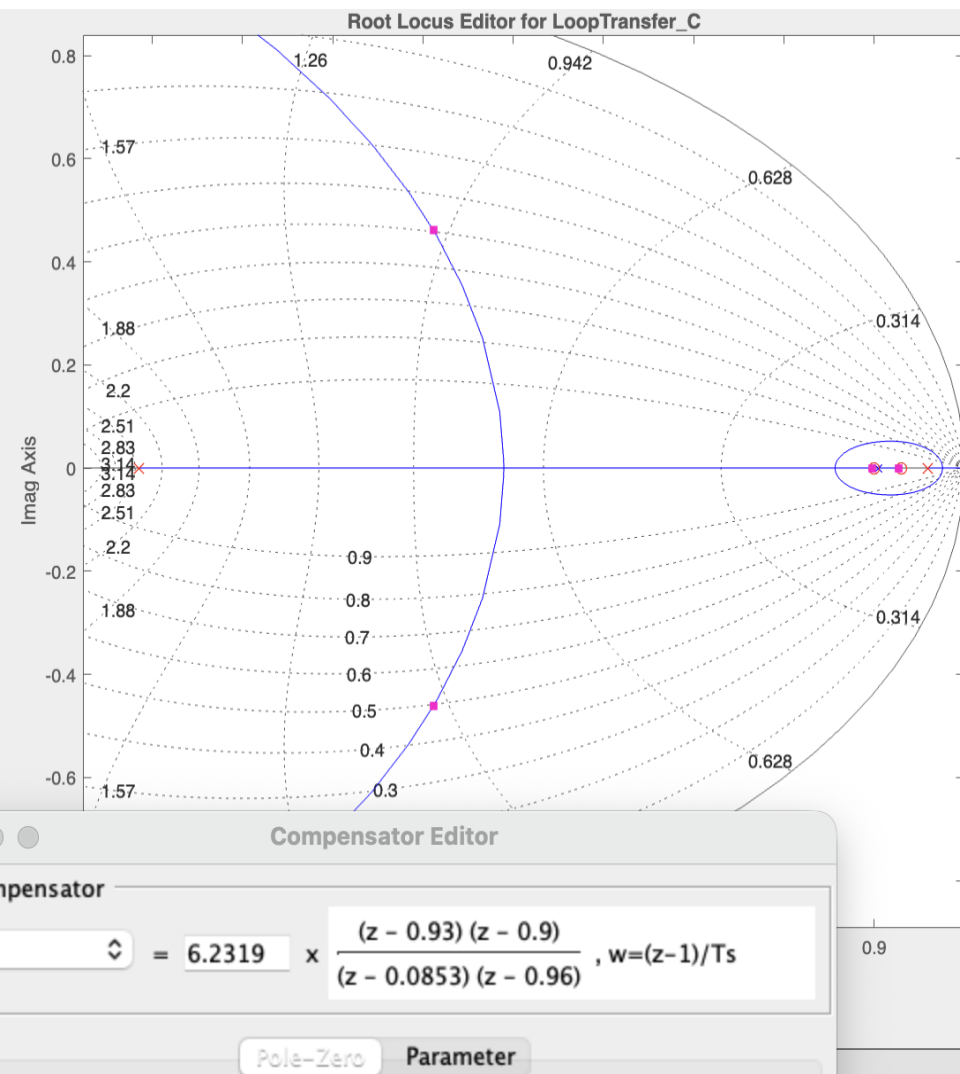
$D(z)_{z=1} = 7.1671 \cdot (1 - 0.9048) = 0.6823$ **NOK!**

... 7.3 - Projeto LGR digital –Avanço 3



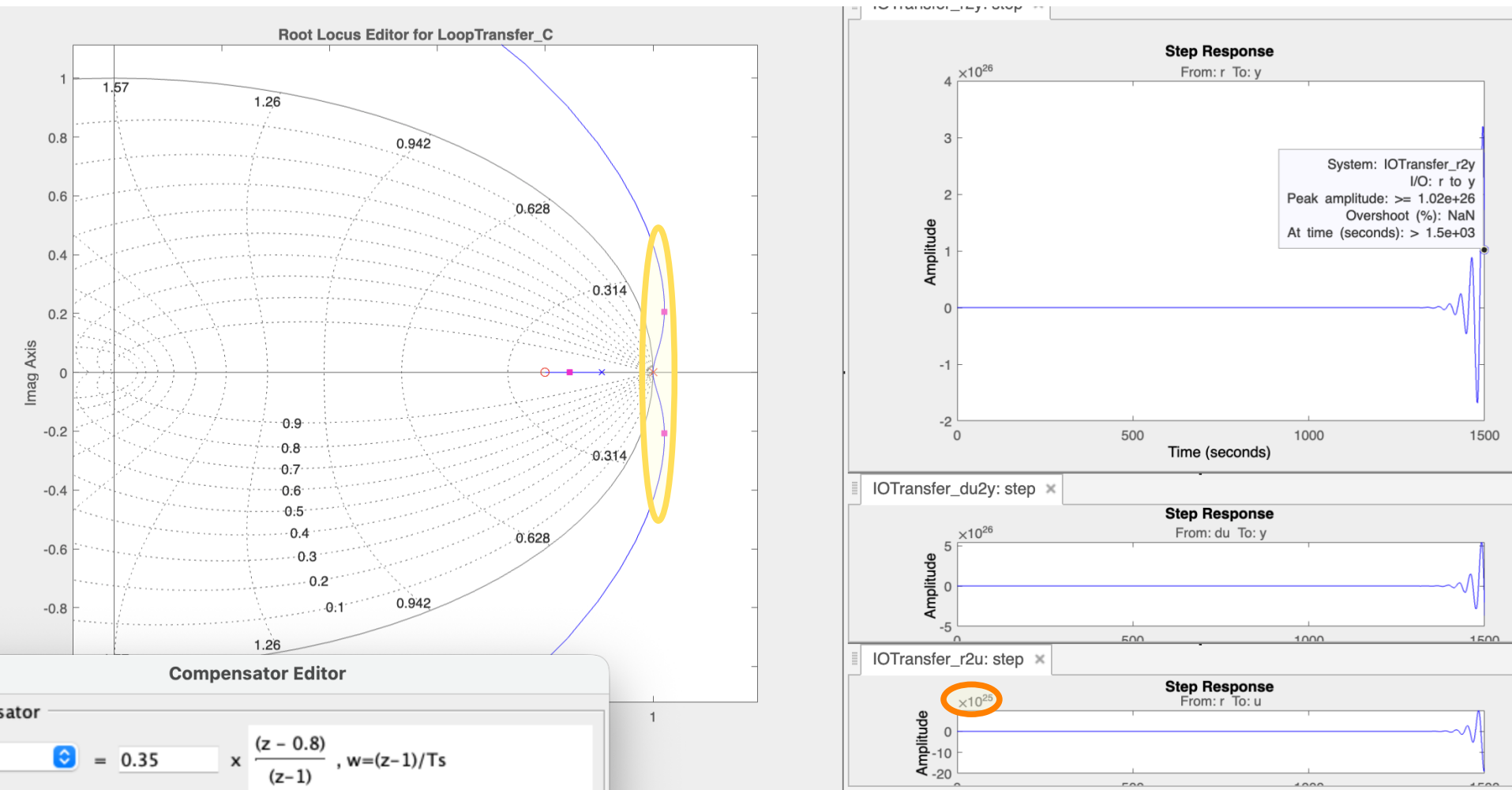
$$D(z)_{z=1} = 6.1 \cdot (1 - 0.93) = 0.4668 \text{ NOK!}$$

... 7.3 - Projeto LGR digital –Av-Atr



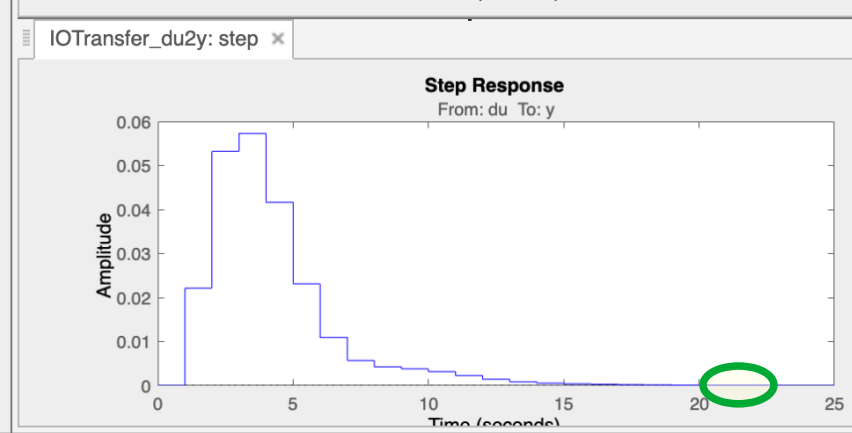
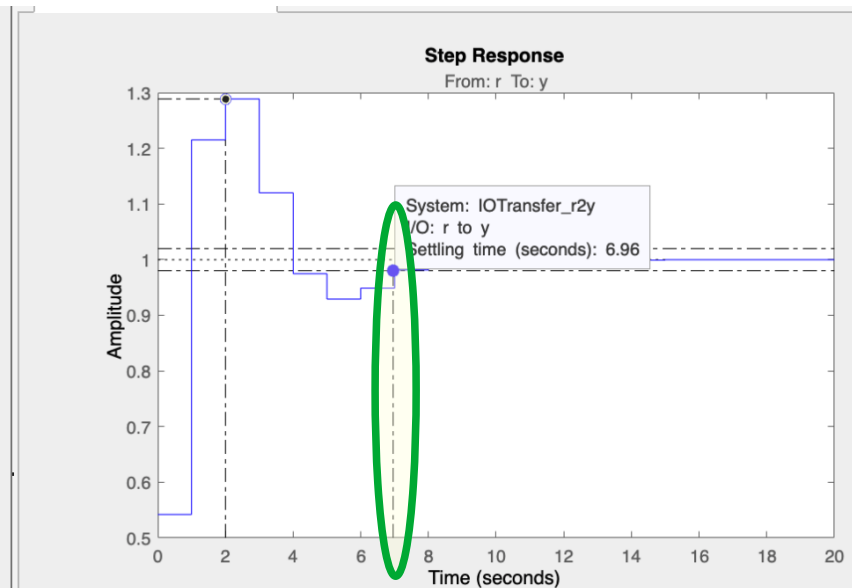
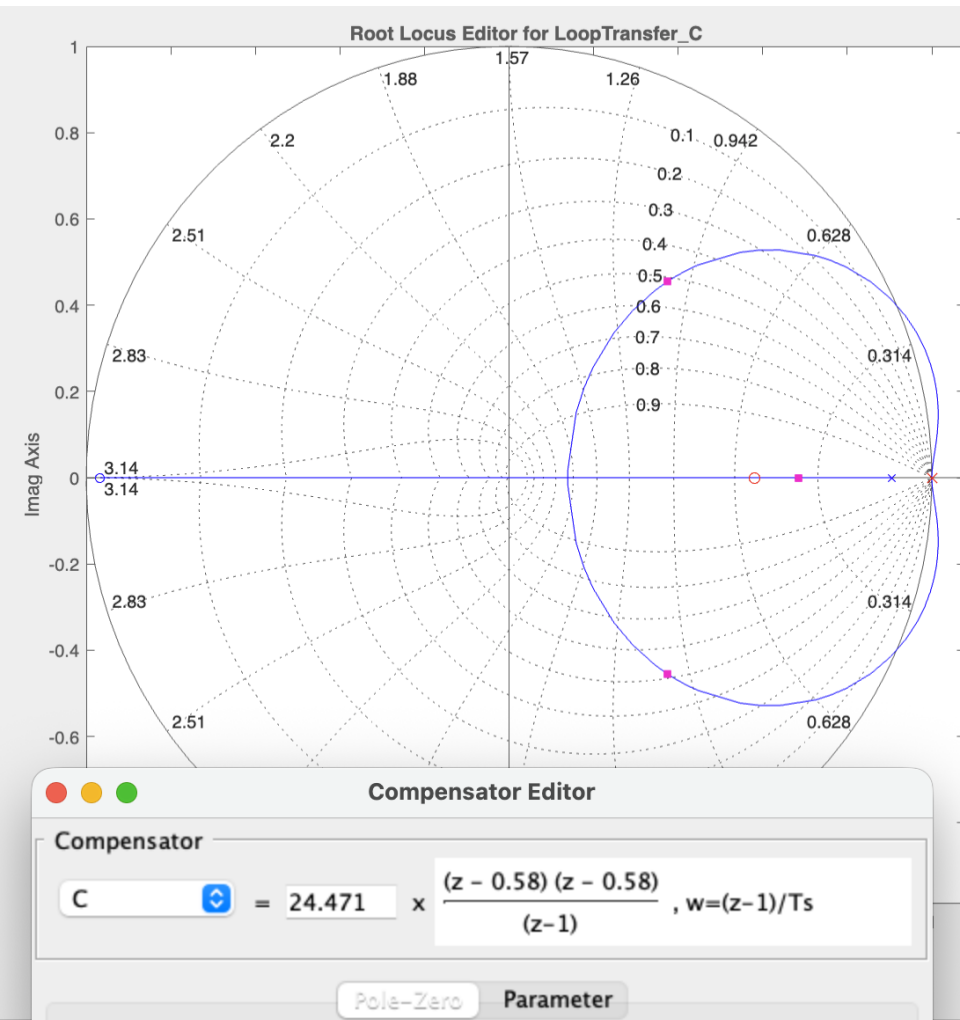
$$D(z)_{z=1} = 6.2319 \cdot (1-0.93) \cdot (1-0.9) / ((1-0.0853) \cdot (1-0.96)) = 1.1923 \quad \text{OK!}$$

... 7.3 - Projeto LGR digital -PI



Instável **NOK!**

... 7.3 - Projeto LGR digital -PID $(1-ze)^2$

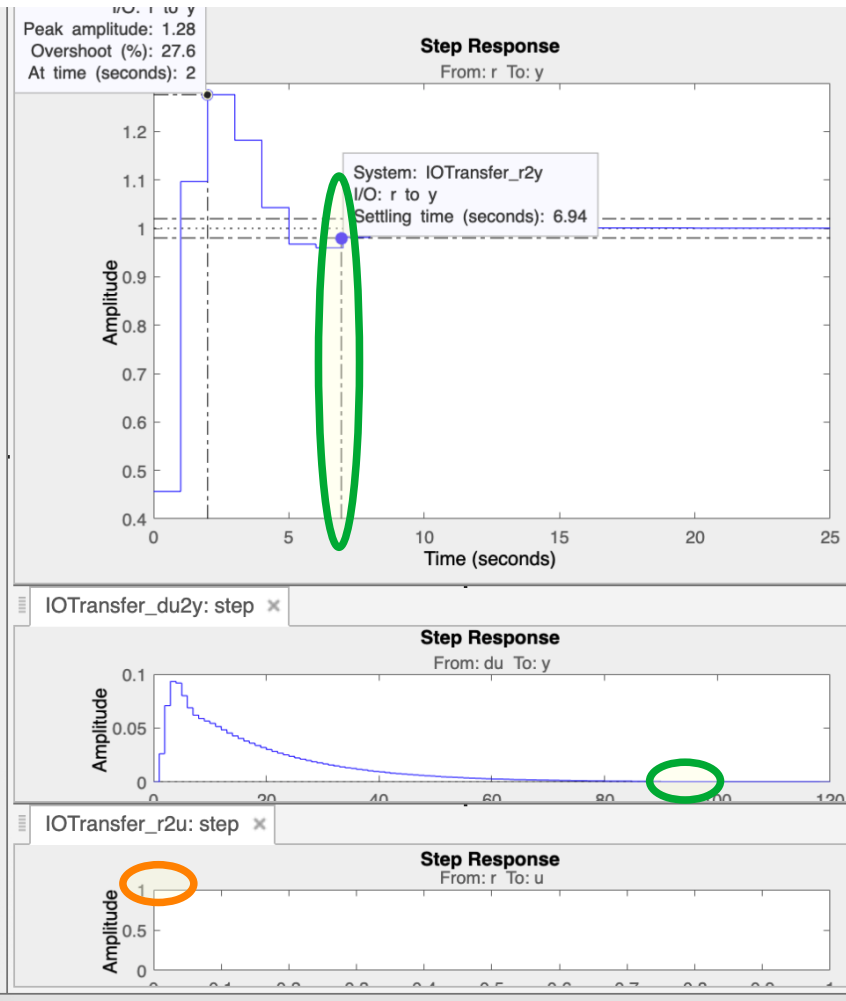
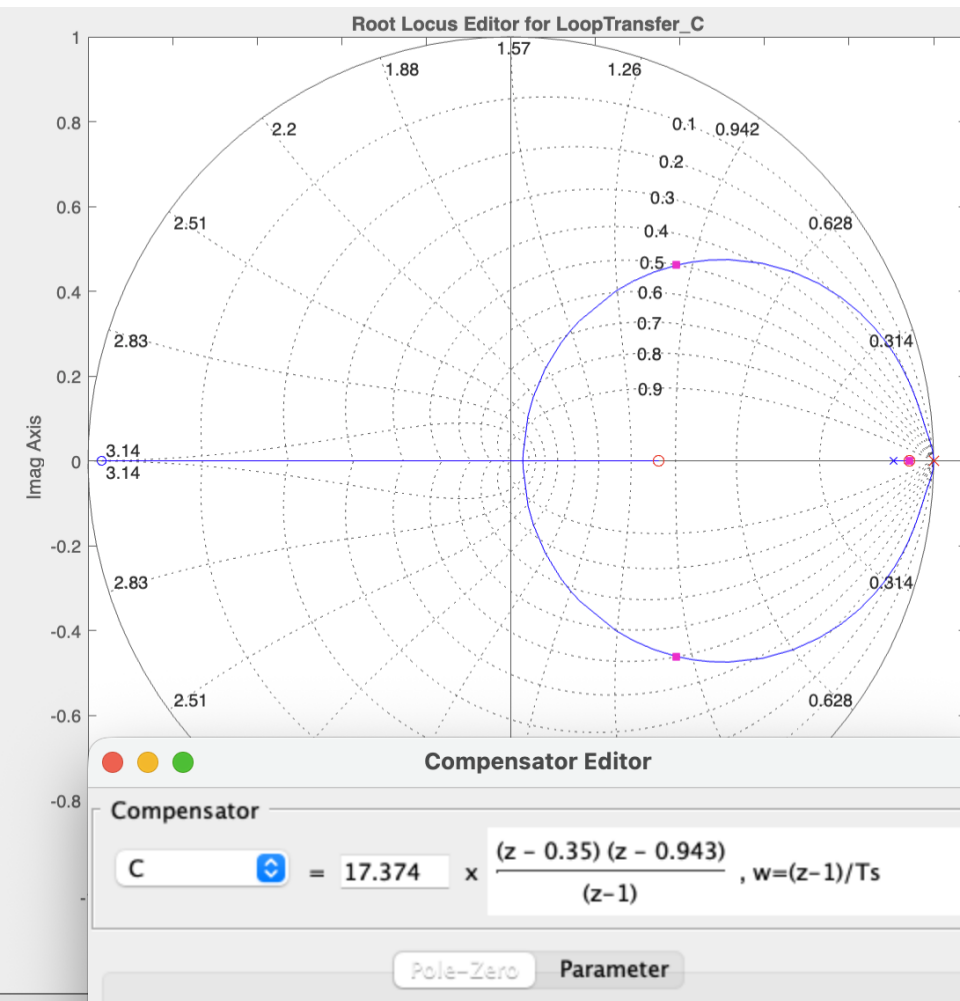


$D(z)_{z=1} \rightarrow \text{infinito} \Rightarrow e_{ss} = 0$, para a rampa!

CDig-ENE/UnB

Rejeita perturbações constantes!

... 7.3 - Projeto LGR digital -PID

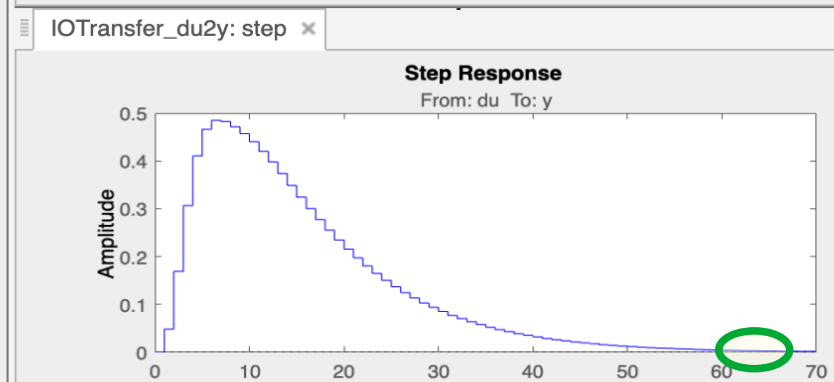
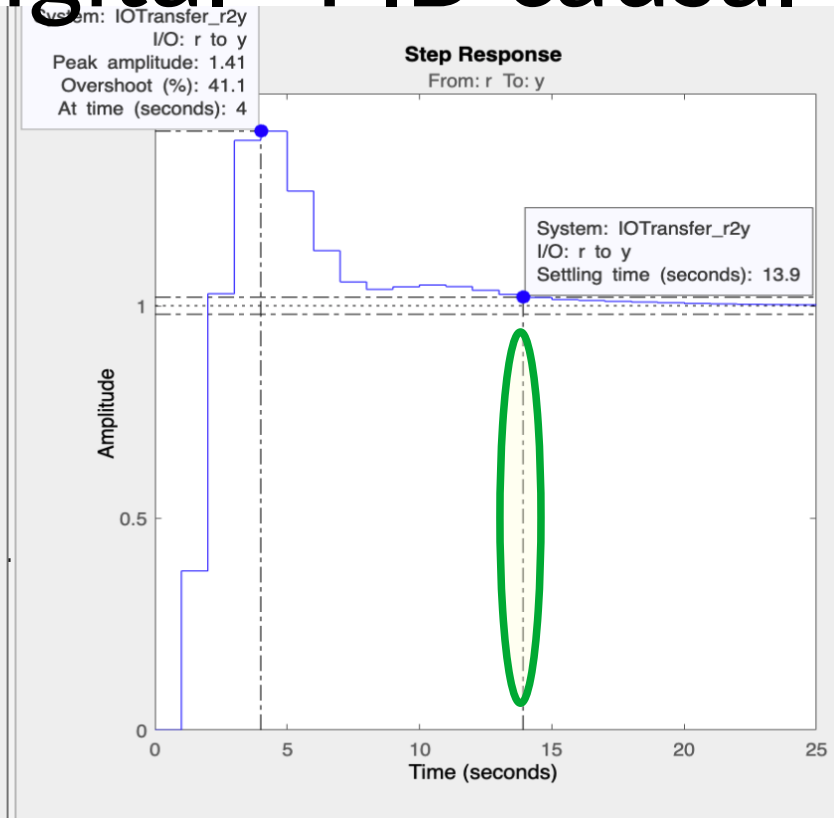
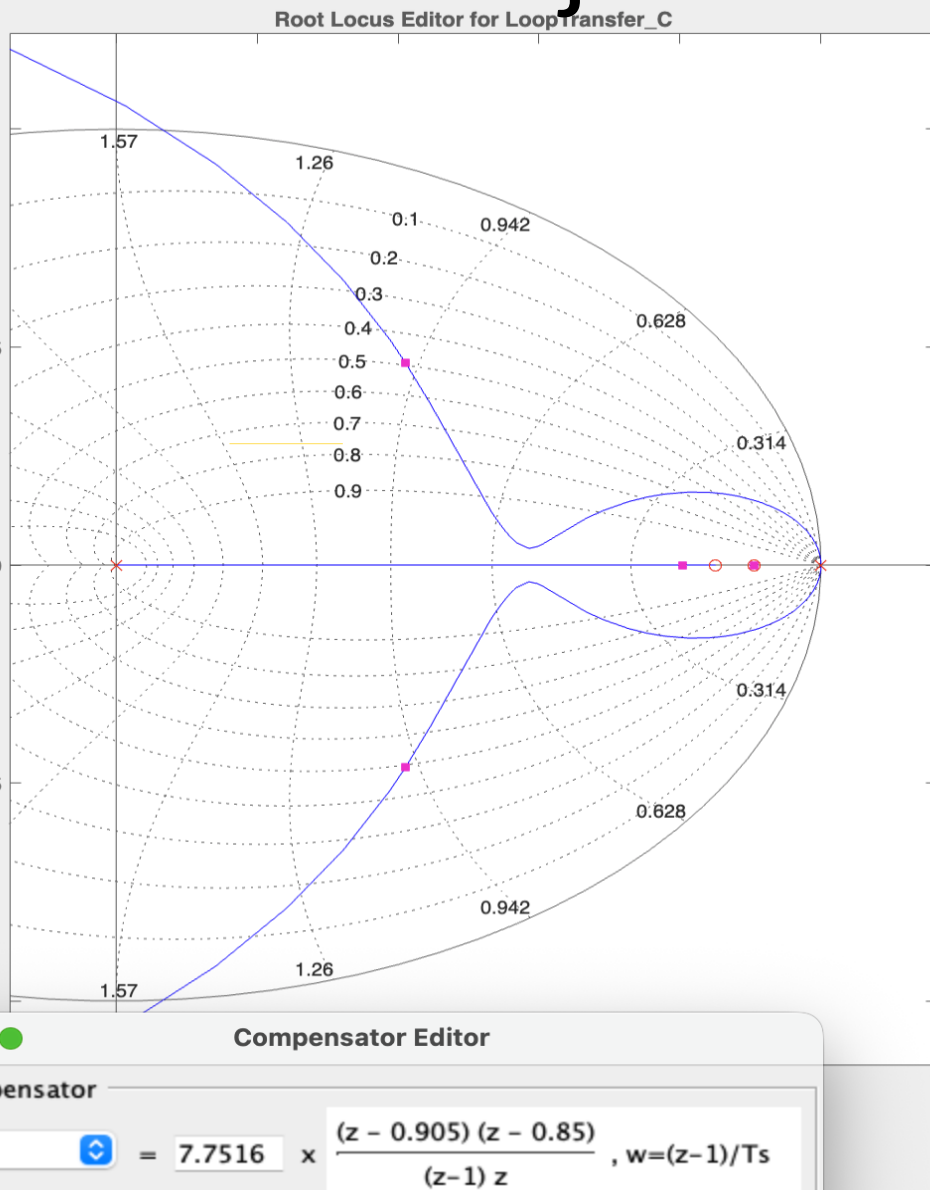


$D(z)_{z=1} \rightarrow \text{infinito} \Rightarrow e_{ss} = 0$, para a rampa!

CDig-ENE/UnB

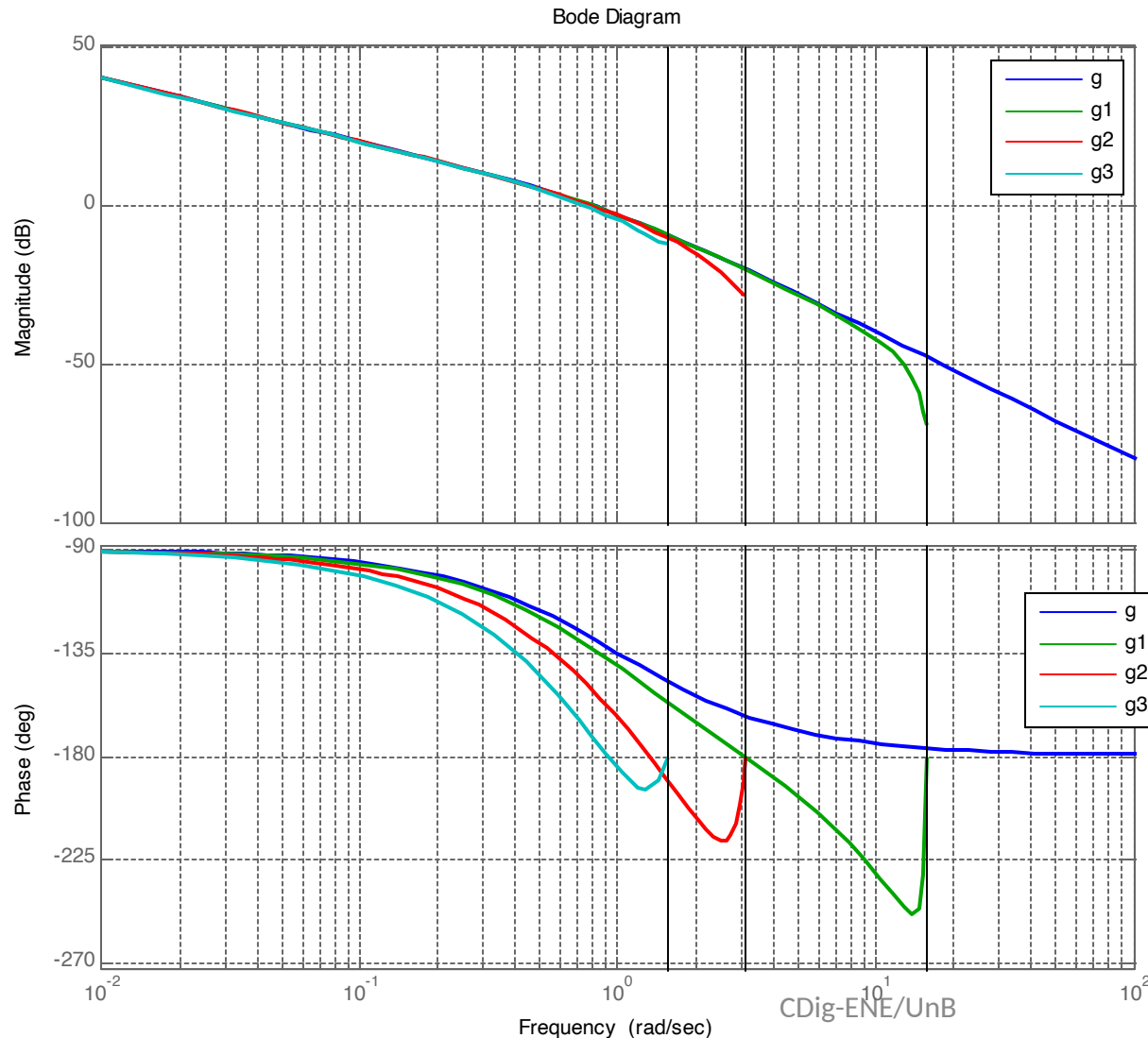
Rejeita perturbações constantes!

... 7.3 - Projeto LGR digital - PID causal



Realizável! $U/E = K_p(1 - z^{-1})(1 - z^{-2})/(1 - z^{-1}) \rightarrow U(1 - z^{-1}) = E K_p\{1 - (z^{-1} + z^{-2})\}$

7.4 Métodos no Domínio da Frequência



$$g(s) = \frac{1}{s^2 + s}$$

$$g_1(s) = c_2 d(g, 0.2)$$

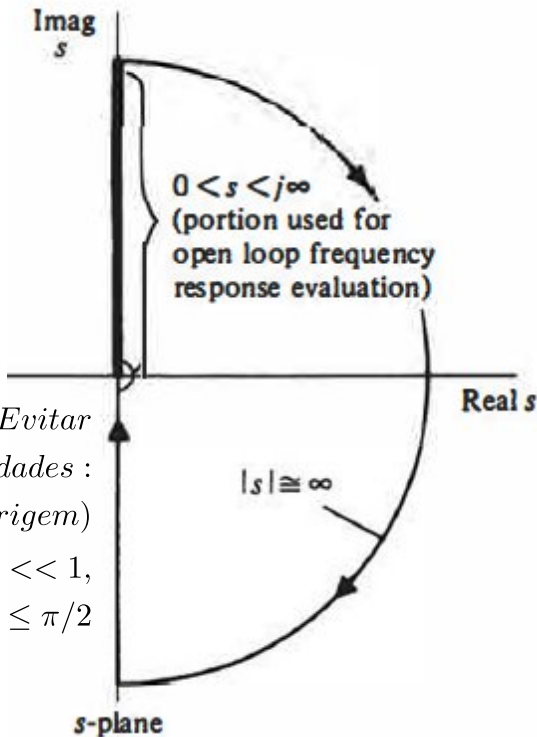
$$g_2(s) = c_2 d(g, 1.0)$$

$$g_3(s) = c_2 d(g, 2.0)$$

$$\Delta\phi = \frac{\omega T}{2}$$

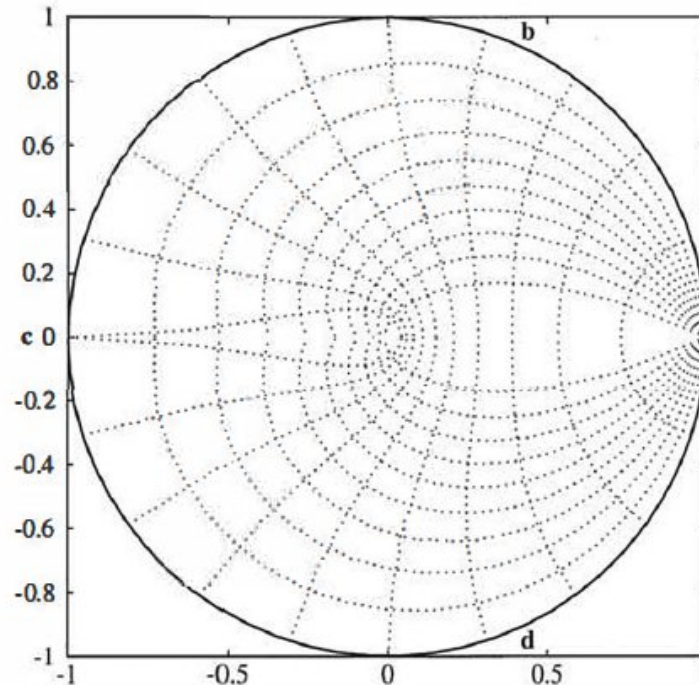
Critério de Estabilidade de Nyquist

$$1 + KD(s)G(s) = 0$$



Evitar
singularidades :
(polo origem)
 $s = re^{j\phi}$, $r \ll 1$,
 $-\pi/2 \leq \phi \leq \pi/2$

$$1 + KD(z)G(z) = 0$$



trecho e → a
contorna
eventuais
polos em $z=1$

$$Z = P + N$$

Z – raízes da eq. carac fora do circ. unit.

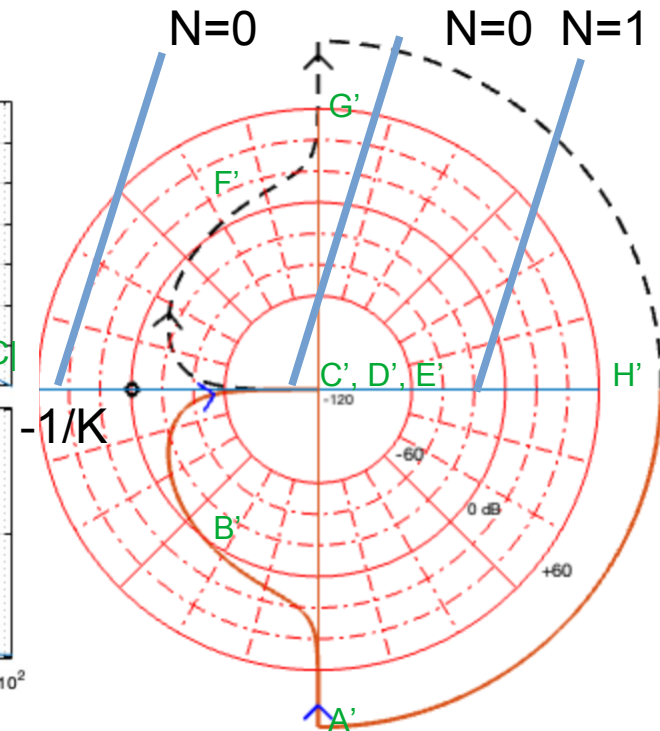
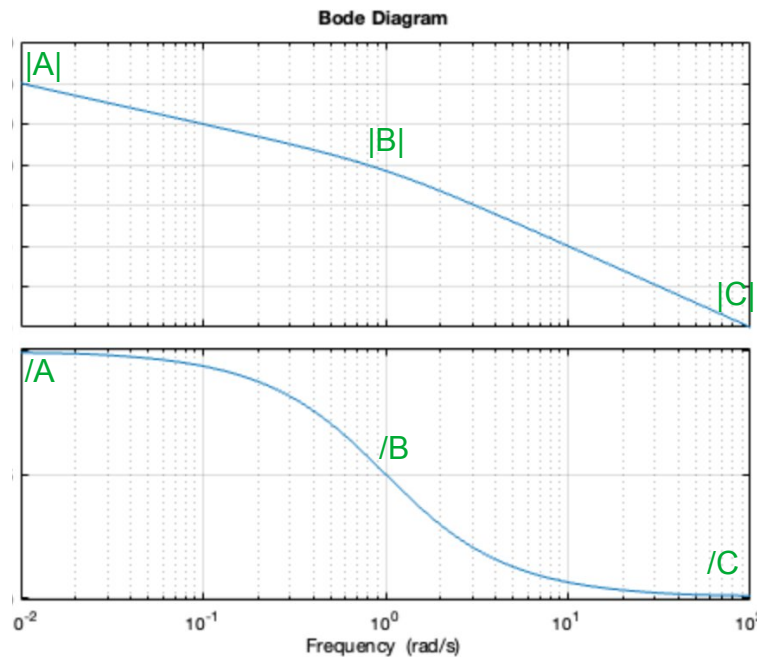
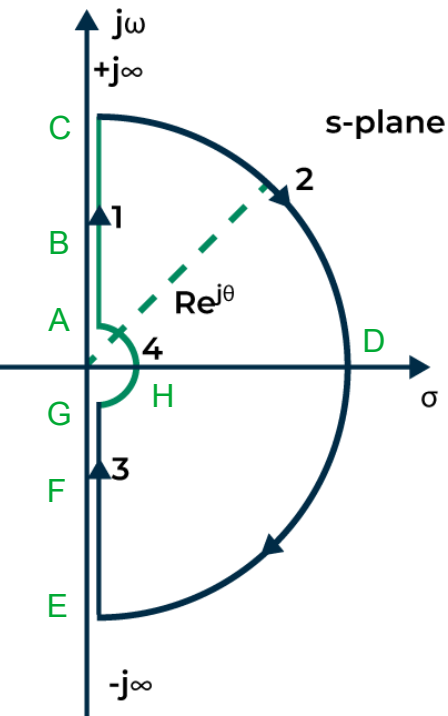
P – # pólos fora circ. unit. em MA

N – # envoltimentos (-1) sentido horário

(~ Nyquist
tradicional em s
 $Z=P-N$)

Exemplo - Estabilidade via Nyquist (s)

$$G(s) = \frac{1}{s(s+1)}$$

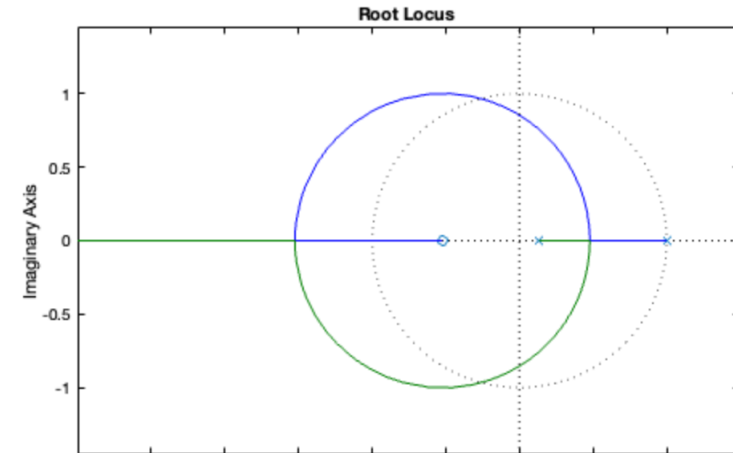


$$1 + KD(s)G(s) = 0; \quad Z = P + N \rightarrow (\text{Horario!})$$

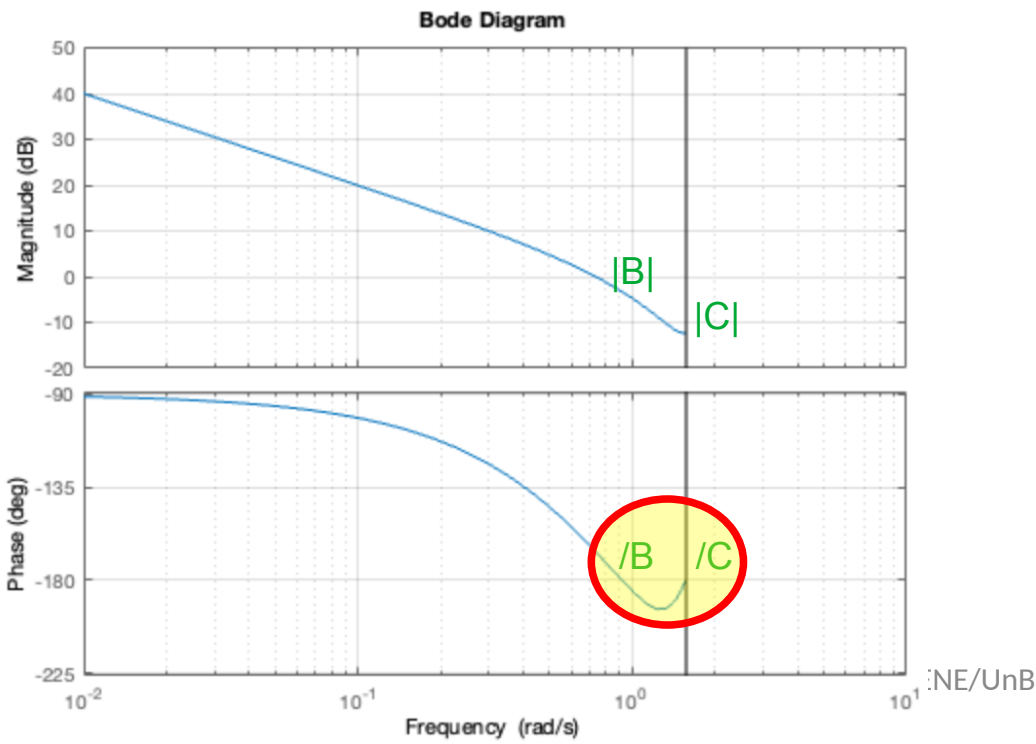
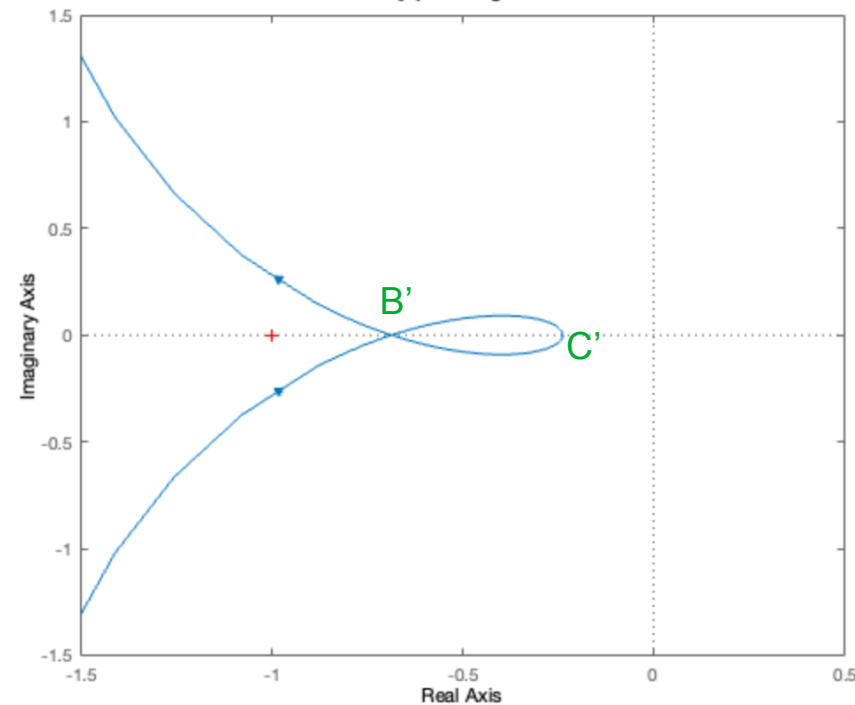
Estável para $0 \leq K \leq \infty$

Exemplo - Estabilidade via Nyquist (z)

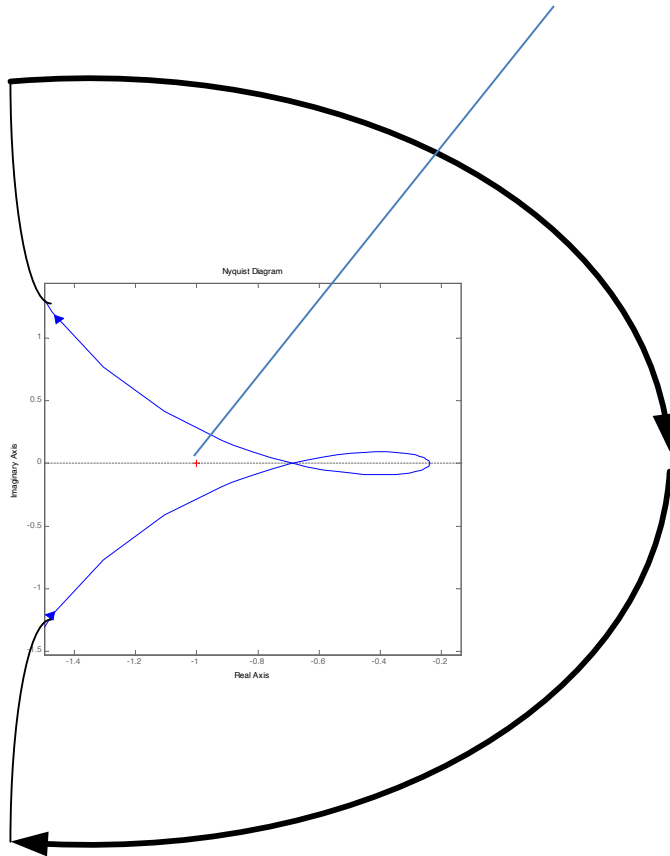
$$G(s) = \frac{1}{s(s+1)} \xrightarrow{T=2s} G(z) = \frac{1.1353(z+0.5232)}{(z-1)(z-0.1353)}$$



Nyquist Diagram



Exemplo - Estabilidade via Nyquist



$$G(z) = \frac{1.1353(z + 0.5232)}{(z - 1)(z - 0.1353)}$$

$$1 + KD(z)G(z) = 0$$

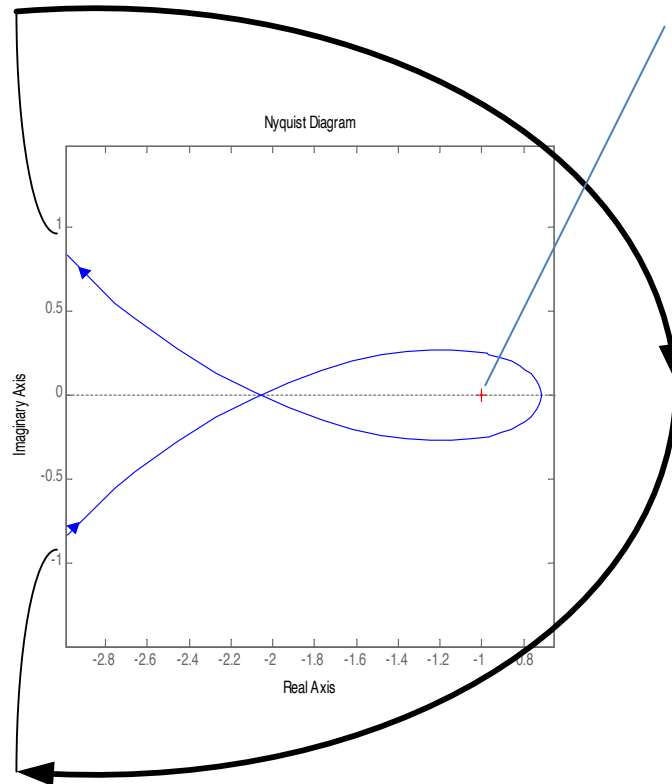
$$Z = P + N$$

$$N = 0$$

$$P = 0$$

$$Z = 0 \Rightarrow \text{Sistema Estável}$$

Exemplo - Estabilidade via Nyquist



$$G(z) = 3 \frac{1.1353 (z + 0.5232)}{(z - 1)(z - 0.1353)}$$

$$1 + KD(z)G(z) = 0$$

$$Z = P + N$$

$$N = 2$$

$$P = 0$$

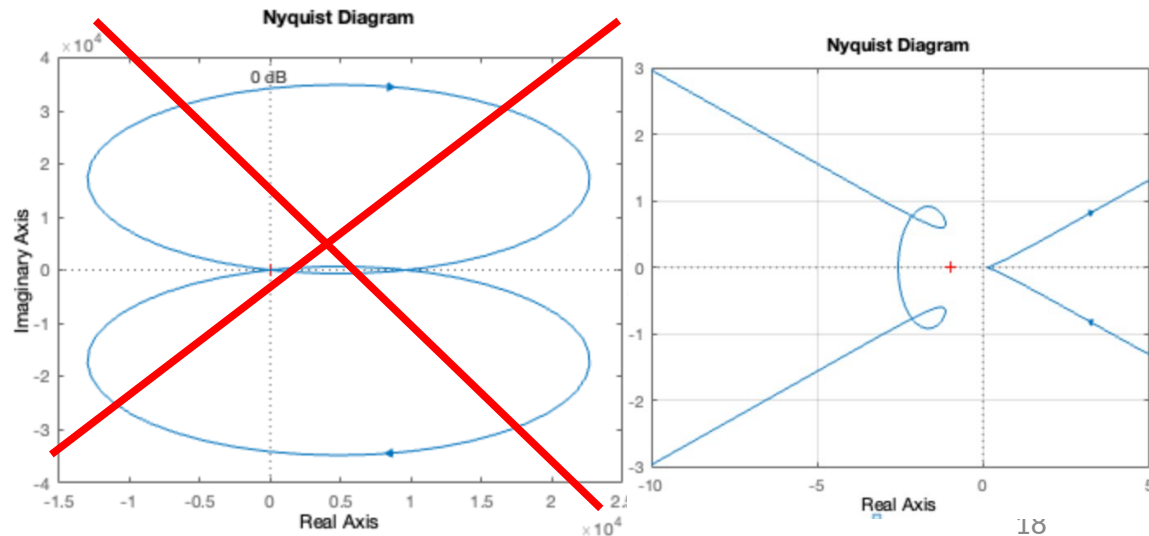
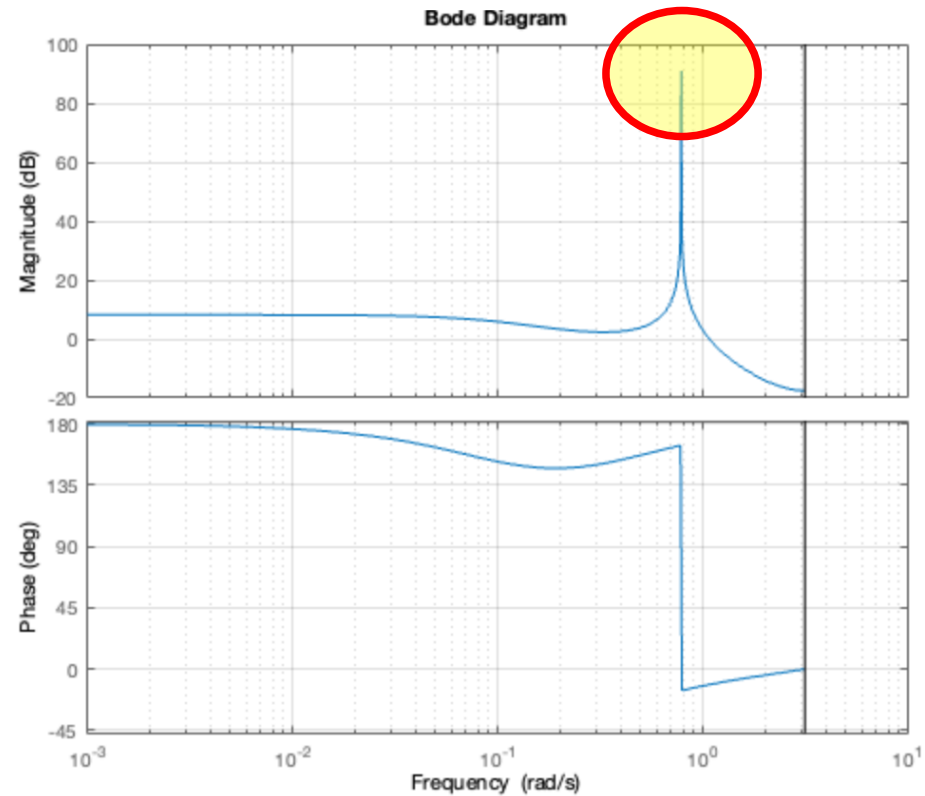
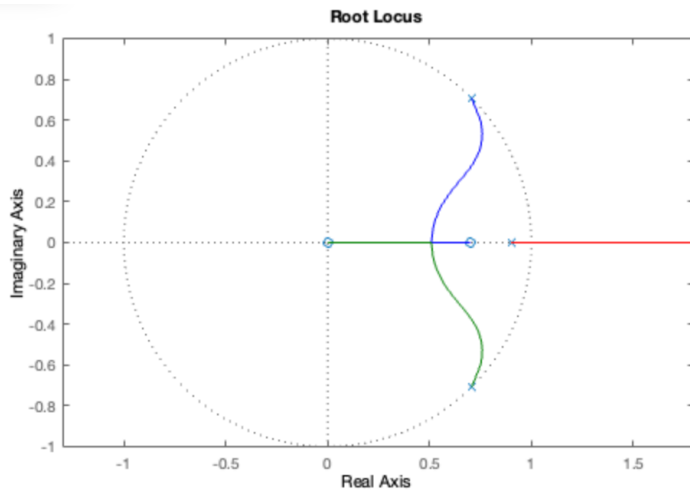
$$Z = 2$$

*Dois zeros da eq. característica
fora do círculo unitário
= dois pólos de MF
fora do círculo unitário
⇒ Sistema Instável*

Exemplo

Polos sobre o círculo unitário

$$G(z) = \frac{-0.5z(z - 0.7)}{(z - 0.9)(z^2 - 1.414z + 1)}$$

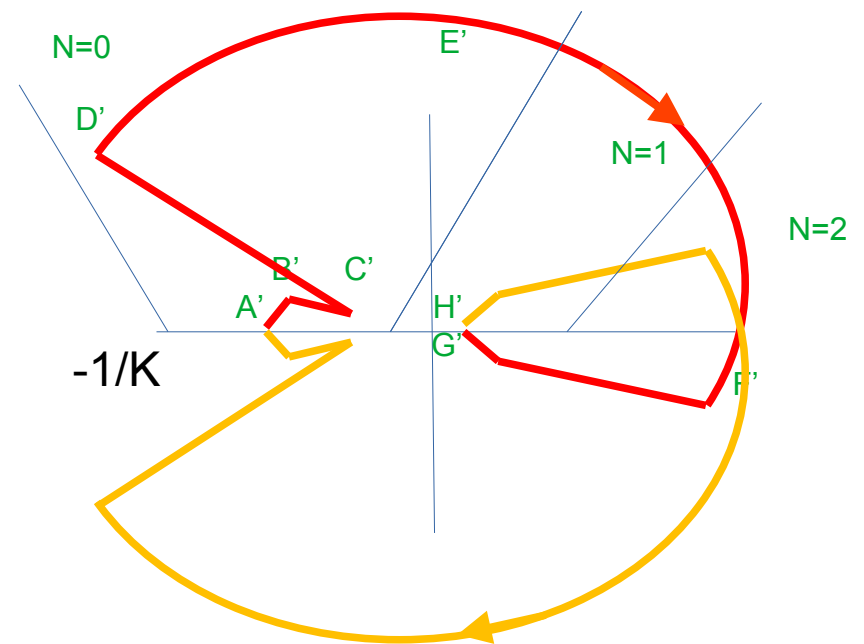
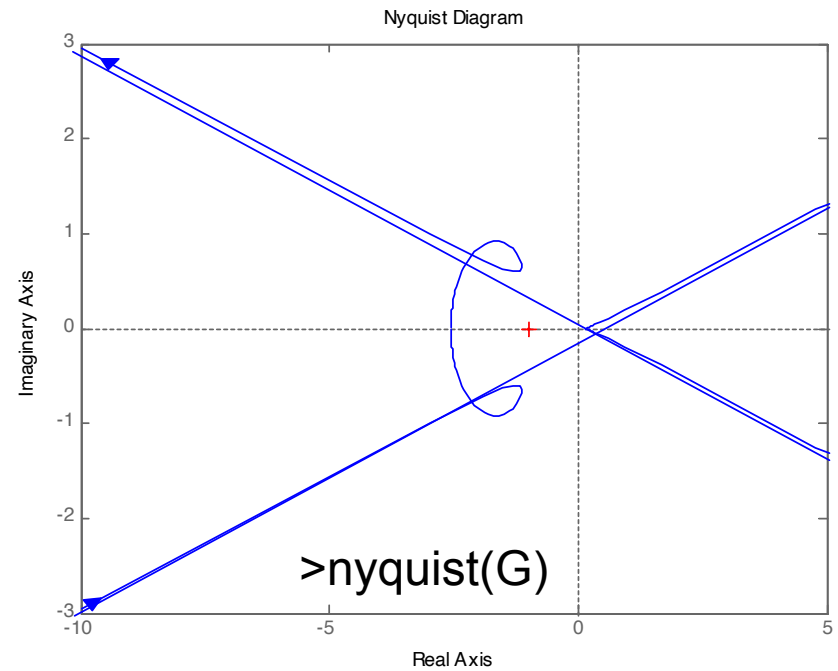
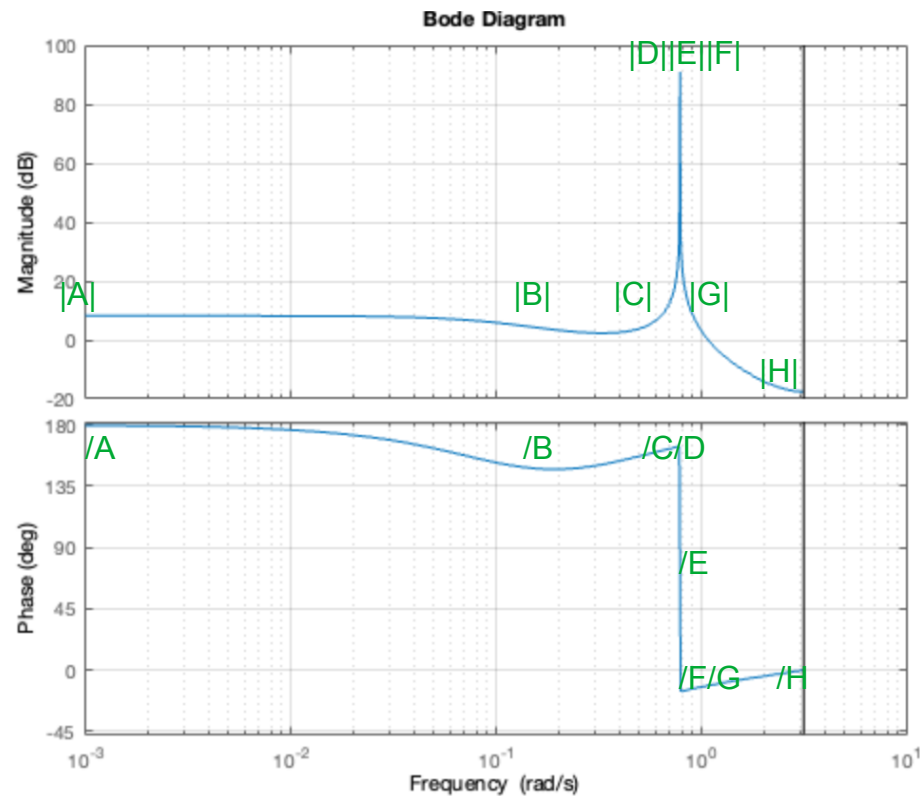


>nyquist(G)

Exemplo

Polos sobre o círculo unitário

$$G(z) = \frac{-0.5z(z - 0.7)}{(z - 0.9)(z^2 - 1.414z + 1)}$$



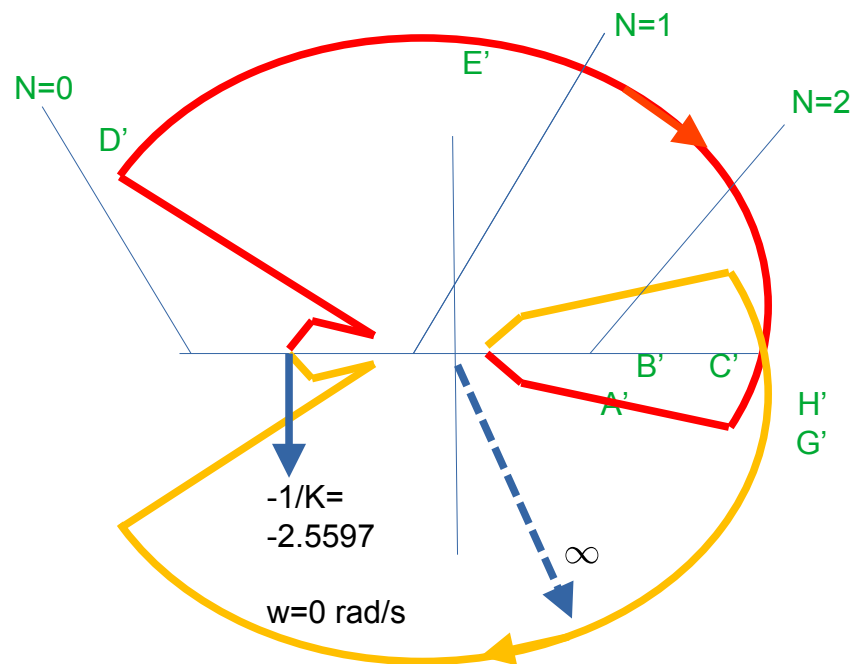
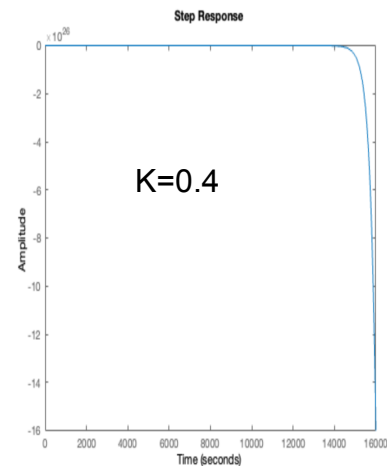
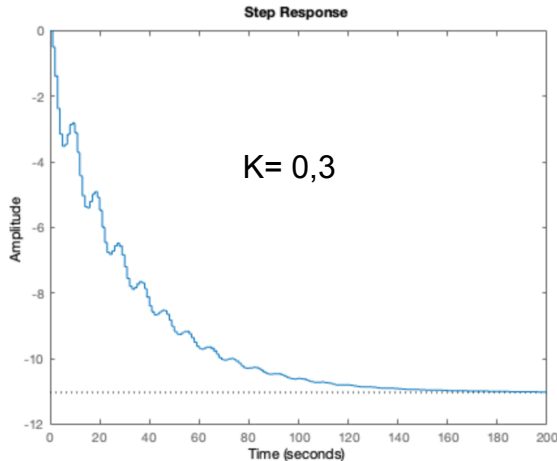
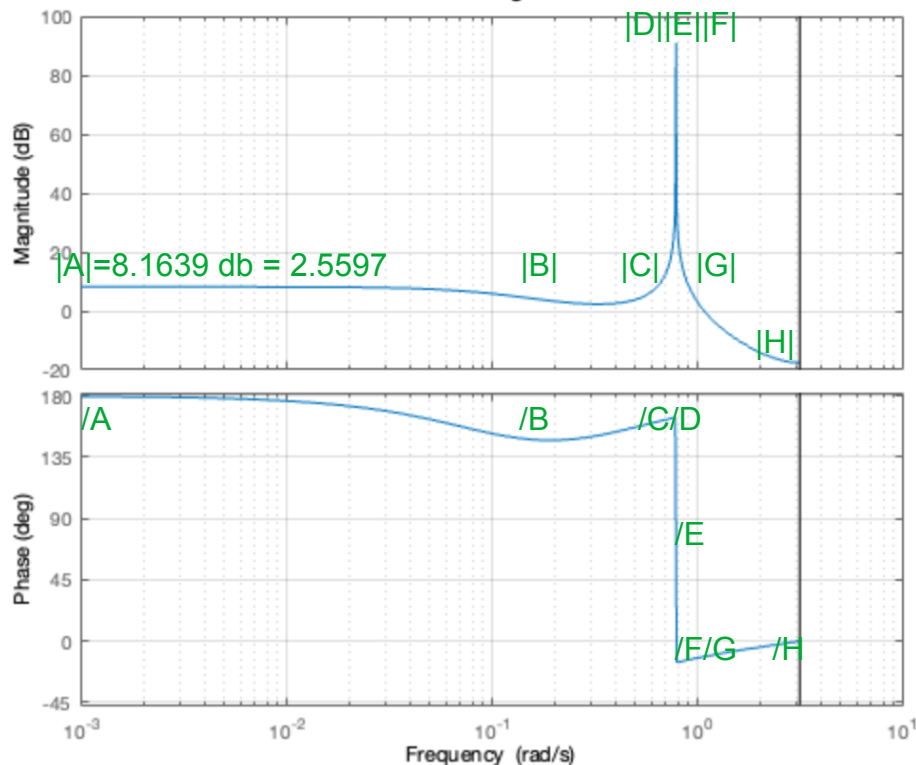
Exemplo

Polos sobre o círculo unitário

$$G(z) = \frac{-0.5z(z - 0.7)}{(z - 0.9)(z^2 - 1.414z + 1)}$$

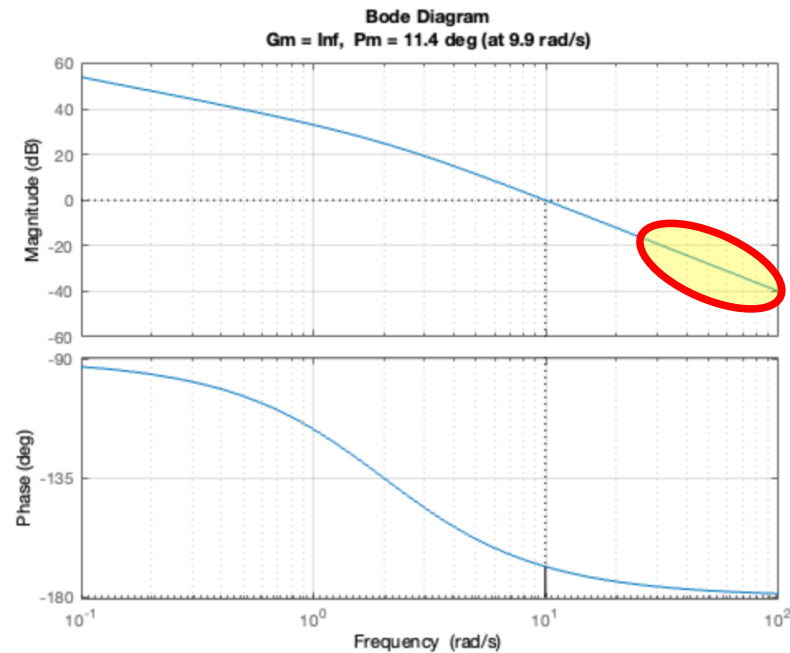
$$G(z \rightarrow 1) = -2.5597 \quad (\text{inversor})$$

Bode Diagram



Estável para $0 < K < 0,3907$

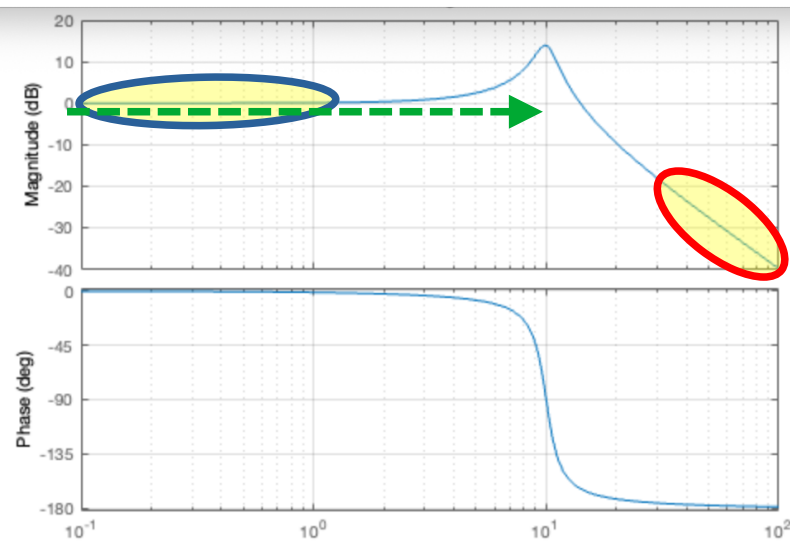
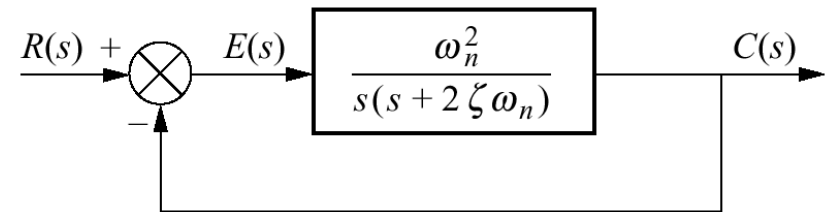
Especificações no domínio- ω (Sistema Contínuo)



$G(s)$

Banda Passante: -3 dB

$\uparrow \omega_{BP} \Rightarrow \downarrow \text{tempos } t_r, t_s, t_p$



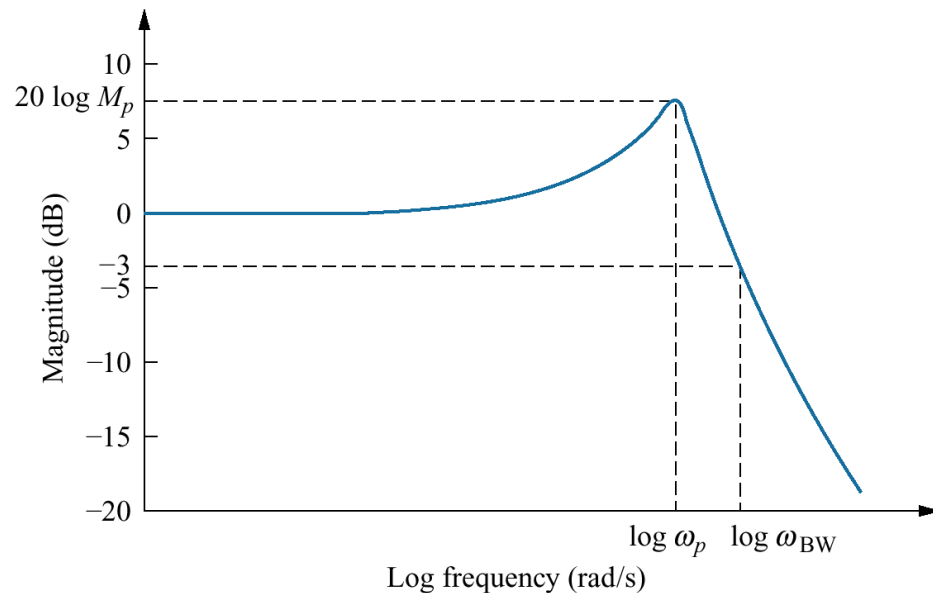
$KG(s)$

$1+KG(s)$

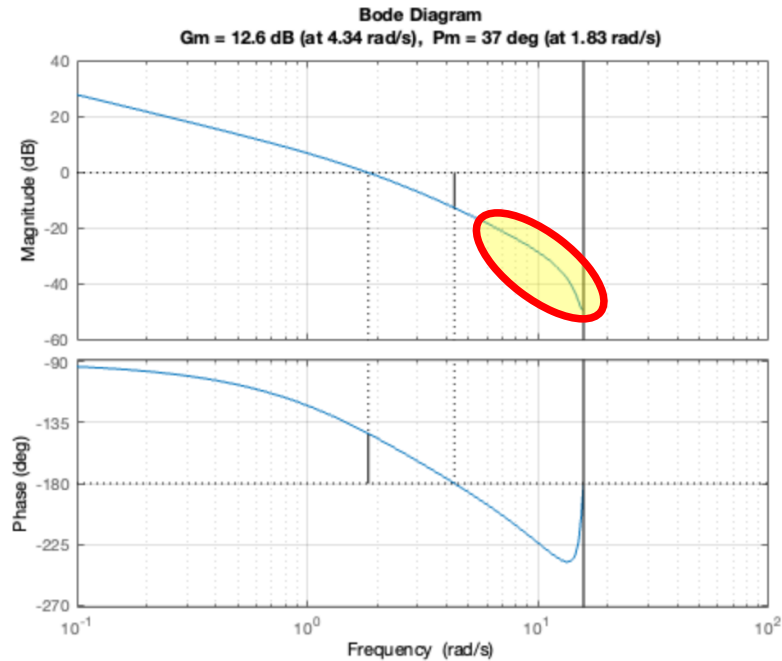
$\omega \rightarrow 0$

$\omega \rightarrow \infty$

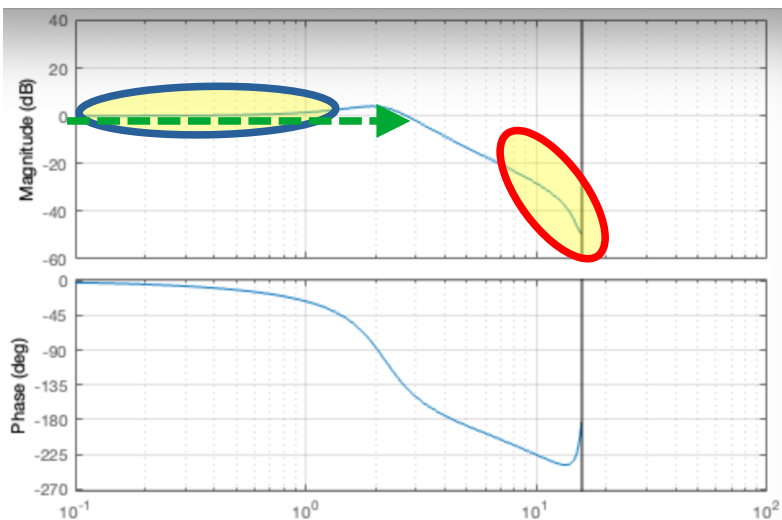
Prof. Adolfo Bauchspies



Especificações no domínio- ω (Sistema Discreto)



$G(s)$



$KG(s)$

$1+KG(s)$

$w \rightarrow 0$

$w \rightarrow \pi/T$

Banda Passante: -3 dB

$\uparrow \omega_{BP} \Rightarrow \downarrow \text{tempos } t_r, t_s, t_p$

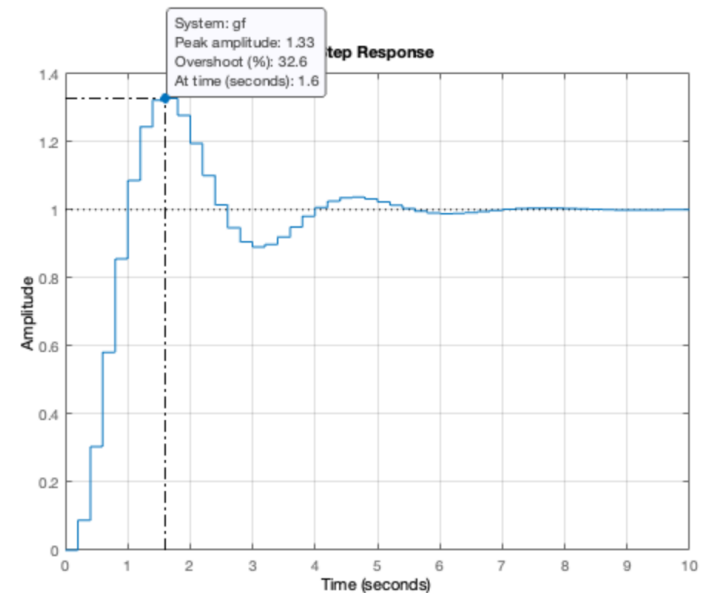
$$G(s) = \frac{100}{s^2 + 2s}$$

$$MF \rightarrow K = 0.05;$$

$$\omega_s = 10 * \pi, T_s = 2 * \pi / \omega_s,$$

$$Gd = c2d(K * G, T_s),$$

$$Gf = \text{feedback}(Gd, 1)$$

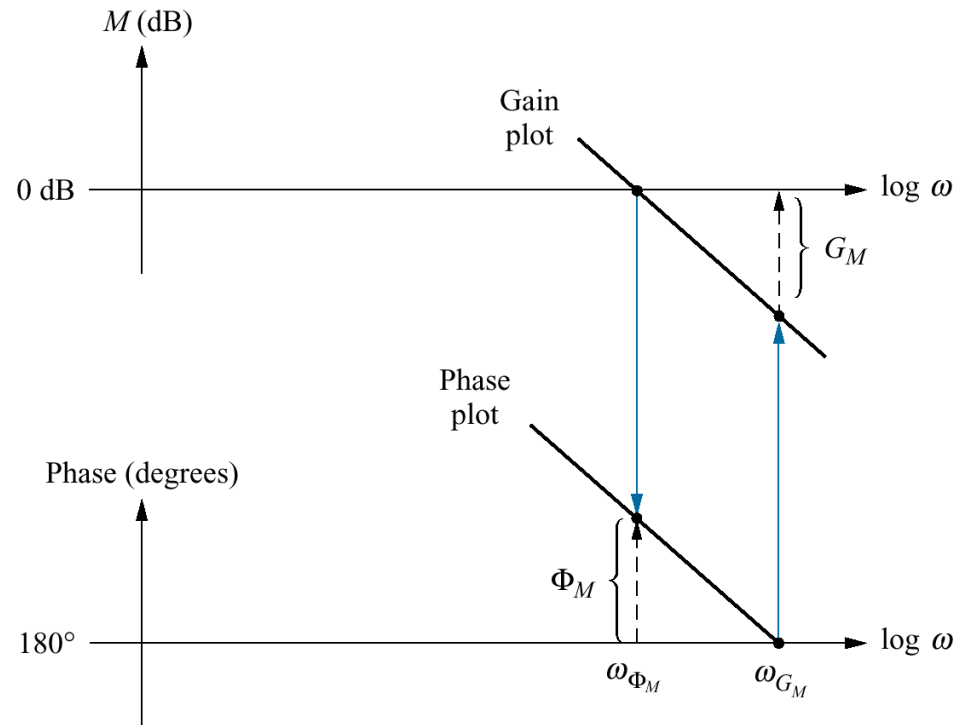
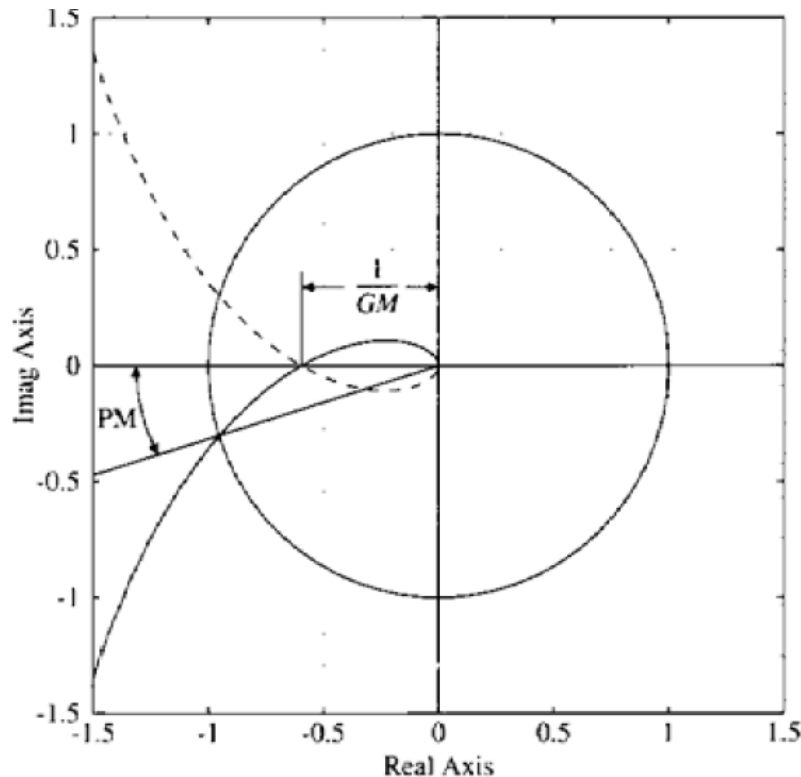


Especificações no domínio- ω

Margem de Ganho MG e Margem de Fase MF

O que “falta” para em o sistema fique instável

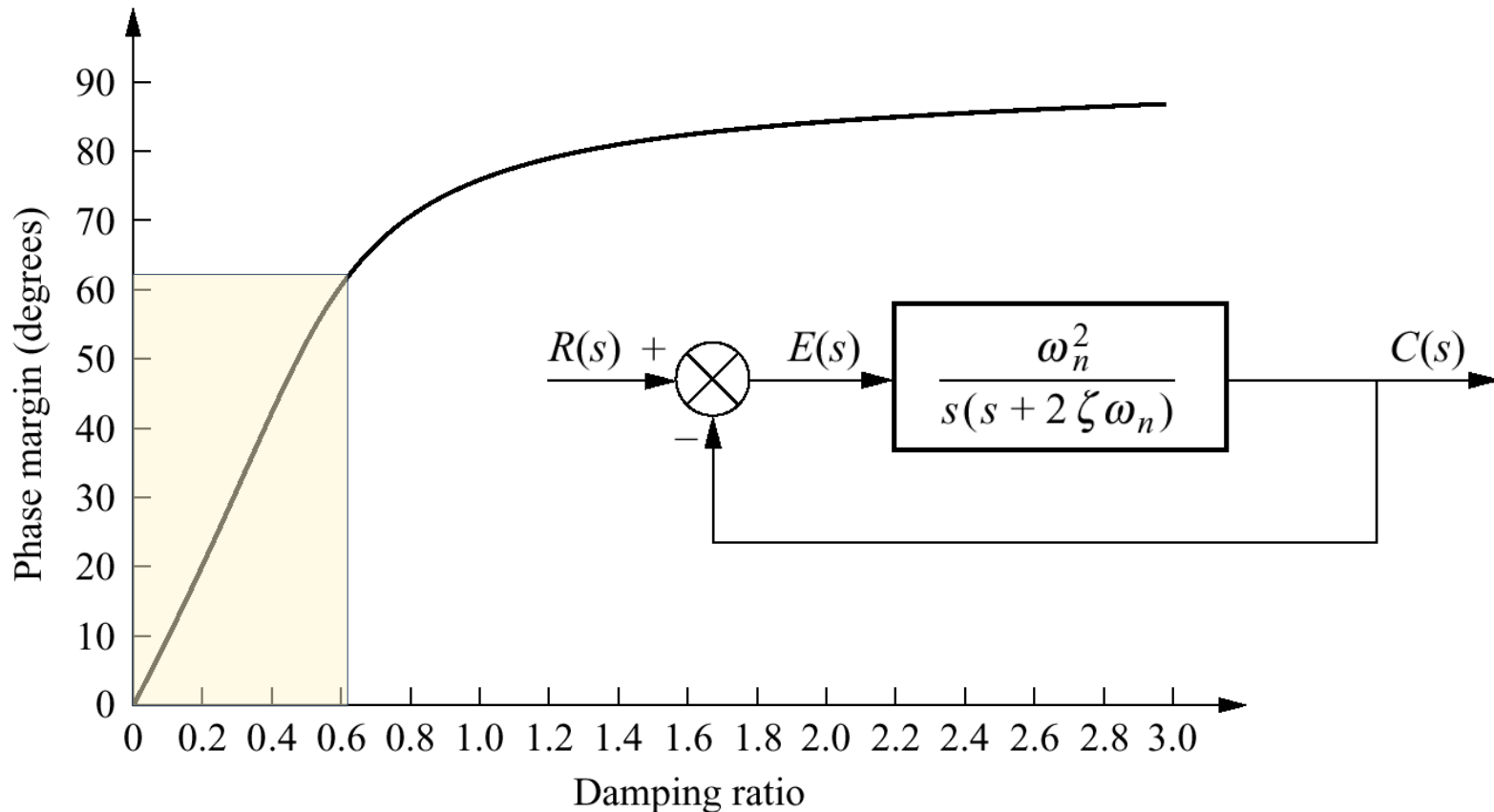
$$GM |G(j\omega_F)| = 1 \quad \angle G(j\omega_G) + \Phi_M = 180^\circ$$



Especificações no domínio- ω

Margem de Fase x Fator de amortecimento

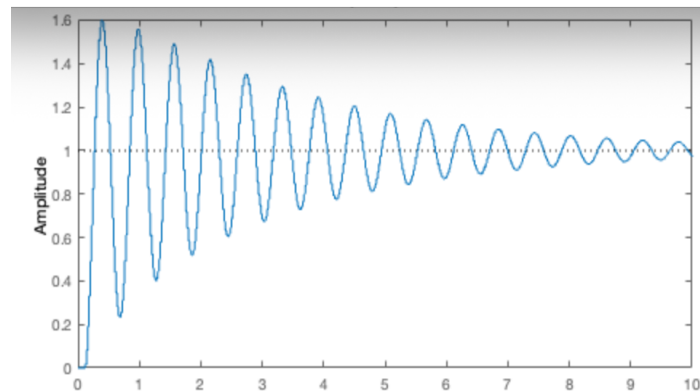
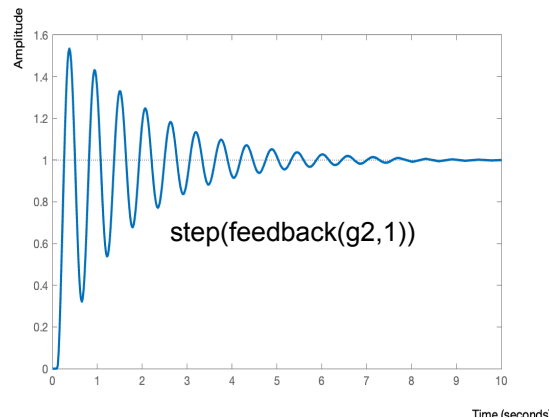
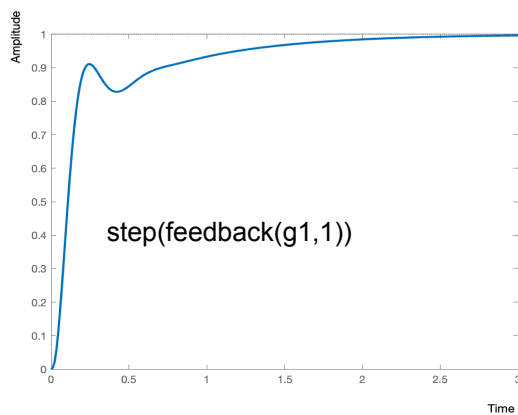
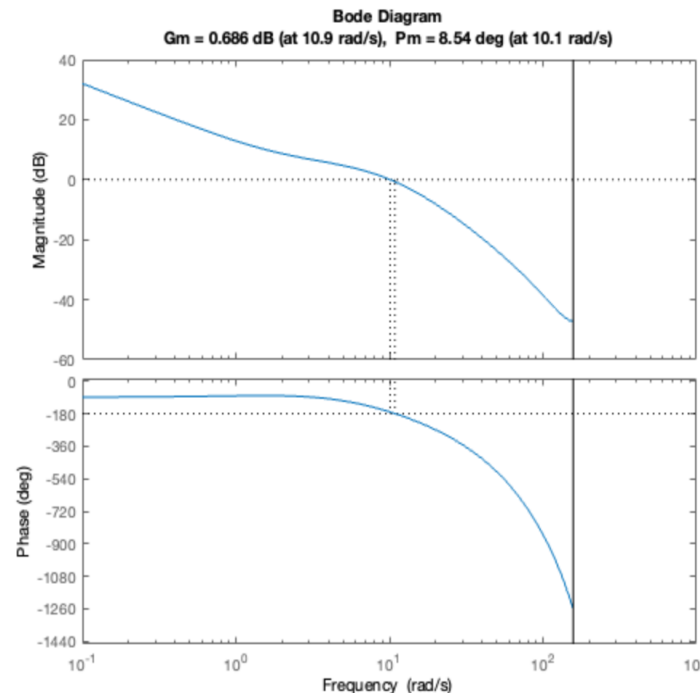
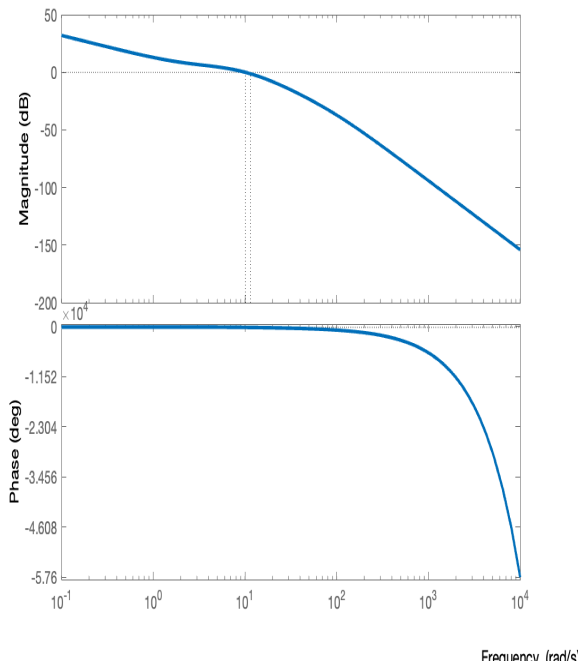
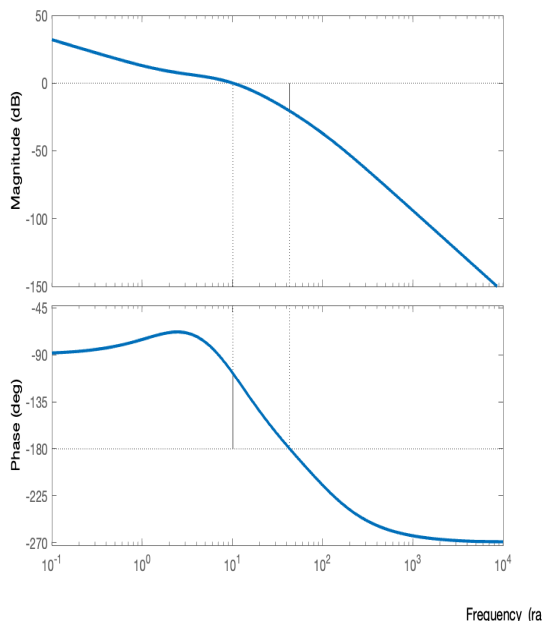
$$MF \sim 100\zeta$$



Sistemas com atraso de transporte no domínio- ω

$g1=zpk([-2],[0 \ -10 \ -10 \ -100],20000); g2.InputDelay=.1; (G_3(s)e^{-0.1s})$

$g3=c2d(g2,0.02)$

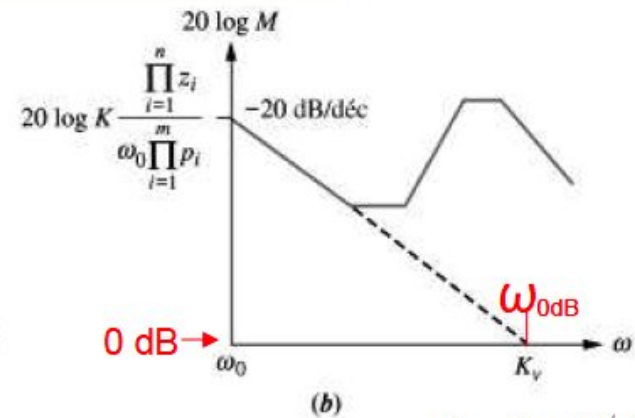
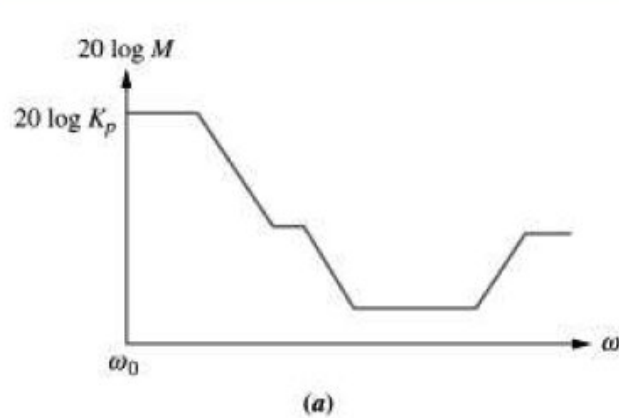


Especificações no domínio- ω

Erro em Regime Permanentes: K_p , K_v , K_a

Método 1 (Nise) encontrar ω_{0dB}

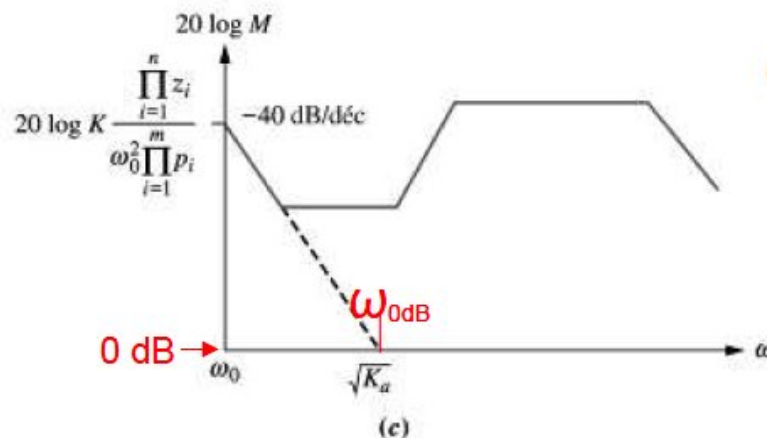
$a. \text{Tipo } 0: K_p = \lim_{s \rightarrow 0} G(s)$
 $b. \text{Tipo } 1: K_v = \lim_{s \rightarrow 0} sG(s)$
 $c. \text{Tipo } 2: K_a = \lim_{s \rightarrow 0} s^2 G(s)$
 $\vdots$



$$K_v = \lim_{s \rightarrow 0} sG(s)$$

em $|G(j\omega_{0dB})| = 0 \text{ dB}$

$$K_v = \omega_{0dB}$$



$$K_a = \lim_{s \rightarrow 0} s^2 G(s)$$

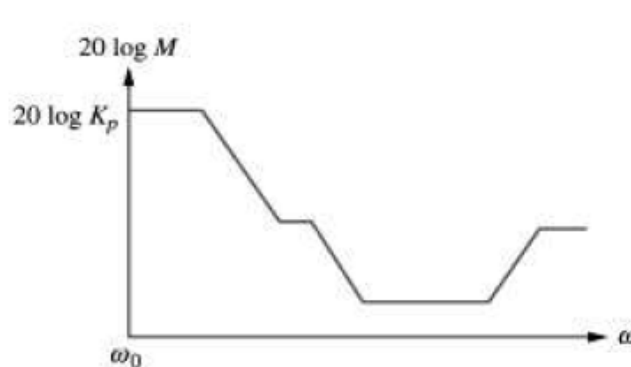
$$K_a = \omega_{0dB}^2$$

Especificações no domínio- ω

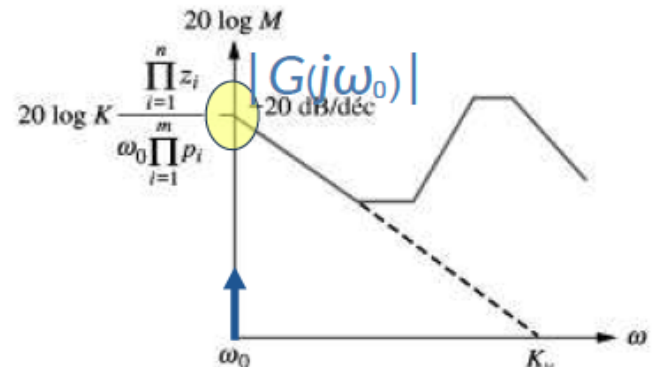
Erro em Regime Permanentes: K_p , K_v , K_a

Método 2 => encontrar $|G(j\omega_0)|$

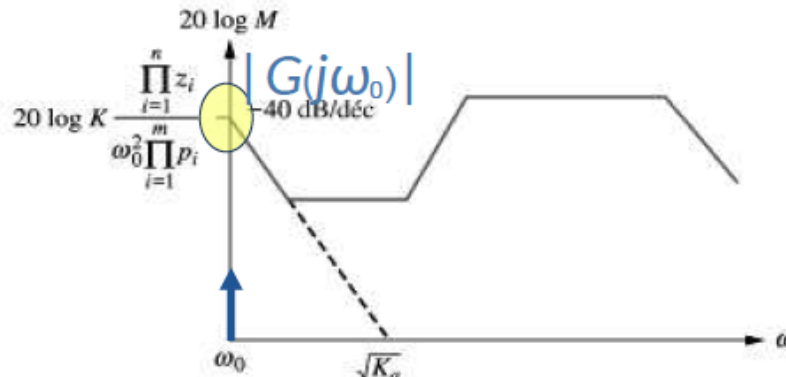
onde ω_0 é a menor freq. disponível no diagrama de Bode



(a)



(b)



(c)

$$K_v = \lim_{s \rightarrow 0} sG(s)$$

$$em |G(j\omega_0)|$$

$$K_v = \omega_0 |G(j\omega_0)|$$

$$K_a = \lim_{s \rightarrow 0} s^2 G(s)$$

$$K_a = \omega_0^2 |G(j\omega_0)|$$

a. Tipo 0: $K_p = \lim_{s \rightarrow 0} G(j\omega_0)$

b. Tipo 1: $K_v = \lim_{s \rightarrow 0} j\omega_0 G(j\omega_0)$

c. Tipo 2: $K_a = \lim_{s \rightarrow 0} j\omega_0^2 G(j\omega_0)$

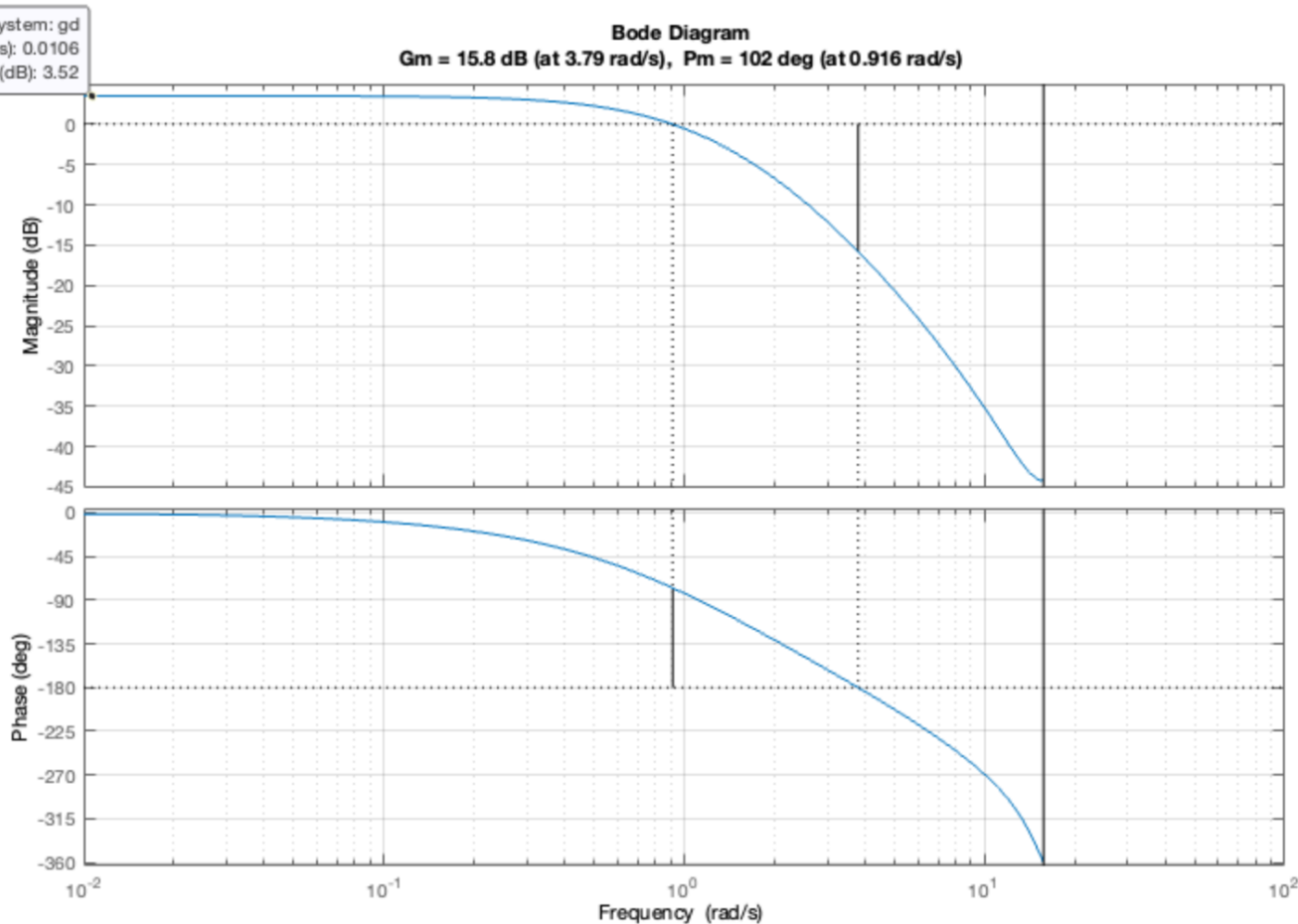
Especificações no domínio- ω

Erro em Regime Permanentes: K_p

$$\text{Tipo 0: } K_p = \lim_{s \rightarrow 0} G(j\omega_0)$$

$$K_p = (3,52 \text{ dB})$$

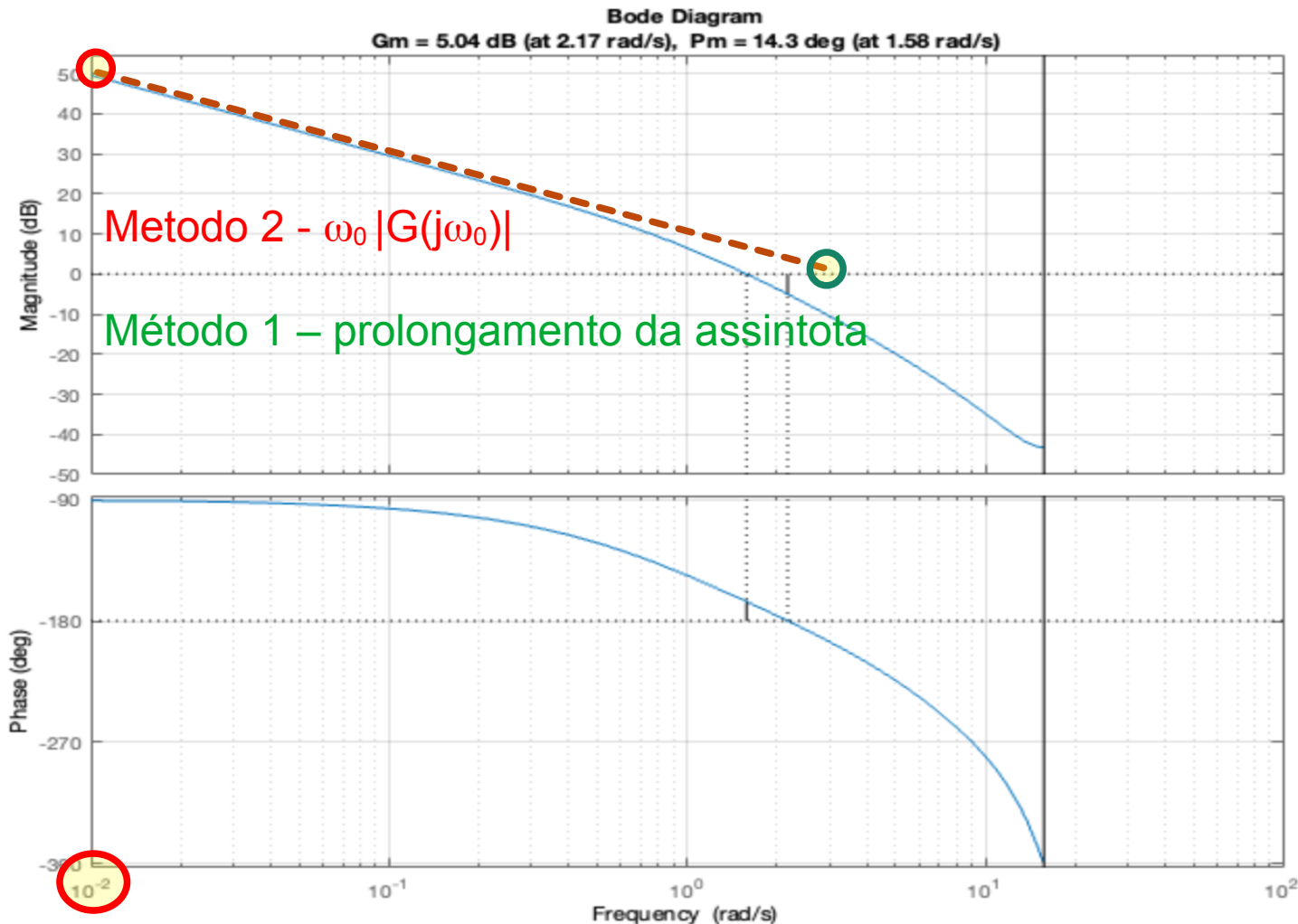
$$K_p = 10^{(3,52/20)} = 1,5$$



Erro em Regime Permanentes: K_v

$$K_v = \lim_{z \rightarrow 1} \frac{(z-1)D(z)G(z)}{Tz}$$

Mas, só temos
o Diag. Bode!!
(dB, °, w)



Tipo 1 : $K_v = \lim_{s \rightarrow 0} sG(s)$
 $s = j\omega_0$

$$K_v = \omega_0 |G(j\omega_0)|$$

$$K_v = 0.01 \times 10^{50/20}$$

$$K_v = 3.1623$$

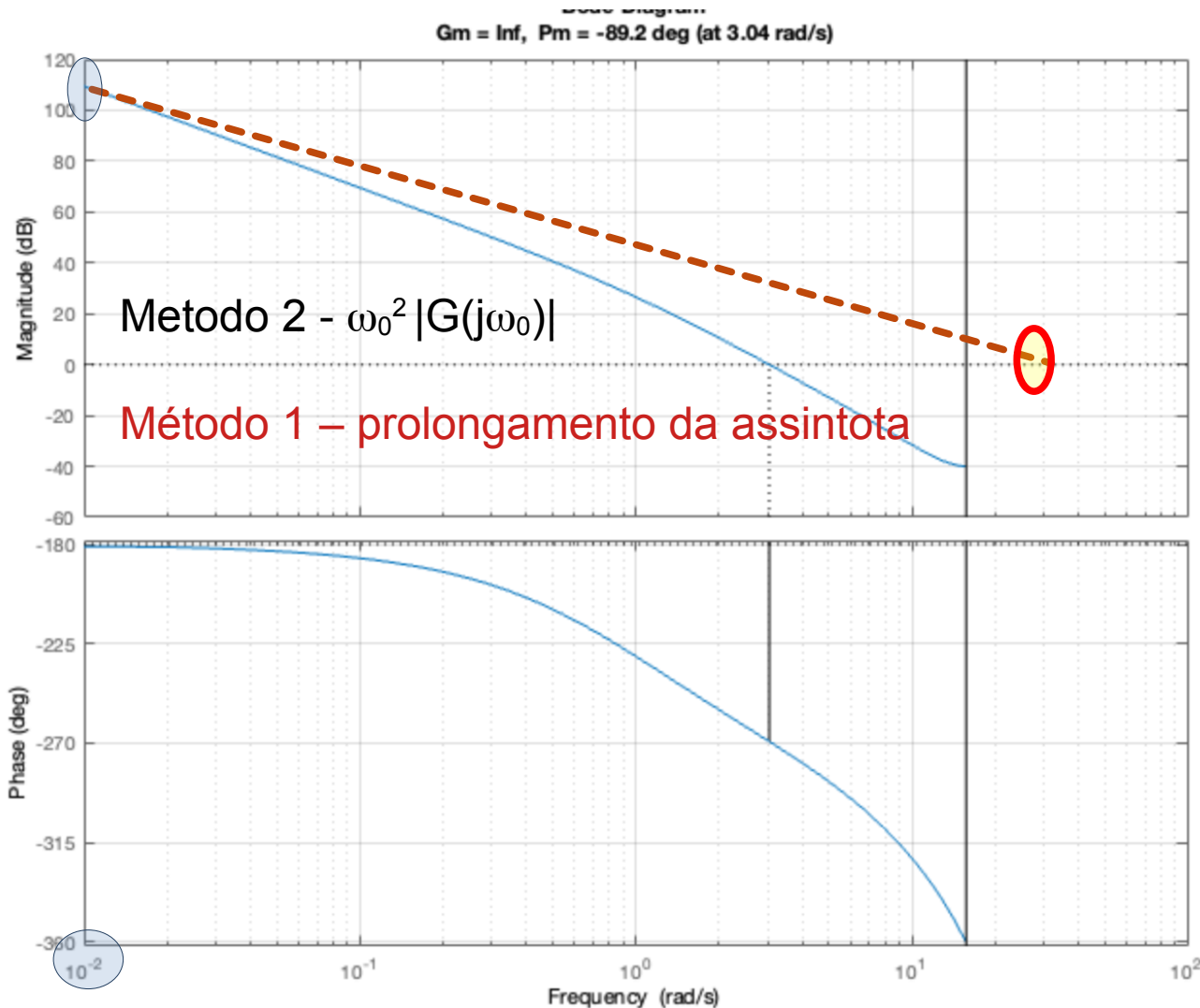
Erro em Regime Permanentes: K_a

Tipo2 : $K_a = \lim_{s \rightarrow 0} s^2 G(j\omega_0)$

$$K_a = \omega_0^2 |G(j\omega_0)|$$

$$K_a = 0.01^2 \times 10^{110/20}$$

$$K_a = 31.6228$$

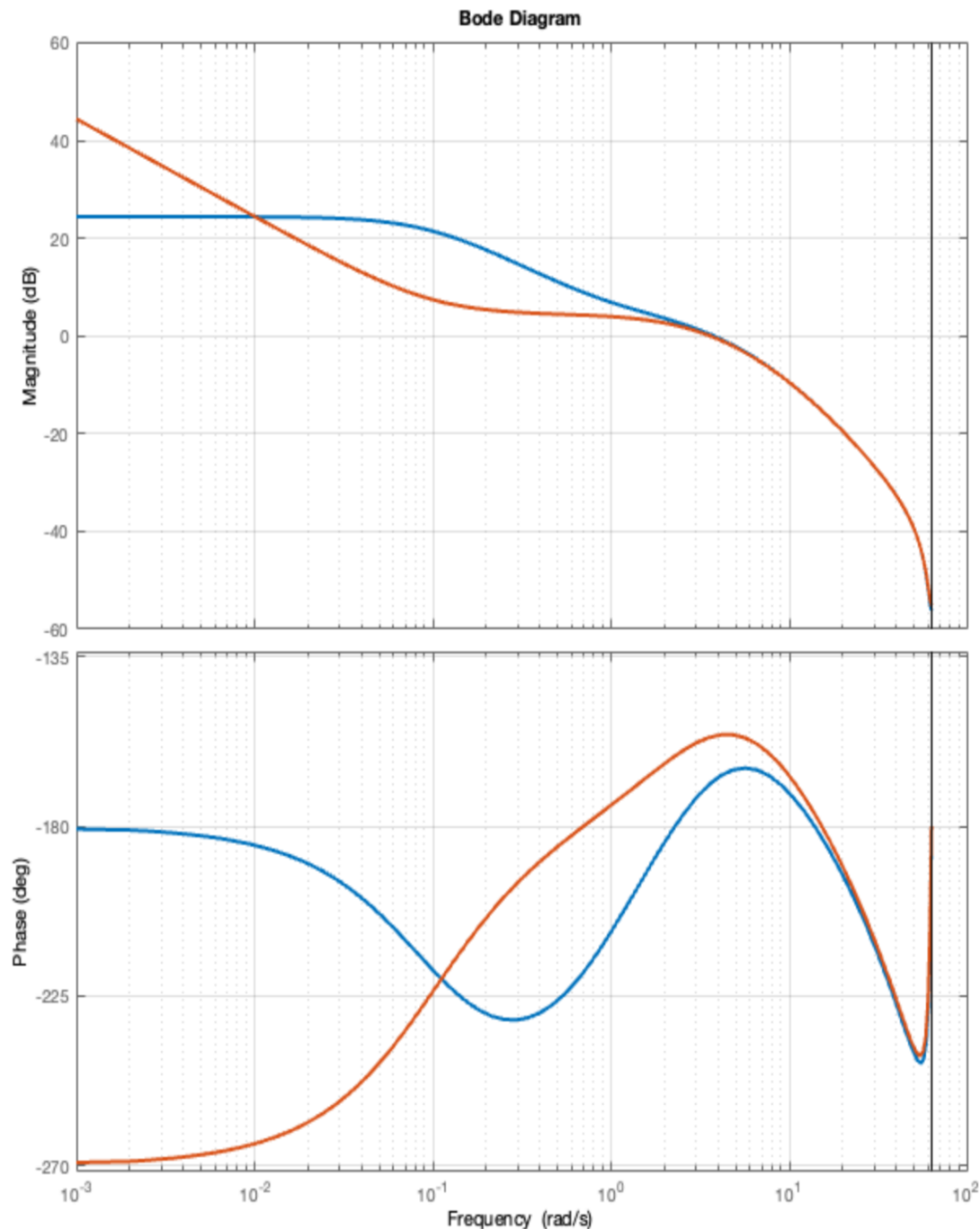


Exercício: Controle P

Considere a resposta em frequência $G(j\omega)$ de dois sistemas que tem um polo no SPD.

Para (I) e (II):

- Esboce os diagramas de Nyquist correspondentes.
- Em malha fechada, quais valores de K , $-\infty < K < \infty$, produzem respostas estáveis?
- Para quais valores de K se tem a maior margem de fase possível?
- Nas condições do item c), qual o erro em regime permanente para um degrau de referência.
- Para quais valores de K o erro em regime permanente para um degrau de referência é $e_{ss} \leq 0,01$.

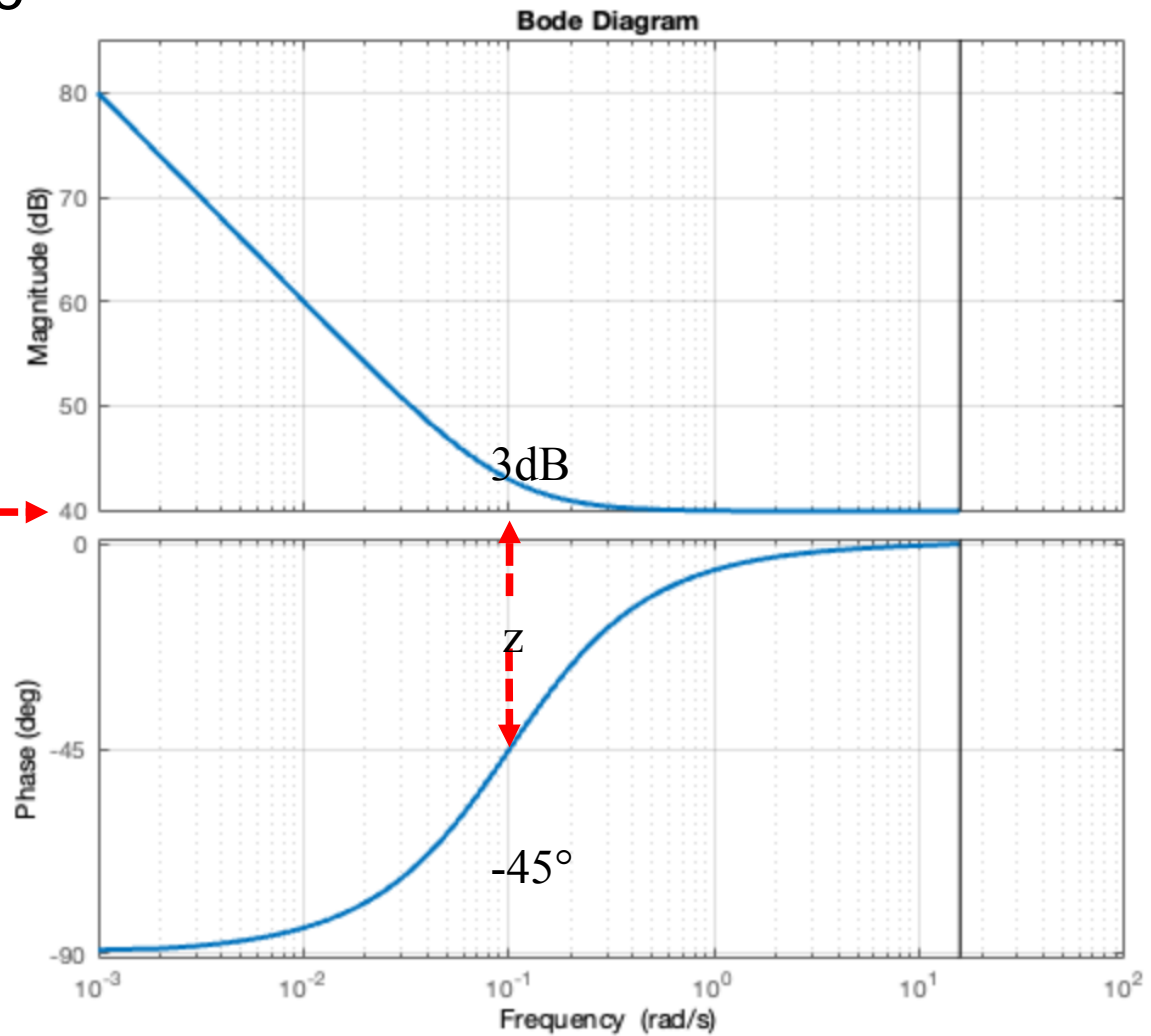


➤ Compensador PI

Muito utilizado

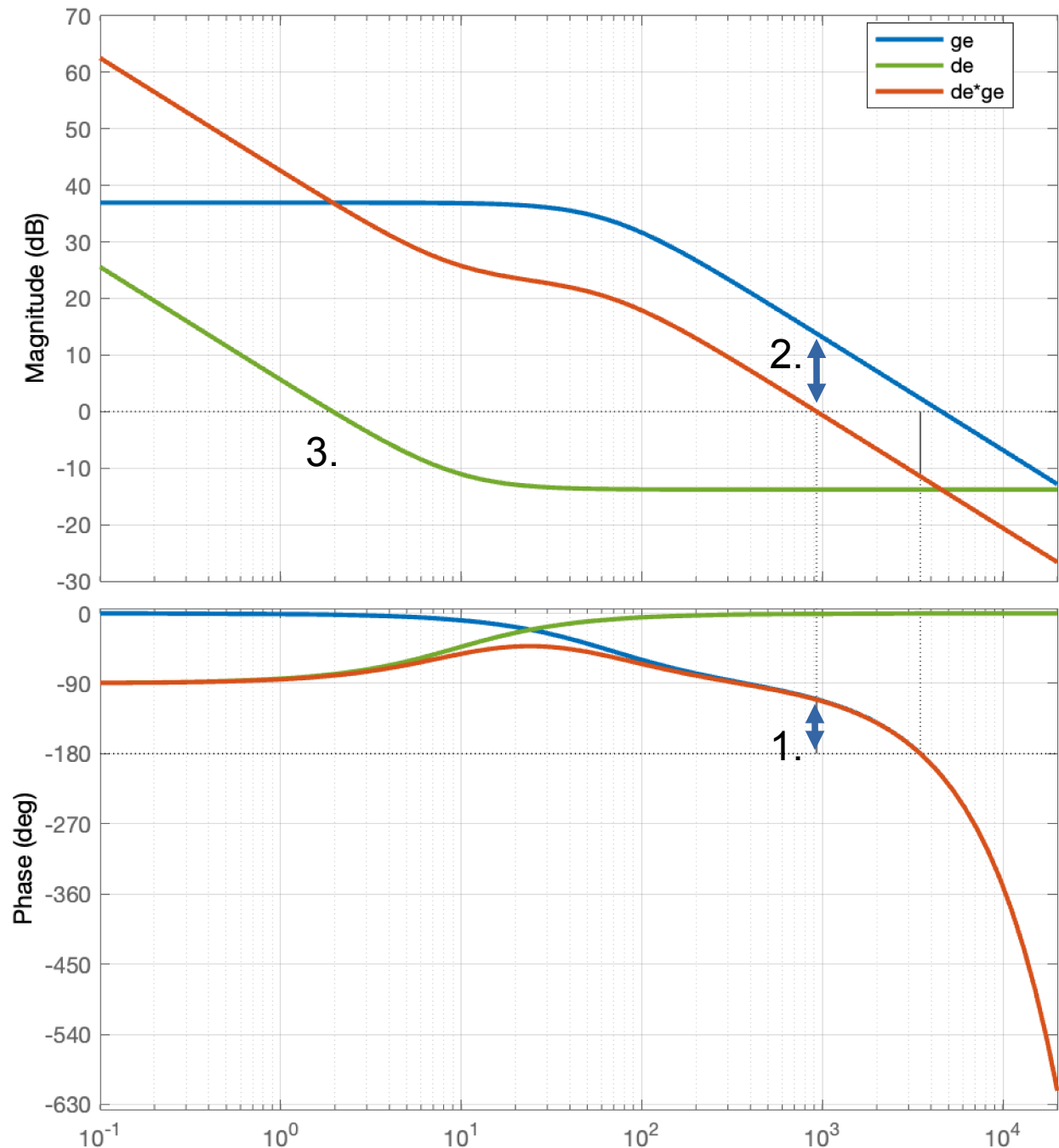
$$D(s) = \frac{K(s + ze)}{s} \rightarrow^T D(z) = \frac{K_d(z - zd)}{z - 1}$$

$s \rightarrow \infty, K_{db}$



➤ Metodologia Projeto PI

- 1. Buscar a fase especificada em ω_F
- 2. Buscar o ajuste de ganho $K \rightarrow 0\text{dB}$ em ω_F
- 3. Posicionar $z_e = 0.01\omega_F$
- ($\angle D(j0.01\omega_F) = 0.8^\circ$)



➤ Projeto PI (em s)

➤ Especificações:

- $MF \geq 70^\circ$

➤ - max K_v

➤ A $MF = 70^\circ$ ocorre em $\omega = 932 \text{ rad/s}$

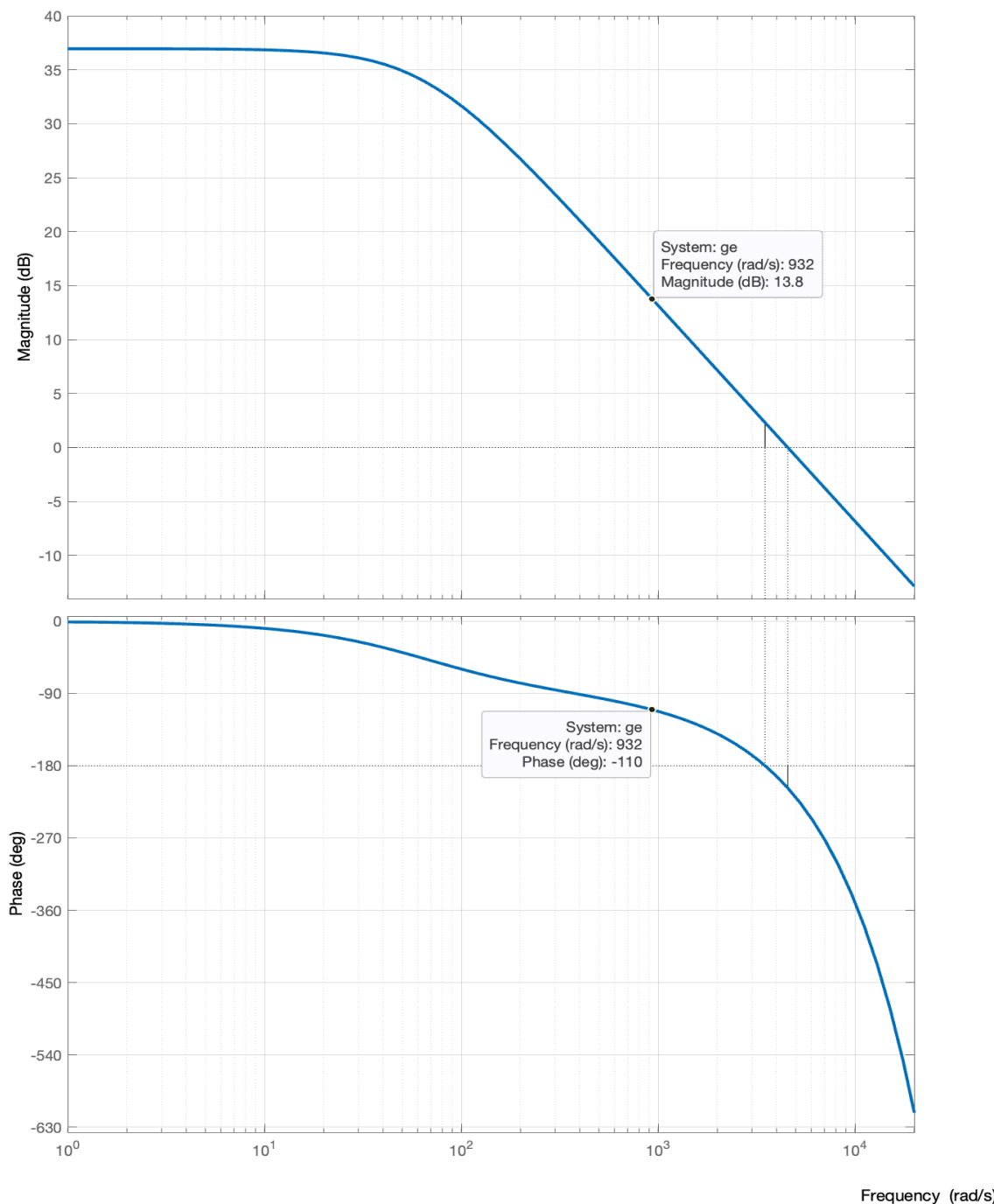
➤ → atenuar ganho 13,8 dB (0,2042)

➤ A posição do zero:

Quanto maior $z \rightarrow$ maior K_v

Para $z = 0.1 \omega_c \Rightarrow MF -5.73^\circ$ nOK

Para $z = 0.01 \omega_c \Rightarrow MF -0,825^\circ$ OK

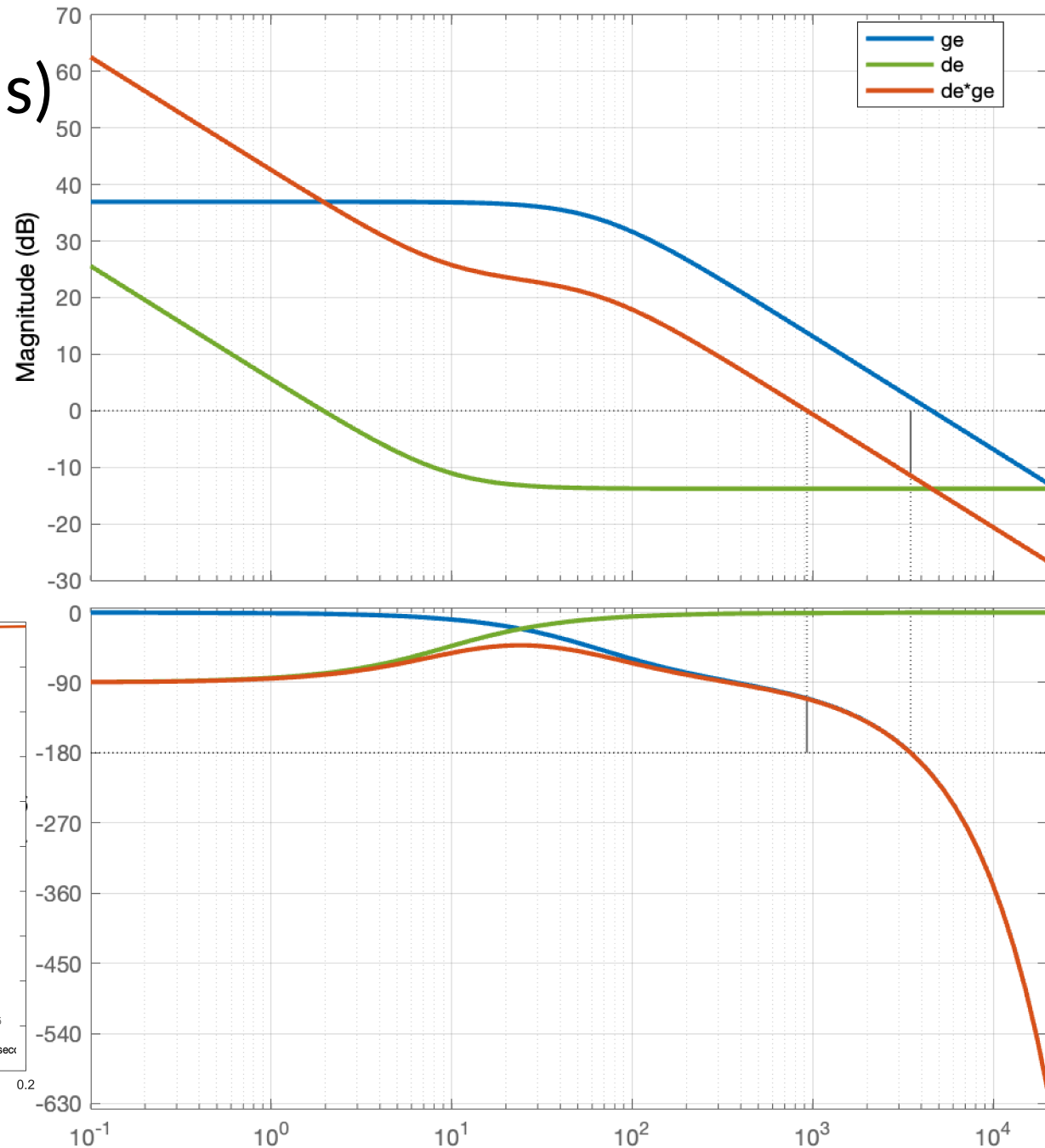
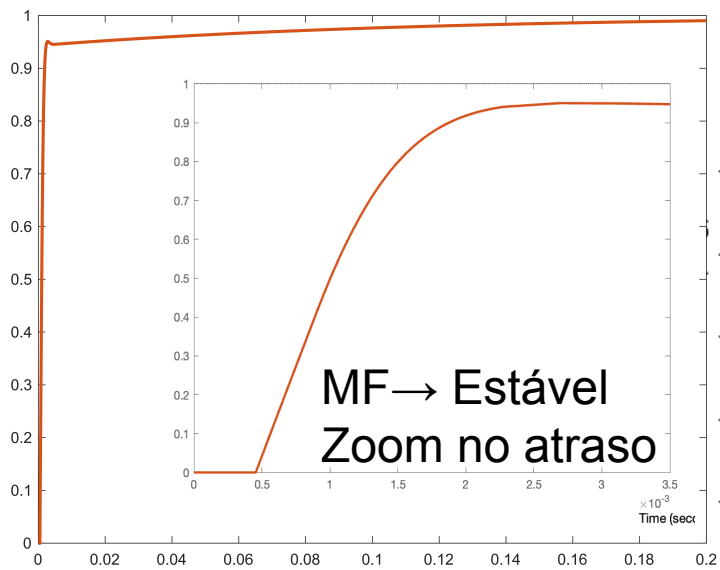
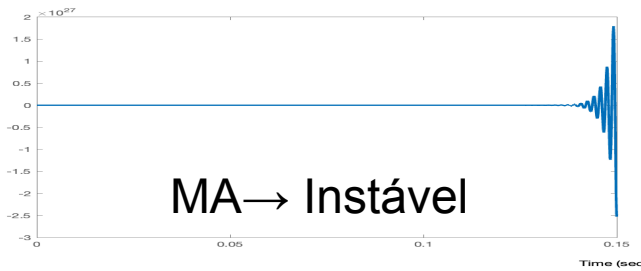


➤ Projeto PI (em s)

➤ Especificações:

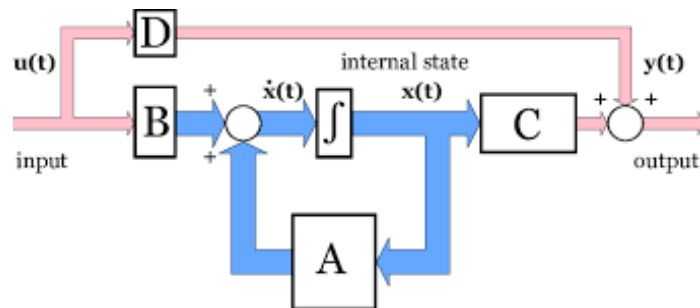
- MF $\geq 70^\circ$

➤ - max Kv



8 - Controle no Espaço-de-Estados

Notação:



Ogata:

$$\dot{\mathbf{x}}(t) = \mathbf{A}\mathbf{x}(t) + \mathbf{B}u(t)$$

$$y(t) = \mathbf{C}\mathbf{x}(t) + \mathbf{D}u(t)$$

$$\mathbf{x}(k+1) = \mathbf{G}\mathbf{x}(k) + \mathbf{H}u(k)$$

$$y(k) = \mathbf{C}\mathbf{x}(k) + \mathbf{D}u(k)$$

Franklin:

$$\dot{\mathbf{x}}(t) = \mathbf{F}\mathbf{x}(t) + \mathbf{G}u(t)$$

$$y(t) = \mathbf{H}\mathbf{x}(t) + \mathbf{J}u(t)$$

$$\mathbf{x}(k+1) = \Phi\mathbf{x}(k) + \Gamma u(k)$$

$$y(k) = \mathbf{H}\mathbf{x}(k) + \mathbf{J}u(k)$$

matlab, Aguirre, Isermann, Samantha,...:

$$\dot{\mathbf{x}}(t) = \mathbf{A}\mathbf{x}(t) + \mathbf{B}u(t)$$

$$y(t) = \mathbf{C}\mathbf{x}(t) + \mathbf{D}u(t)$$

$$\mathbf{x}(k+1) = \mathbf{A}\mathbf{x}(k) + \mathbf{B}u(k)$$

$$y(k) = \mathbf{C}\mathbf{x}(k) + \mathbf{D}u(k)$$

Controle no Espaço-de-Estados

➤ Modelo Discreto Equivalente

Eq. de Estados
$$\begin{cases} \dot{\mathbf{x}} = \mathbf{F}\mathbf{x} + \mathbf{G}u \\ y = \mathbf{H}\mathbf{x} + \mathbf{J}u \end{cases}$$



Solução Homogênea $\dot{\mathbf{x}}_h = \mathbf{F}\mathbf{x}_h(t), \quad \mathbf{x}_h(t_0) = \mathbf{x}_0$

$$\mathbf{x}_h(t) = e^{\mathbf{F}(t-t_0)} \mathbf{x}(t_0)$$

$$\begin{aligned} e^{\mathbf{F}(t-t_0)} &= \mathbf{I} + \mathbf{F}(t-t_0) + \mathbf{F}^2 \frac{(t-t_0)^2}{2!} + \mathbf{F}^3 \frac{(t-t_0)^3}{3!} + \dots \\ &= \sum_{k=0}^{\infty} \mathbf{F}^k \frac{(t-t_0)^k}{k!}. \end{aligned} \quad (\text{Exponencial da Matriz})$$

$$\frac{Y(s)}{U(s)} = \frac{1}{s-a}$$

$$\begin{aligned} \dot{x}(t) &= ax(t) + u(t) \\ y(t) &= x(t) \end{aligned}$$

CDig-ENE/UnB

C.I. : $x(t) = e^{a(t-t_0)} x(t_0)$

Propriedades – Exponencial da Matriz

$$\mathbf{x}_h(t) = e^{\mathbf{F}(t-t_0)}\mathbf{x}(t_0)$$

$$\left\{ \begin{array}{l} \mathbf{x}(t_1) = e^{\mathbf{F}(t_1-t_0)}\mathbf{x}(t_0) \\ \mathbf{x}(t_2) = e^{\mathbf{F}(t_2-t_0)}\mathbf{x}(t_0) \end{array} \right. \Rightarrow \mathbf{x}(t_2) = e^{\mathbf{F}(t_2-t_1)}\mathbf{x}(t_1)$$

$$\mathbf{x}(t_2) = e^{\mathbf{F}(t_2-t_1)}e^{\mathbf{F}(t_1-t_0)}\mathbf{x}(t_0)$$

$$e^{\mathbf{F}(t_2-t_0)} = e^{\mathbf{F}(t_2-t_1)}e^{\mathbf{F}(t_1-t_0)}$$

$$\mathbf{I} = e^{-\mathbf{F}(t_1-t_0)}e^{\mathbf{F}(t_1-t_0)}$$

Solução Completa

$$\mathbf{x}(t) = e^{\mathbf{F}(t-t_0)}\mathbf{x}(t_0) + \int_{t_0}^t e^{\mathbf{F}(t-\tau)}\mathbf{G}u(\tau)d\tau$$

Para um período de amostragem:

$$t = kT + T \text{ and } t_0 = kT$$

$$\mathbf{x}(kT + T) = e^{\mathbf{F}T}\mathbf{x}(kT) + \int_{kT}^{kT+T} e^{\mathbf{F}(kT+T-\tau)}\mathbf{G}u(\tau)d\tau$$

$$\text{ZOH: } u(\tau) = u(kT), \quad kT \leq \tau < kT + T$$

Solução Completa

$$\mathbf{x}(kT + T) = e^{\mathbf{F}T} \mathbf{x}(kT) + \int_{kT}^{kT+T} e^{\mathbf{F}(kT+T-\tau)} \mathbf{G}u(\tau) d\tau$$

$$\text{ZOH: } u(\tau) = u(kT), \quad kT \leq \tau < kT + T$$

$$\mathbf{x}(kT + T) = e^{\mathbf{F}T} \mathbf{x}(kT) + \int_0^T e^{\mathbf{F}\eta} d\eta \mathbf{G}u(kT) \quad \eta = kT + T - \tau$$

Definindo:

$$\begin{aligned} \Phi &= e^{\mathbf{F}T} \\ \Gamma &= \int_0^T e^{\mathbf{F}\eta} d\eta \mathbf{G}. \end{aligned}$$

$$\begin{aligned} \mathbf{x}(k+1) &= \Phi \mathbf{x}(k) + \Gamma u(k) \\ y(k) &= \mathbf{H} \mathbf{x}(k) + J u(k) \end{aligned}$$

*Espaço de Estados Discreto
Equivalente ZOH*

Exemplo EE

$$G(s) = \frac{1}{s^2} \quad \ddot{\theta} = u.$$

$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \end{bmatrix} = \underbrace{\begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix}}_{\mathbf{F}} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + \underbrace{\begin{bmatrix} 0 \\ 1 \end{bmatrix}}_{\mathbf{G}} u.$$

$$\theta = y = \underbrace{\begin{bmatrix} 1 & 0 \end{bmatrix}}_{\mathbf{H}} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix}$$

$$\begin{aligned} \mathbf{x}(k+1) &= \mathbf{\Phi} \mathbf{x}(k) + \mathbf{\Gamma} u(k) \\ y(k) &= \mathbf{H} \mathbf{x}(k) + J u(k) \end{aligned}$$

$$\begin{aligned} \mathbf{\Phi} &= \mathbf{I} + \mathbf{F}T + \frac{\mathbf{F}^2 T^2}{2!} + \dots \\ &= \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} + \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix} T = \begin{bmatrix} 1 & T \\ 0 & 1 \end{bmatrix} \end{aligned}$$

Cálculo das Matrizes

$$\Phi = \mathbf{I} + \mathbf{F}T\mathbf{\Psi}$$

onde $\mathbf{\Psi} = \mathbf{I} + \frac{\mathbf{F}T}{2!} + \frac{\mathbf{F}^2T^2}{3!} + \dots$

$$\begin{aligned}\Gamma &= \sum_{k=0}^{\infty} \frac{\mathbf{F}^k T^{k+1}}{(k+1)!} \mathbf{G} \\ &= \sum_{k=0}^{\infty} \frac{\mathbf{F}^k T^k}{(k+1)!} T \mathbf{G} \\ &= \mathbf{\Psi} T \mathbf{G}.\end{aligned}$$

Cálculo das Matrizes

$$\begin{aligned}\Gamma &= \sum_{k=0}^{\infty} \frac{\mathbf{F}^k T^{k+1}}{(k+1)!} \mathbf{G} \\ &= \sum_{k=0}^{\infty} \frac{\mathbf{F}^k T^k}{(k+1)!} T \mathbf{G} \\ &= \Psi T \mathbf{G}.\end{aligned}\quad \text{onde} \quad \Psi = \mathbf{I} + \frac{\mathbf{F}T}{2!} + \frac{\mathbf{F}^2 T^2}{3!} + \dots$$

$$\begin{aligned}\Phi &= \mathbf{I} + \mathbf{F}T + \frac{\mathbf{F}^2 T^2}{2!} + \dots \\ &= \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} + \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix} T = \begin{bmatrix} 1 & T \\ 0 & 1 \end{bmatrix} \\ \Gamma &= \left[\mathbf{I}T + \mathbf{F} \frac{T^2}{2!} + \frac{\mathbf{F}^2 T^3}{3!} \right] \mathbf{G} \\ &= \left\{ \begin{bmatrix} T & 0 \\ 0 & T \end{bmatrix} + \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix} \frac{T^2}{2} \right\} \begin{bmatrix} 0 \\ 1 \end{bmatrix} = \begin{bmatrix} T^2/2 \\ T \end{bmatrix}.\end{aligned}$$

Exemplo: Função de Transferência

$$\frac{Y(z)}{U(z)} = \mathbf{H}[z\mathbf{I} - \mathbf{\Phi}]^{-1}\mathbf{\Gamma}$$

$$\begin{aligned}\frac{Y(z)}{U(z)} &= [1 \quad 0] \left\{ z \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} - \begin{bmatrix} 1 & T \\ 0 & 1 \end{bmatrix} \right\}^{-1} \begin{bmatrix} T^2/2 \\ T \end{bmatrix} \\ &= \frac{T^2}{2} \frac{(z+1)}{(z-1)^2},\end{aligned}$$

Controle no Espaço de Estados

➤ Projeto direto - Emulação (em "s")

$$\dot{\mathbf{x}} = \mathbf{F}\mathbf{x} + \mathbf{G}u$$

$$y = \mathbf{H}\mathbf{x} + \mathbf{J}u$$



$$\mathbf{x}(k+1) = \mathbf{\Phi}\mathbf{x}(k) + \mathbf{\Gamma}u(k)$$

$$y(k) = \mathbf{H}\mathbf{x}(k) + \mathbf{J}u(k)$$

Onde:

$$\mathbf{\Phi} = e^{\mathbf{F}T},$$

$$\mathbf{\Gamma} = \int_0^T e^{\mathbf{F}\eta} d\eta \mathbf{G}.$$

> sysC = ss(F,G,H,J)

> sysD = c2d(sysC,T)

> [sysD.A, sysD.B, sysD.C, sysD.D]

Projeto da Lei de Controle

- Realimentação de Estados

$$u = -\mathbf{K}\mathbf{x} = -\begin{bmatrix} K_1 & K_2 & \cdots \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ \vdots \end{bmatrix}$$

$$\mathbf{x}(k+1) = \mathbf{\Phi}\mathbf{x}(k) - \mathbf{\Gamma}\mathbf{K}\mathbf{x}(k)$$

$$(z\mathbf{I} - \mathbf{\Phi} + \mathbf{\Gamma}\mathbf{K})\mathbf{X}(z) = 0$$

$$|z\mathbf{I} - \mathbf{\Phi} + \mathbf{\Gamma}\mathbf{K}| = 0 \quad \text{Eq. característica}$$

$$\alpha_c(z) = (z - \beta_1)(z - \beta_2) \cdots (z - \beta_n) = 0 \quad \begin{array}{l} \text{Eq. Característica desejada} \\ \rightarrow \text{igualar potências} \end{array}$$

Exemplo

$$G(s) = \frac{1}{s^2} \quad \ddot{\theta} = u$$

$$\Phi = \begin{bmatrix} 1 & T \\ 0 & 1 \end{bmatrix} \quad \text{and} \quad \Gamma = \begin{bmatrix} T^2/2 \\ T \end{bmatrix}$$

Equação característica desejada: $z^2 - 1.6z + 0.70 = 0$

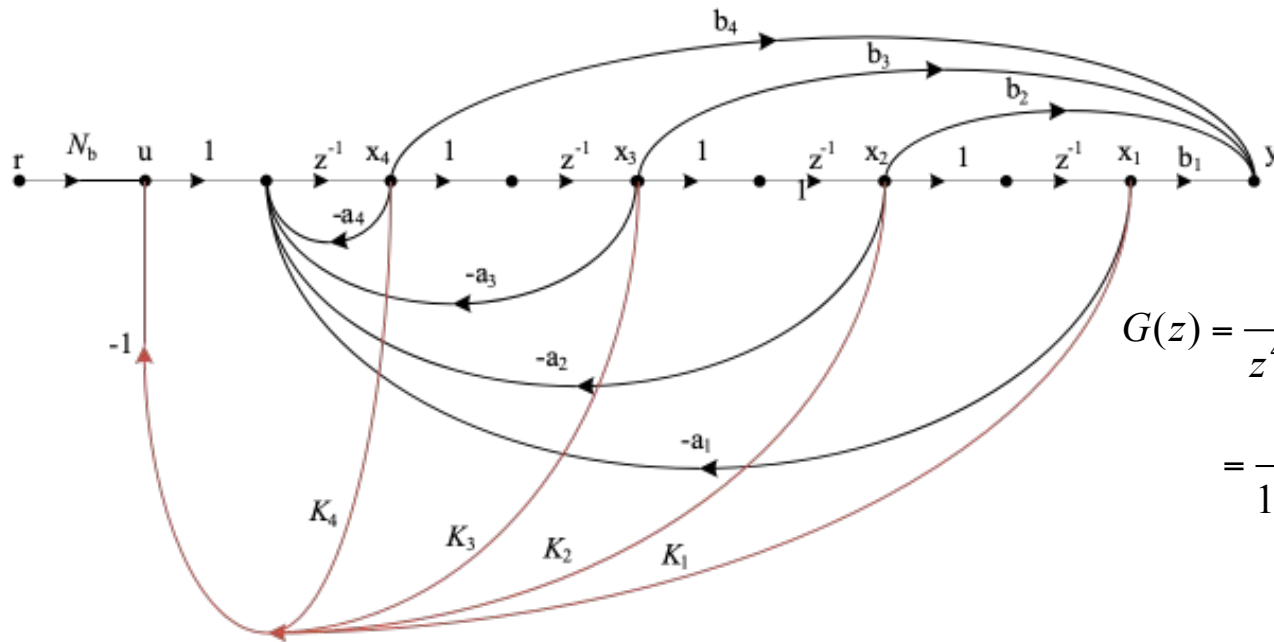
$$\left| z \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} - \begin{bmatrix} 1 & T \\ 0 & 1 \end{bmatrix} + \begin{bmatrix} T^2/2 \\ T \end{bmatrix} [K_1 \quad K_2] \right| = 0$$

$$z^2 + (TK_2 + (T^2/2)K_1 - 2)z + (T^2/2)K_1 - TK_2 + 1 = 0$$

$$\begin{aligned} TK_2 + (T^2/2)K_1 - 2 &= -1.6, \\ (T^2/2)K_1 - TK_2 + 1 &= 0.70, \end{aligned}$$

$$K_1 = \frac{0.10}{T^2} = 10, \quad K_2 = \frac{0.35}{T} = 3.5$$

Utilizando a forma canônica controlável



$$G(z) = \frac{b_4 z^3 + b_3 z^2 + b_2 z + b_1}{z^4 + a_4 z^3 + a_3 z^2 + a_2 z + a_1}$$

$$= \frac{b_4 z^{-1} + b_3 z^{-2} + b_2 z^{-3} + b_1 z^{-4}}{1 + a_4 z^{-1} + a_3 z^{-2} + a_2 z^{-3} + a_1 z^{-4}}$$

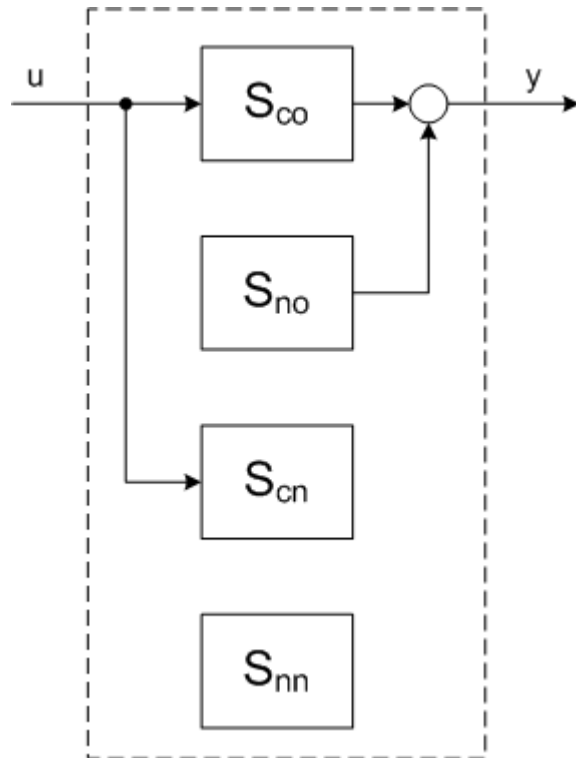
Eq. desejada $\alpha(z) = z^4 + \alpha_4 z^3 + \alpha_3 z^2 + \alpha_2 z + \alpha_1$

$$\Phi_c - \Gamma_c K = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ -a_1 - K_1 & -a_2 - K_2 & -a_3 - K_3 & -a_4 - K_4 \end{bmatrix}$$

$$\det(zI - \Phi_c + \Gamma_c K) = z^4 + (a_4 + K_4)z^3 + (a_3 + K_3)z^2 + (a_2 + K_2)z + (a_1 + K_1)$$

$$\begin{aligned} K_1 &= \alpha_1 - a_1 \\ K_2 &= \alpha_2 - a_2 \\ K_3 &= \alpha_3 - a_3 \\ K_4 &= \alpha_4 - a_4 \end{aligned}$$

Possíveis componentes de um sistema



Subsistemas

- Controlável e Observável
- Não controlável, Observável
- Controlável, Não observável
- Não Controlável e Não Observável

Função de Transferência

$$Y(z)/U(z) = S_{co} \text{ (pólos)}$$

Autovalores consideram

$$S_{co}, S_{no}, S_{cn}, S_{nn} \text{ (estabilidade)}$$

Controlabilidade

- Em malha aberta

$$|z\mathbf{I} - \mathbf{\Phi}| = 0$$

Na forma canônica de Jordan (pólos distintos)

$$\mathbf{x}(k+1) = \begin{bmatrix} \lambda_1 & 0 & \cdots & 0 \\ 0 & \lambda_2 & \cdots & 0 \\ 0 & 0 & \ddots & 0 \\ 0 & 0 & \cdots & \lambda_n \end{bmatrix} \mathbf{x}(k) + \begin{bmatrix} \Gamma_1 \\ \Gamma_2 \\ \vdots \\ \Gamma_n \end{bmatrix} u(k)$$

Controlabilidade $\rightarrow$ Nenhum Γ_i pode ser nulo

Posicionamento de pólos CACSD

- Para s e para z
 - >> place
 - >> acker

Fórmula de Ackermann (1972), SISO, até n=10, pólos repetidos

$$\mathbf{K} = [0 \ \cdots \ 1] \underbrace{[\Gamma \ \Phi\Gamma \ \Phi^2\Gamma \ \cdots \ \Phi^{n-1}\Gamma]^{-1}}_{\text{Matriz de controlabilidade}} \alpha_c(\Phi)$$

Matriz de controlabilidade

$$\alpha_c(z) = |z\mathbf{I} - \Phi + \Gamma\mathbf{K}| = z^n + \alpha_1 z^{n-1} + \cdots + \alpha_n$$

$$\alpha_c(\Phi) = \Phi^n + \alpha_1 \Phi^{n-1} + \alpha_2 \Phi^{n-2} + \cdots + \alpha_n \mathbf{I} \quad \text{Eq. Carcterística desejada}$$

Posicionamento de pólos CACSD

- Place (Kautsky, Nichols & Von Dooren, 1985)
MIMO, ordem elevada, não aceita pólos repetidos

Exemplo: antena $1/s^2$

$$T = .1$$

$$\Phi = [1 \ T; 0 \ 1]$$

$$G = [T^2/2; T]$$

$$p = [.8 + i*.25; .8 - i*.25]$$

$$K = \text{acker}(\Phi, G, p)$$

$$K = [10.25 \ 3.4875]$$

Observabilidade

- Também via Fórmula de Ackermann» $L_p' = \text{acker}(\Phi', H', p_e)$

$$\tilde{\mathbf{x}}(k+1) = [\Phi - \mathbf{L}_p \mathbf{H}] \tilde{\mathbf{x}}(k) \quad \text{erro de observação (estimação)}$$

$$[\Phi - \mathbf{L}_p \mathbf{H}]^T = \Phi^T - \mathbf{H}^T \mathbf{L}_p^T$$

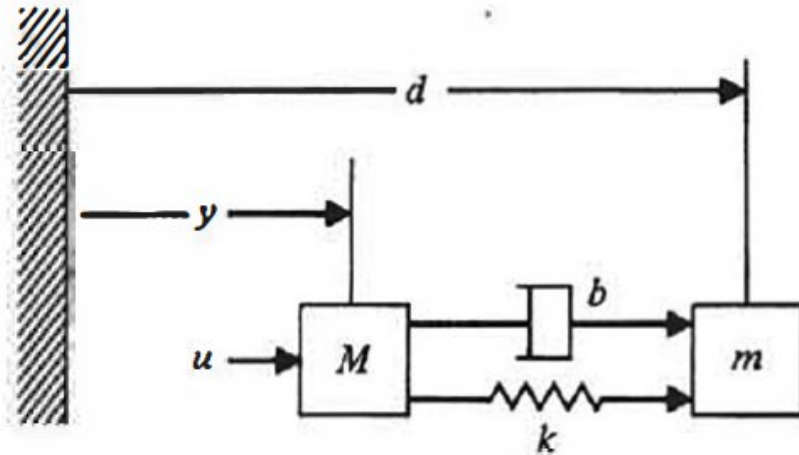
$$\mathbf{L}_p = \alpha_e(\Phi) \underbrace{\begin{bmatrix} \mathbf{H} \\ \mathbf{H}\Phi \\ \mathbf{H}\Phi^2 \\ \vdots \\ \mathbf{H}\Phi^{n-1} \end{bmatrix}^{-1}}_{\text{Matriz de Observabilidade}} \begin{bmatrix} 0 \\ 0 \\ \vdots \\ 0 \\ 1 \end{bmatrix},$$

Matriz de Observabilidade

$$\alpha_e(\Phi) = \Phi^n + \alpha_1 \Phi^{n-1} + \alpha_2 \Phi^{n-2} + \dots + \alpha_n \mathbf{I}$$

$$\alpha_e(z) = (z - \beta_1)(z - \beta_2) \dots (z - \beta_n) = z^n + \alpha_1 z^{n-1} + \dots + \alpha_n \quad \text{Eq. Car. desejada}$$

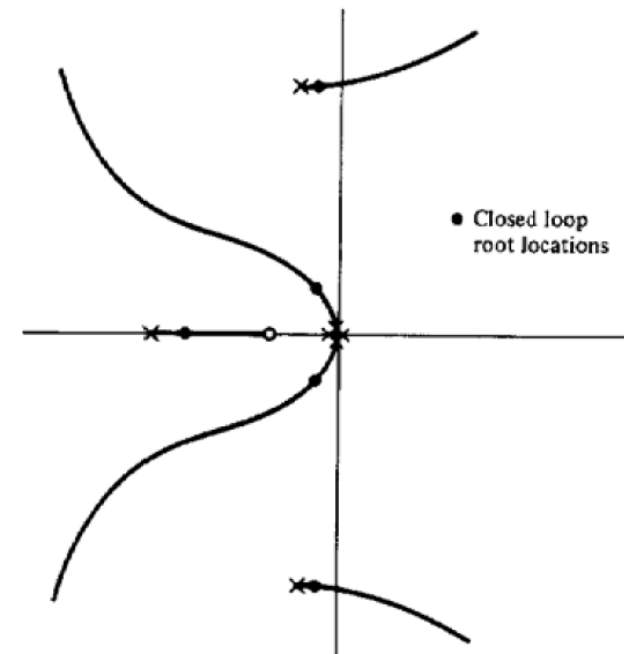
Exemplo: Processo Massa-Mola-Amortecedor (4ª ordem)



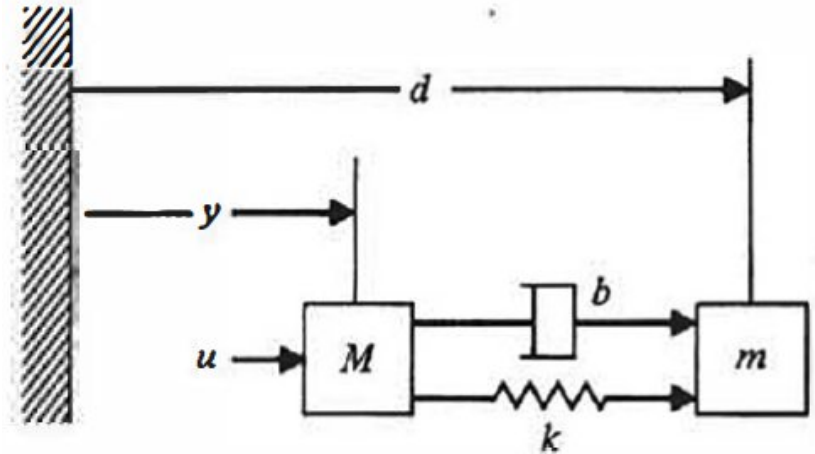
m, M = masses
 b = damping coefficient
 k = spring constant
 F = applied control force

$$\begin{aligned}
 M\ddot{y} + (\dot{y} - \dot{d})b + (y - d)k &= u \\
 m\ddot{d} + (\dot{d} - \dot{y})b + (d - y)k &= 0
 \end{aligned}$$

$$\begin{bmatrix} Ms^2 + bs + k & -(bs + k) \\ -(bs + k) & ms^2 + bs + k \end{bmatrix} \begin{bmatrix} y(s) \\ d(s) \end{bmatrix} = \begin{bmatrix} 1 \\ 0 \end{bmatrix} u(s)$$



Ex. Processo de 4ª ordem



$$\frac{d(s)}{u(s)} \approx \frac{1}{M} \frac{k/m}{s^2 \left[s^2 + \left(1 + \frac{m}{M} \right) \left(\frac{b}{m}s + \frac{k}{m} \right) \right]}$$

$$\frac{y(s)}{u(s)} = \frac{1}{M} \frac{\left(s^2 + \frac{b}{m}s + \frac{k}{m} \right)}{s^2 \left[s^2 + \left(1 + \frac{m}{M} \right) \left(\frac{b}{m}s + \frac{k}{m} \right) \right]}$$

Cont.

$\omega_n = 1$ rad/sec and damping $\zeta = 0.02$ and select a 10:1 ratio of the two masses. The parameters that provide these characteristics are: $M = 1$ kg, $m = 0.1$ kg, $b = 0.0036$ N-sec/m, and $k = 0.091$ N/m. Pick the sample rate to be 15 times faster than the resonance and show the free response to an initial condition of $d = 1$ m for two cases:

- (a) Pick all the poles at $z = 0.9$.
- (b) Pick the poles at $z = 0.9 \pm j0.05, 0.8 \pm j0.4$.

Solution.

$$x = \begin{bmatrix} d & \dot{d} & y & \dot{y} \end{bmatrix}^T \quad \mathbf{H} = \begin{bmatrix} 1 & 0 & 0 & 0 \end{bmatrix}$$
$$\mathbf{F} = \begin{bmatrix} 0 & 1 & 0 & 0 \\ -0.91 & -0.036 & 0.91 & 0.036 \\ 0 & 0 & 0 & 1 \\ 0.091 & 0.0036 & -0.091 & -0.0036 \end{bmatrix} \quad \mathbf{G} = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 1 \end{bmatrix}, \quad J = 0.$$

The sample rate should be 15 rad/sec which translates to approximately $T = 0.4$ secs.

Cont.

a)

```
sysC = ss(F,G,H,J)
sysD = c2d(sysC,T,'zoh')
p = [.9;.9;.9;.9]
[phi,gam,H,J] = ssdata(sysD)
K=acker(phi,gam,p)      K = [0.650   -0.651   -0.645   0.718]

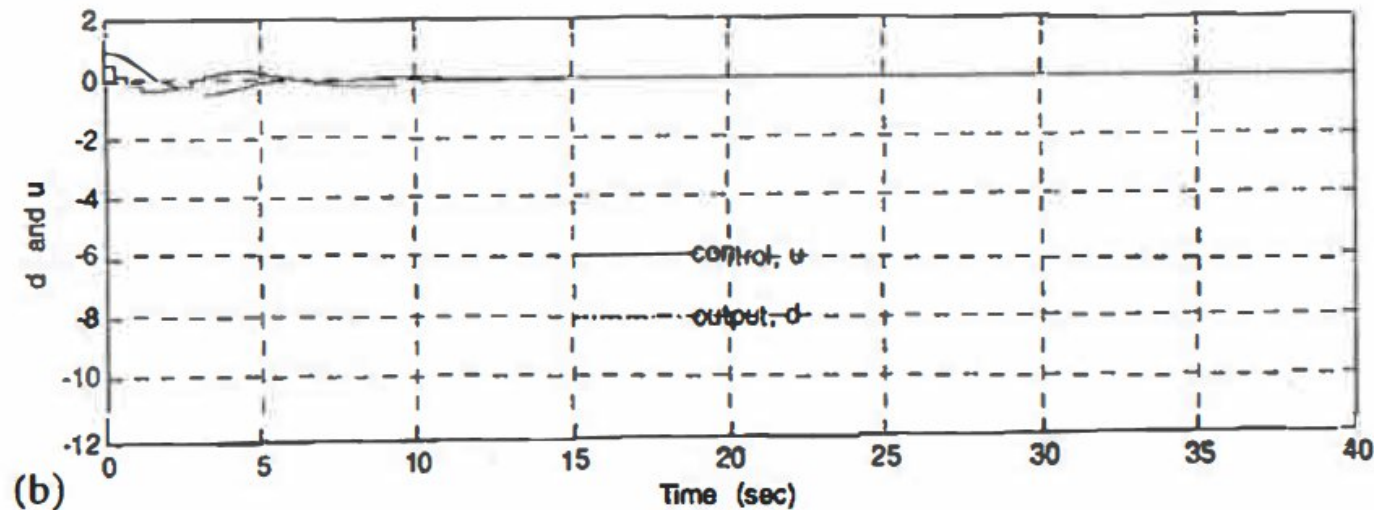
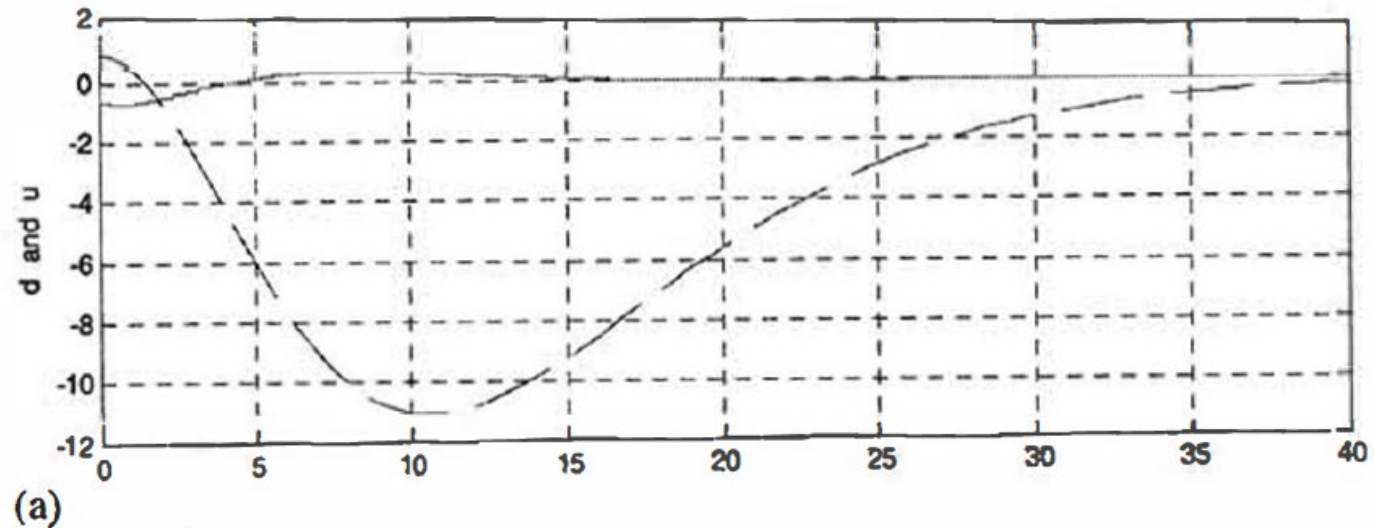
sysCL = feedback(K*sysD,1)
Xo = [1;0;0;0]
d = initial(sysCL,Xo)
```

(b) For the desired poles at $z = 0.9 \pm j0.05, 0.8 \pm j0.4$,

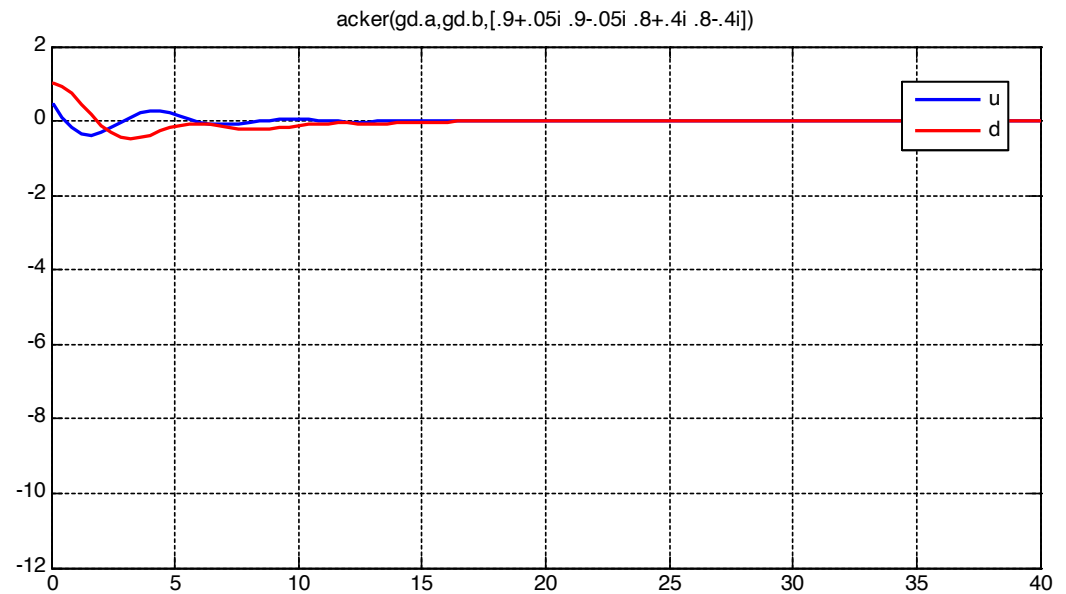
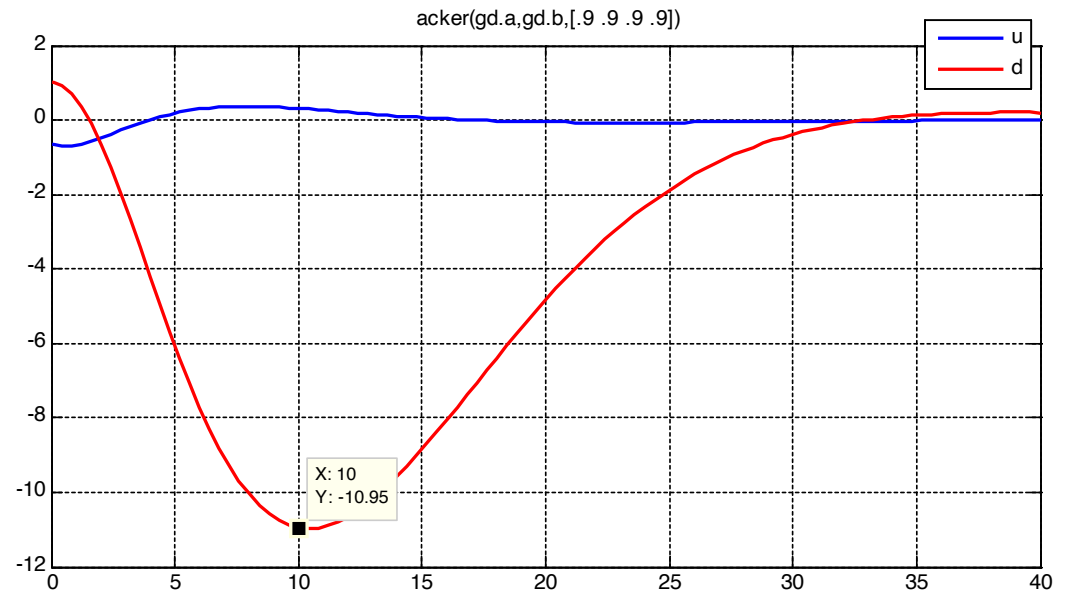
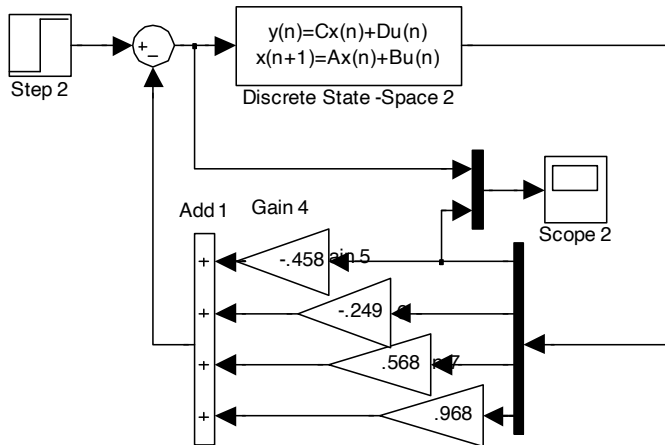
```
p = [.9+i*.05;.9-i*.05;.8+i*.4;.8-i*.4]
```

Resposta à condição inicial

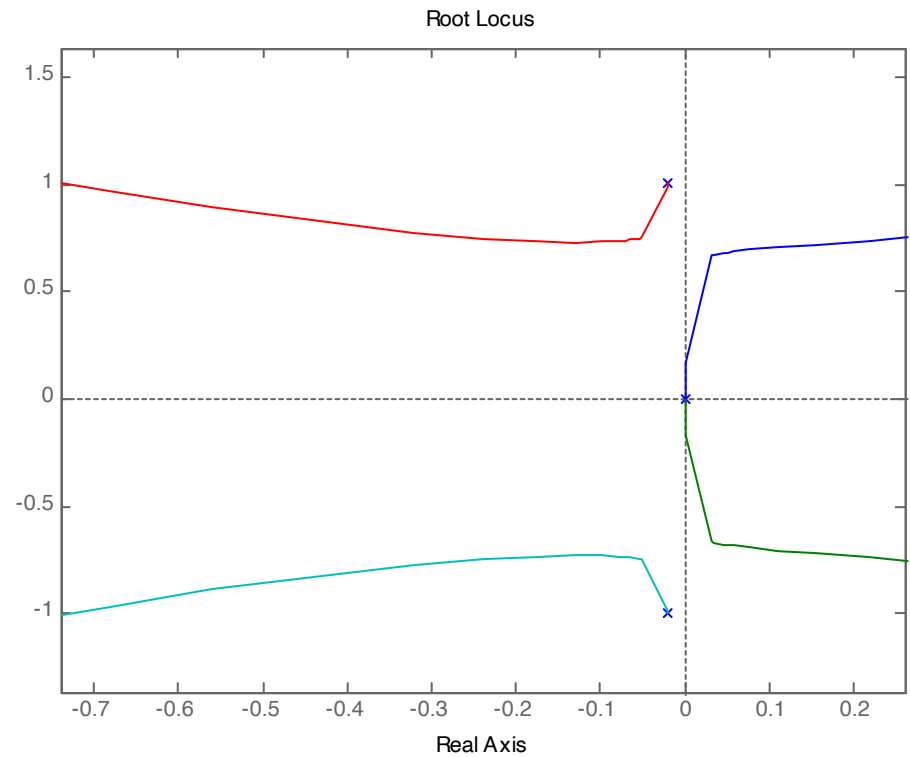
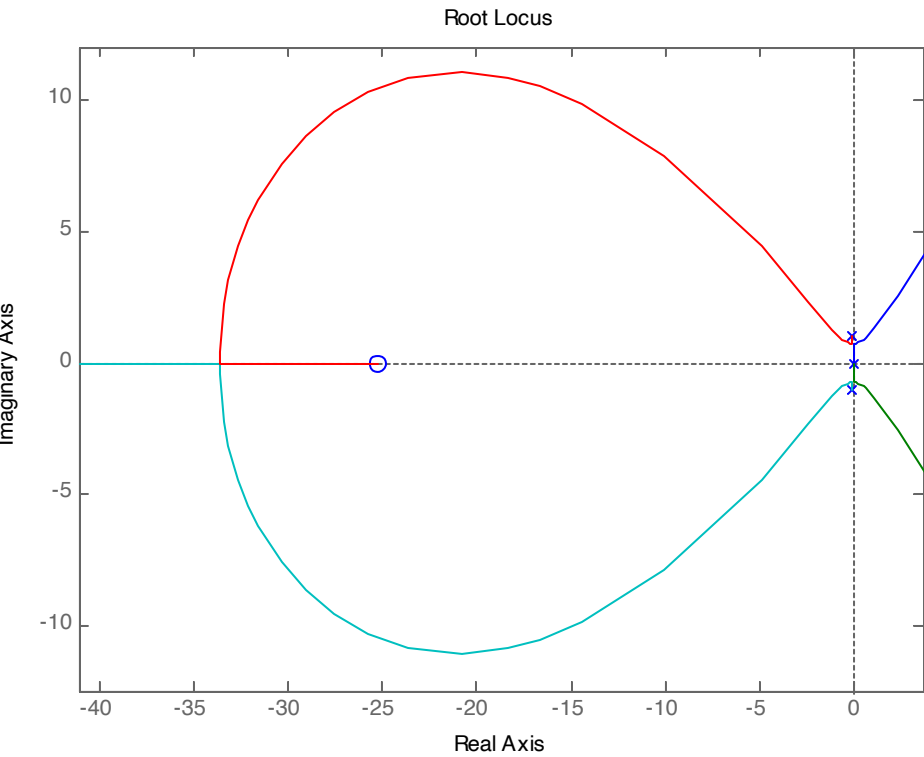
Figure 8.2
Initial condition response
for Example 8.3;
(a) desired poles all at
 $z = 0.9$, and (b) desired
poles at $z = 0.9 \pm j0.05, 0.8 \pm j0.4$



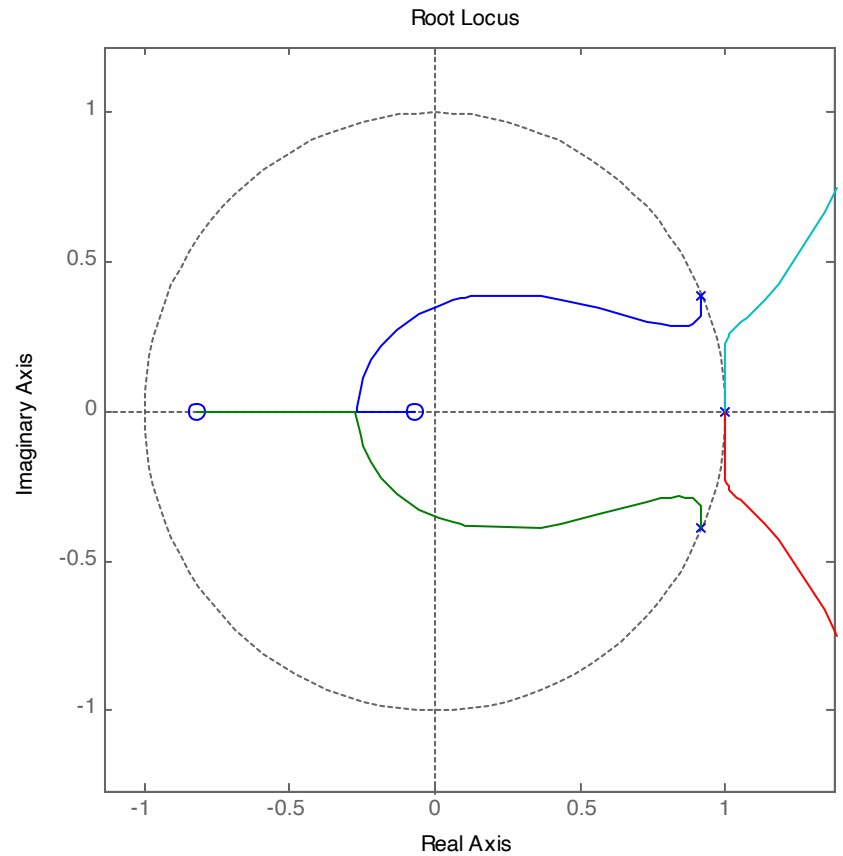
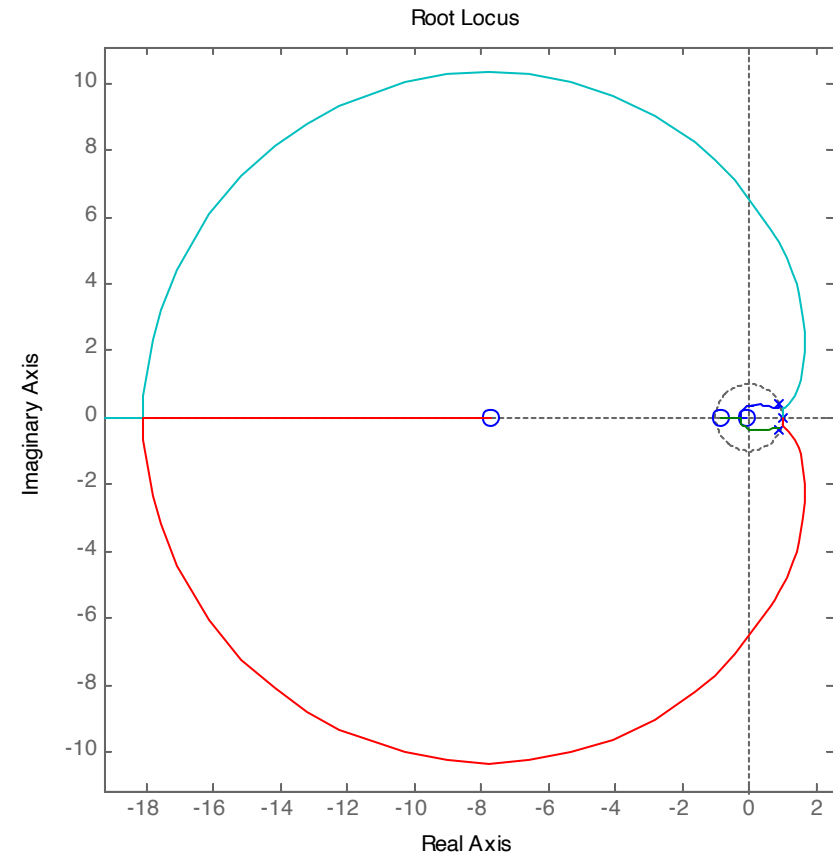
Simulação EE



LGR Contínuo



LGR Discreto



Projeto do Observador de Estados (Estimador)

$\hat{x}(k)$ – estimativa corrente (usa inclusive $y(k)$)

$\bar{x}(k)$ – estimativa predita (usa até $y(k-1)$)

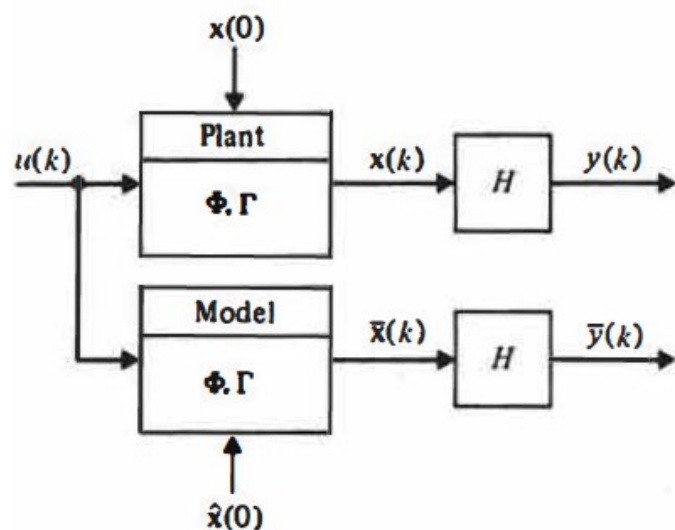
$$\text{Realimentação de Estados} \begin{cases} u(k) = -K\hat{x}(k) \\ u(k) = -K\bar{x}(k) \end{cases}$$

Controle no Espaço-de-Estados

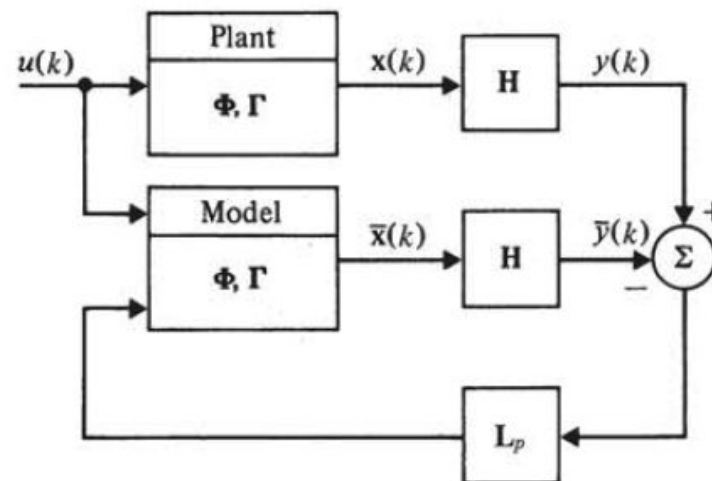
$$y(t) \rightarrow^{\text{der.}} \dot{y}(t) \rightarrow^{\text{der.}} \ddot{y}(t) \dots$$

=> amplifica o ruído

$$y(k) \rightarrow^{\text{dif.}} y(k-1) \rightarrow^{\text{dif.}} y(k-2) \dots$$



Estimador em Malha Aberta



Observador de Estados

Princípio da Separabilidade

- Projetar o controlador como se as variáveis de estados estivessem disponíveis
- Projetar o observador como se não fosse utilizar realimentação de estados

→ Realimentar as variáveis estimadas

Assintoticamente $\hat{\mathbf{x}} \cong \mathbf{x}$

“O observador não é observável”

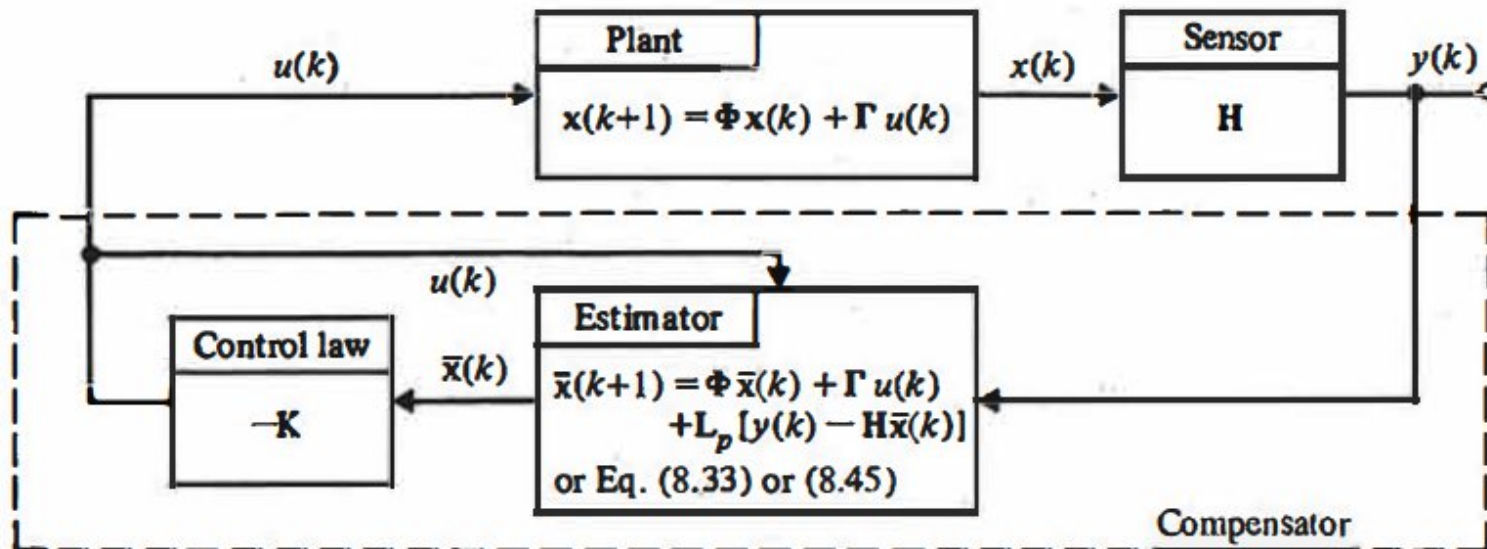
(não aparece na função de transferência)

Princípio da Separabilidade:

$$\begin{bmatrix} \bar{\mathbf{x}}(k+1) \\ \mathbf{x}(k+1) \end{bmatrix} = \begin{bmatrix} \Phi - \mathbf{L}_p \mathbf{H} & 0 \\ \Gamma \mathbf{K} & \Phi - \Gamma \mathbf{K} \end{bmatrix} \begin{bmatrix} \bar{\mathbf{x}}(k) \\ \mathbf{x}(k) \end{bmatrix}$$

$$\begin{vmatrix} z\mathbf{I} - \Phi + \mathbf{L}_p \mathbf{H} & 0 \\ \Gamma \mathbf{K} & z\mathbf{I} - \Phi + \Gamma \mathbf{K} \end{vmatrix} = 0$$

$$\left| z\mathbf{I} - \Phi + \mathbf{L}_p \mathbf{H} \right| \left| z\mathbf{I} - \Phi + \Gamma \mathbf{K} \right| = \alpha_c(z) \alpha_e(z) = 0$$

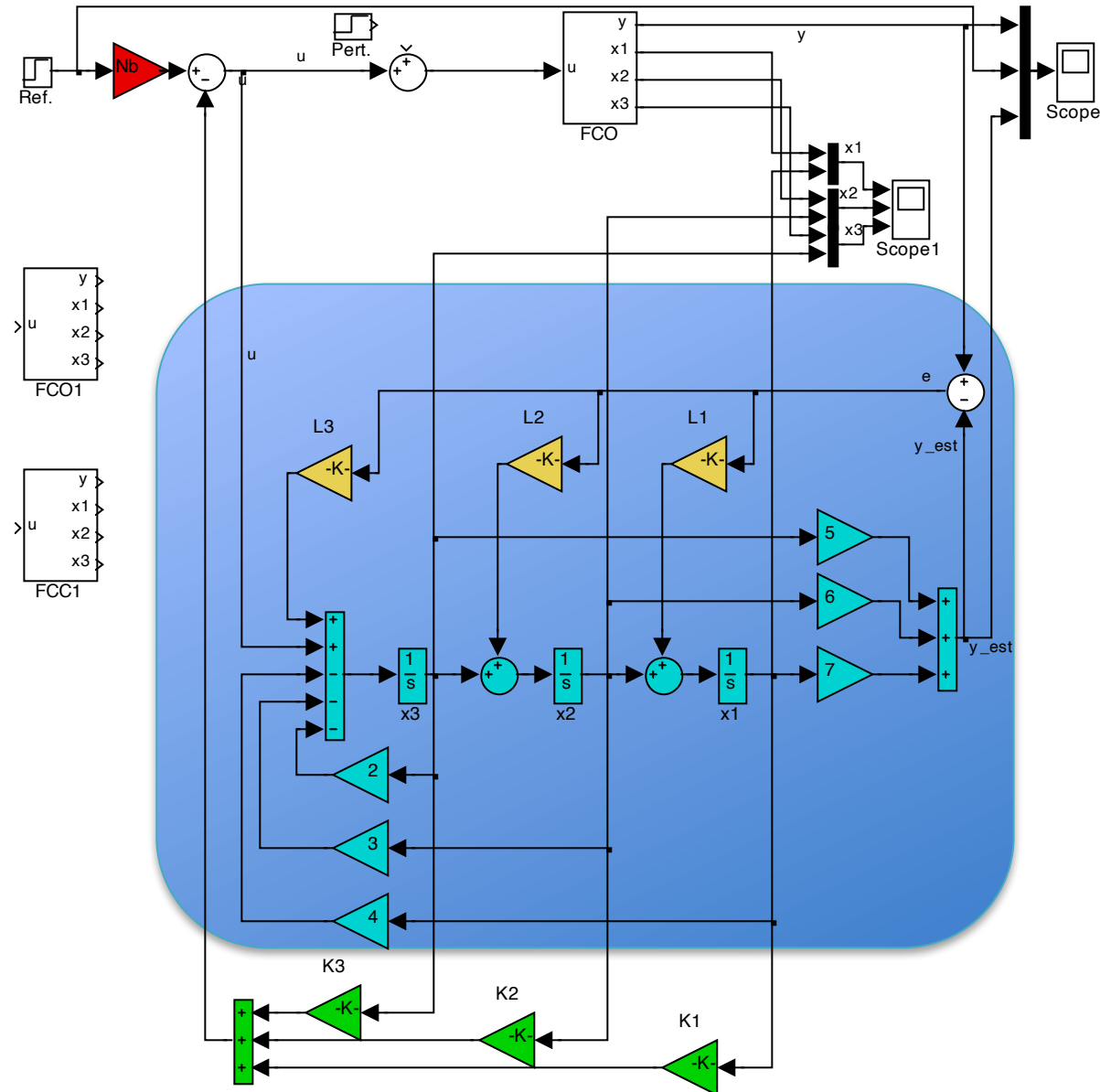


Observador de Estados

Processo

$$G(s) = \frac{5s^2 + 6s + 7}{s^3 + 2s^2 + 3s + 4}$$

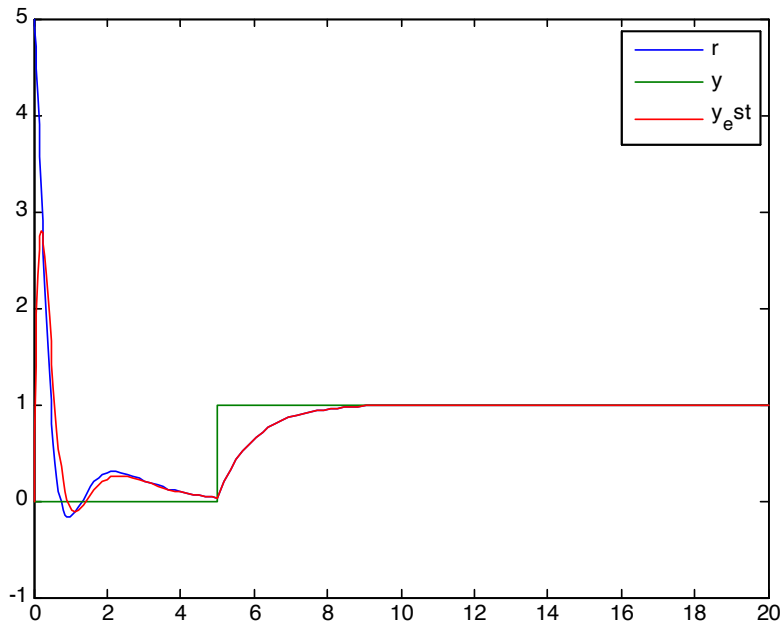
O observador de estados pode utilizar diferentes representações do processo!!



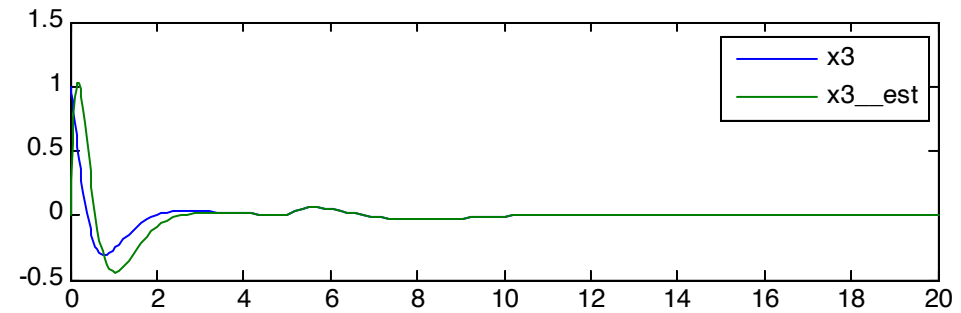
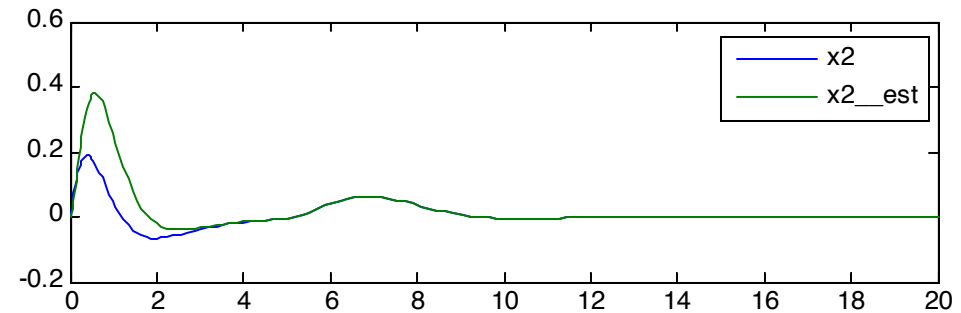
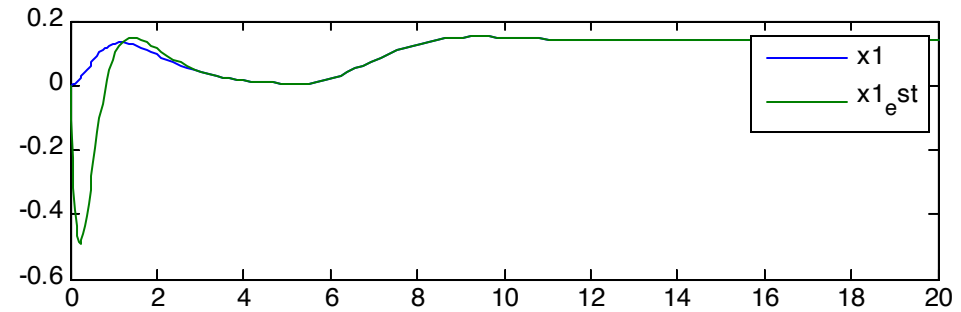
Observador de Estados

Processo

$$G(s) = \frac{5s^2 + 6s + 7}{s^3 + 2s^2 + 3s + 4}$$



A saída estimada (y_{est}) segue a saída real (y)

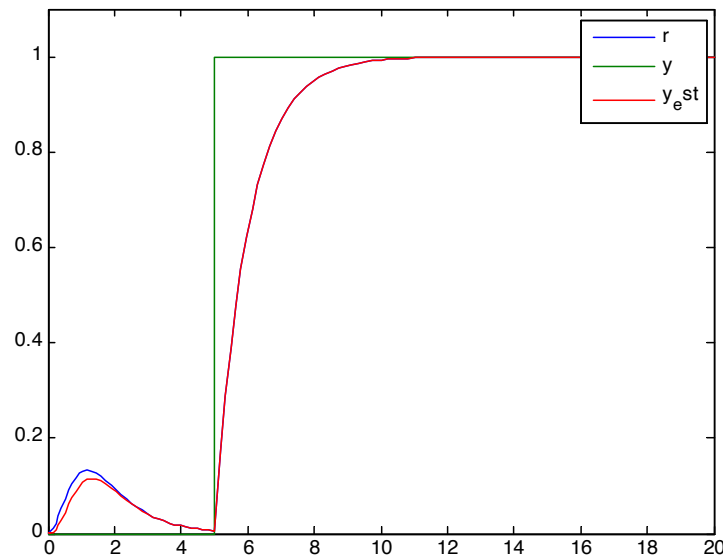


... bem como os estados.
apesar de **Condições Iniciais** distintas!!

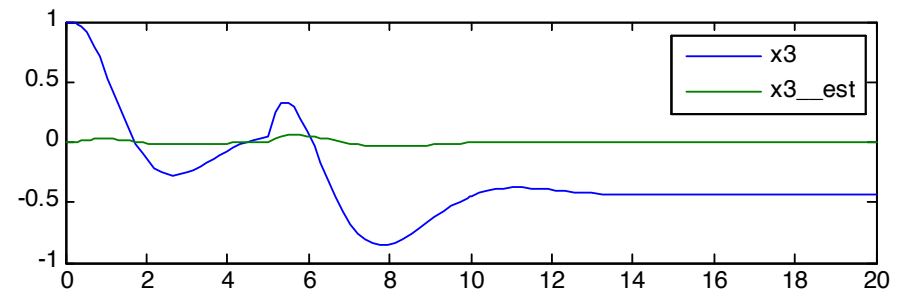
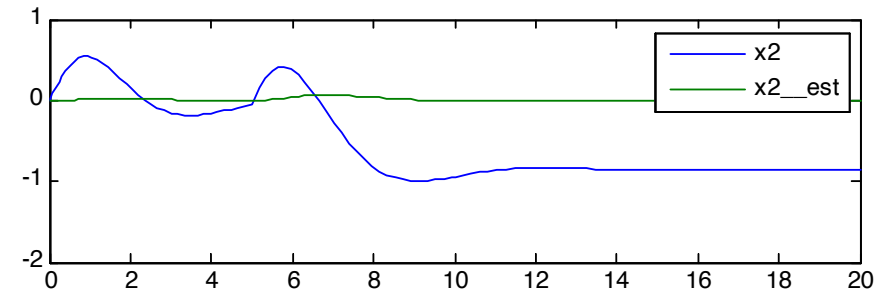
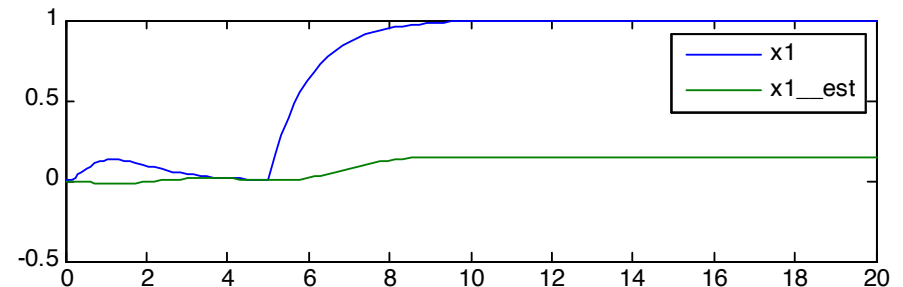
Observador de Estados

Processo

$$G(s) = \frac{5s^2 + 6s + 7}{s^3 + 2s^2 + 3s + 4}$$



A saída estimada (y_{est}) segue a saída real (y)



... apesar de as variáveis de estado estimadas serem distintas das reais!!
(representações distintas)

Fator de Ajuste de Ganho, Nb

$$\frac{Y(z)}{U(z)} = \frac{b_2 z^2 + b_1 z + b_0}{z^3 + a_2 z^2 + a_1 z + a_0}$$

posicionamento de polos $\Rightarrow$

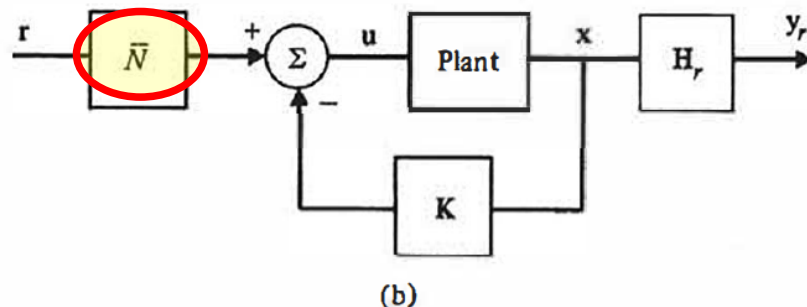
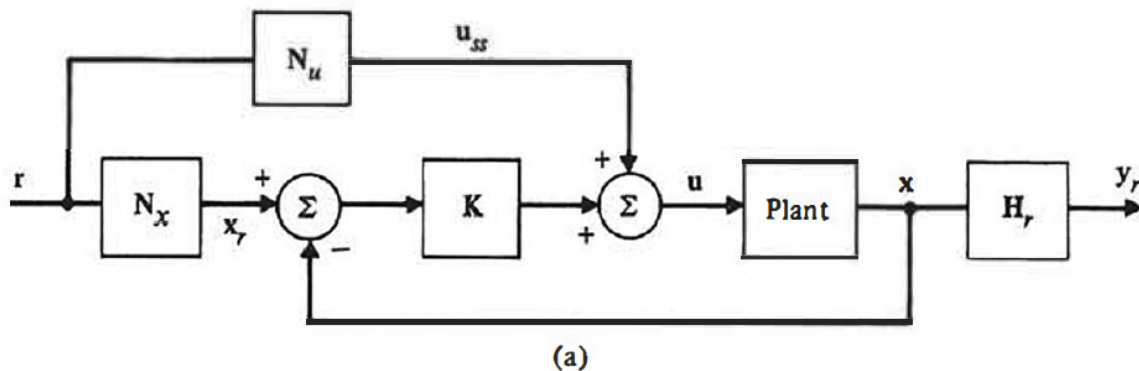
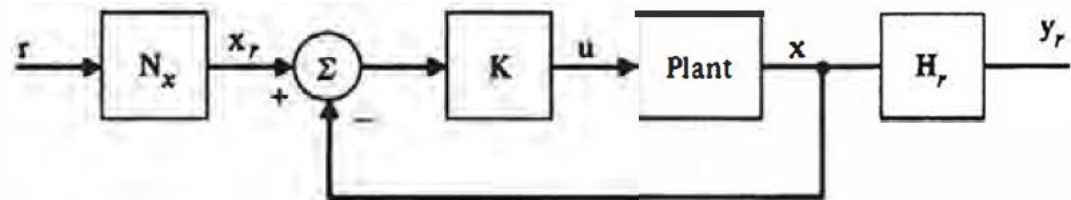
$$\frac{Y(z)}{U(z)} = \frac{b_2 z^2 + b_1 z + b_0}{z^3 + \tilde{a}_2 z^2 + \tilde{a}_1 z + \tilde{a}_0}$$

$$k \rightarrow \infty$$

$$\frac{Y(z)}{U(z)}_{z \rightarrow 1} = \frac{b_2 + b_1 + b_0}{1 + \tilde{a}_2 + \tilde{a}_1 + \tilde{a}_0} = \frac{1}{Nb}$$

$$\frac{Y(z)}{R(z)}_{z \rightarrow 1} = 1$$

ssMF=ss(A-B*K,B,C,D,T);gf=tf(ssMF);
Nb=sum(gf.den{1}(1,:))/sum(gf.num{1}(1,:))



Posicionamento dos Pólos no EE

- Teoricamente seria possível posicionar os pólos no EE de forma arbitrária
- Prática – alterações substanciais da dinâmica do processo levam à saturação do atuador
- Considerar os zeros!! (cancelar polos...)

Projeto do Observador

>> $L_p = \text{acker}(\Phi', H', [\text{polos}_e])'$

Exemplo

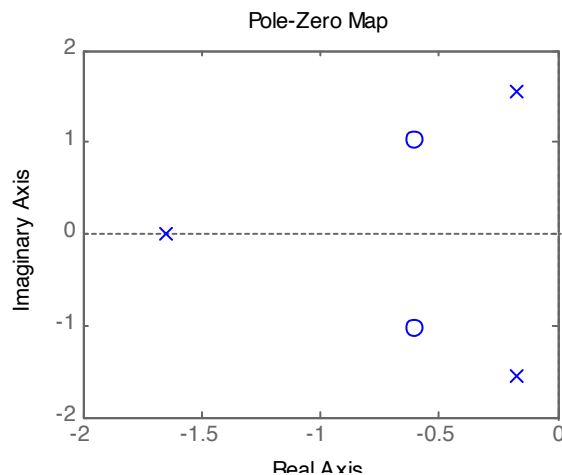
% Forma Canônica Controlável

$A=[0 \ 1 \ 0; 0 \ 0 \ 1; -4 \ -3 \ -2];$

$B=[0; 0; 1]; C=[7 \ 6 \ 5]; D=0;$

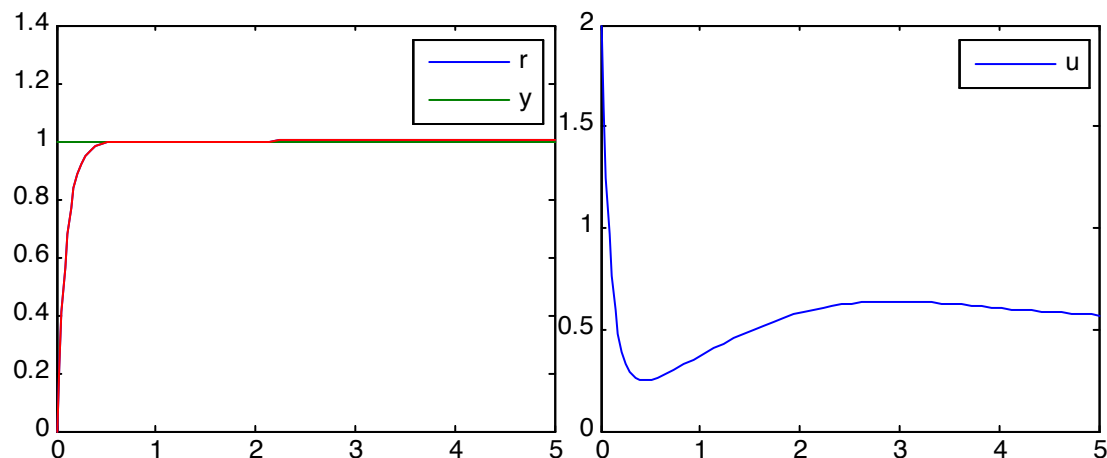
Em malha aberta:

$$G(s) = \frac{5s^2 + 6s + 7}{s^3 + 2s^2 + 3s + 4}$$



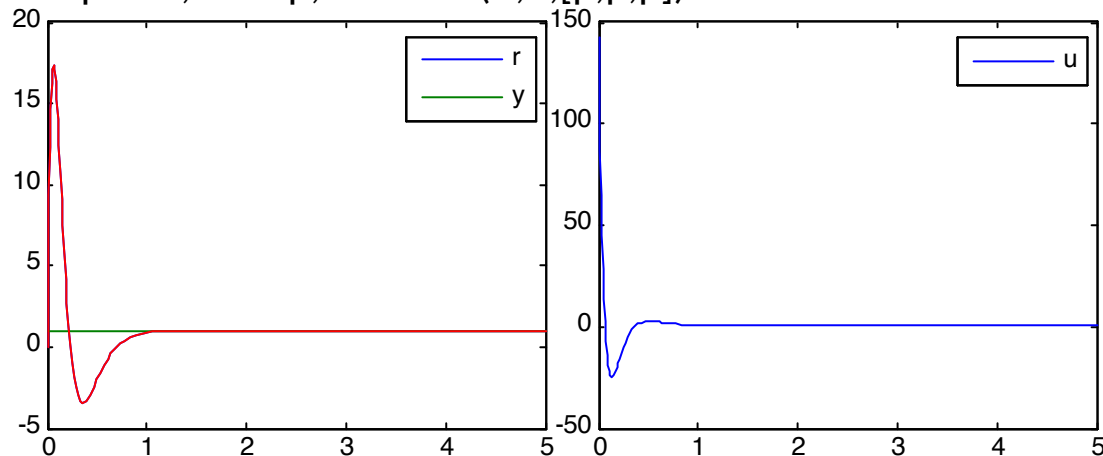
%Projeto 1

$p=-10; o=3*p; K=acker(A,B,[p,-.6-1.02i,-.6+1.02i])$



%Projeto 2

$p=-10; o=3*p; K=acker(A,B,[p,p,p])$



Eqs. do Sistema Completo

$$\text{Processo} \quad \begin{cases} \dot{x} = Ax + Bu + Fw \\ y = Cx \end{cases}$$

$$\text{Observador} \quad \begin{cases} \dot{\hat{x}} = A\hat{x} + Bu + L(y - \hat{y}) \\ \hat{y} = C\hat{x} \end{cases}$$

$$\text{Lei de Controle} : u = -K\hat{x} + N_b r$$

$$\dot{x} = Ax - BK\hat{x} + BN_b r + Fw$$

$$e_x = x - \hat{x}$$

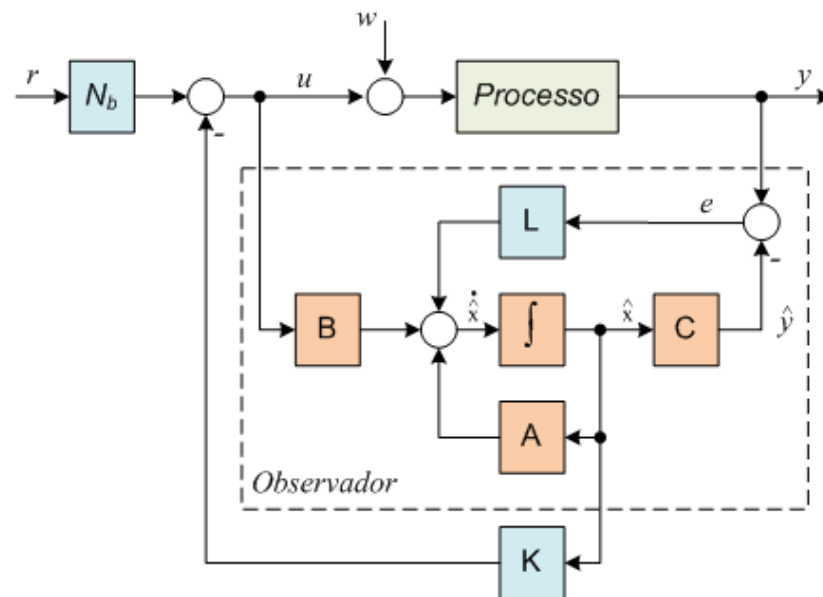
$$\dot{x} - \dot{\hat{x}} = A(x - \hat{x}) + Fw - L(y - \hat{y})$$

$$\dot{x} = (A - BK)x + BKe_x + BN_b r + Fw$$

$$\dot{e}_x = (A - LC)e_x + Fw$$

$$\begin{bmatrix} \dot{x} \\ \dot{e}_x \end{bmatrix} = \begin{bmatrix} (A - BK) & BK \\ 0 & (A - LC) \end{bmatrix} \begin{bmatrix} x \\ e_x \end{bmatrix} + \begin{bmatrix} BN_b \\ 0 \end{bmatrix} r + \begin{bmatrix} F \\ F \end{bmatrix} w$$

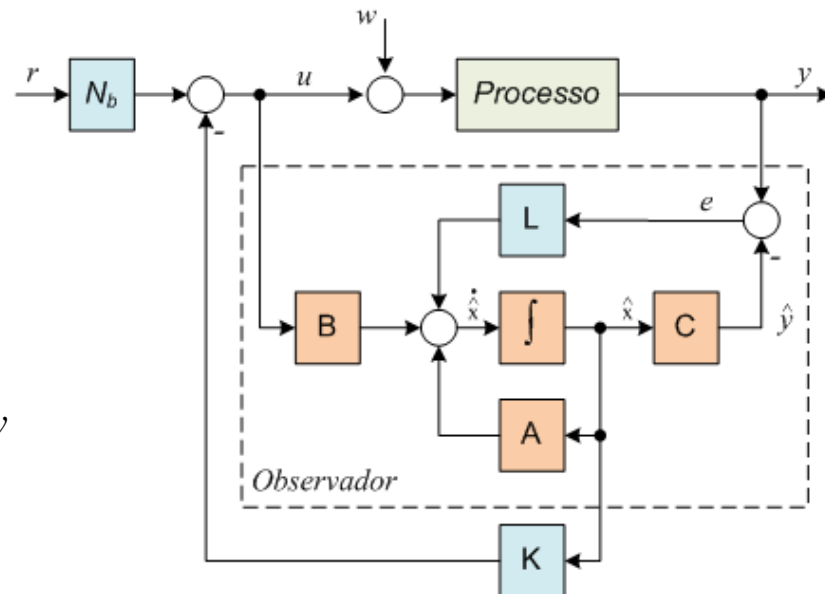
$$y = \begin{bmatrix} C & 0 \end{bmatrix} \begin{bmatrix} x \\ e_x \end{bmatrix}$$



Função de Transferência em MF

$$\begin{bmatrix} \dot{x} \\ \dot{e}_x \end{bmatrix} = \begin{bmatrix} (A - BK) & BK \\ 0 & (A - LC) \end{bmatrix} \begin{bmatrix} x \\ e_x \end{bmatrix} + \begin{bmatrix} BN_b \\ 0 \end{bmatrix} r + \begin{bmatrix} F \\ F \end{bmatrix} w$$

$$y = \begin{bmatrix} C & 0 \end{bmatrix} \begin{bmatrix} x \\ e_x \end{bmatrix}$$



$$\frac{Y(s)}{R(s)} = \begin{bmatrix} C & 0 \end{bmatrix} \begin{bmatrix} (sI - A + BK)^{-1} & -(sI - A + BK)^{-1} BK (sI - A + LC)^{-1} \\ 0 & (sI - A + LC)^{-1} \end{bmatrix} \begin{bmatrix} BN_b \\ 0 \end{bmatrix}$$

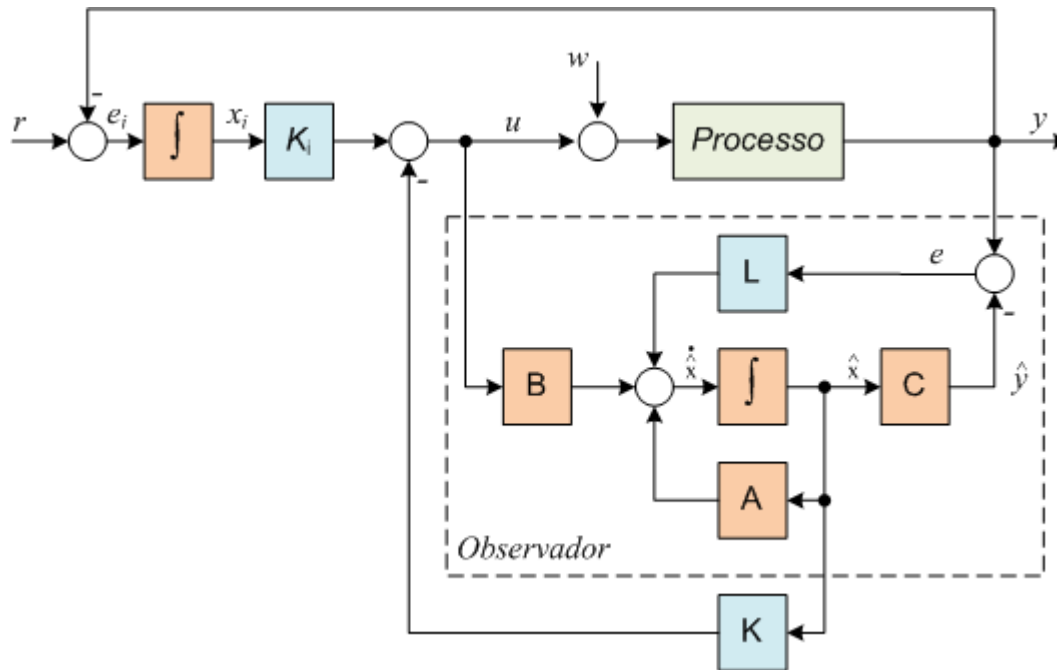
$$\frac{Y(s)}{R(s)} = C(sI - A + BK)^{-1} BN_b$$

A dinâmica do observador não aparece na F.T. !!

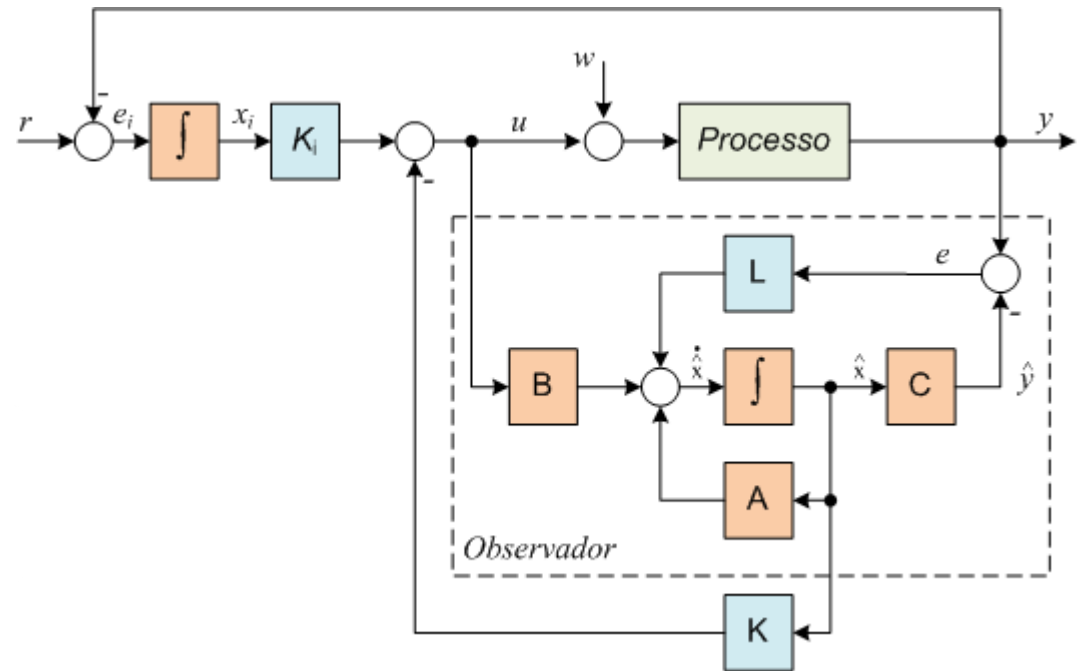
Canal Integral

Para compensar

- Perturbações
- Modelamento inexato (inclusive p/ sistemas não-lineares)



Sistema Aumentado



$$e_i = r - y = r - Cx$$

$$\begin{bmatrix} \dot{x} \\ \dot{x}_i \end{bmatrix} = \begin{bmatrix} (A - BK) & BK_i \\ -C & 0 \end{bmatrix} \begin{bmatrix} x \\ x_i \end{bmatrix} + \begin{bmatrix} 0 \\ 1 \end{bmatrix} r$$

$$y = \begin{bmatrix} C & 0 \end{bmatrix} \begin{bmatrix} x \\ x_i \end{bmatrix}$$

Sistema Aumentado – mais uma variável de estados!
Alocação de $n + 1$ pólos da Equação Característica.

Código:

```
% Controle EE com observador  
g=tf([5 6 7],[1 2 3 4])
```

```
% Forma Canonica Controlavel  
A=[0 1 0;0 0 1;-4 -3 -2];  
B=[0;0;1];C=[7 6 5];D=0;
```

```
%Polos MF - controlador e observador  
p=-1; o=10*p;
```

```
K=acker(A,B,[p,-.6-1.02i,-.6+1.02i])  
L=acker(A',C',[o,o,o]);L=L'
```

```
ssMF=ss(A-B*K,B,C,D);gf=tf(ssMF);  
Nb=gf.den{1}(1,4)/gf.num{1}(1,4)
```

xi:
O que importa é a topologia!
Tanto faz se xi=x1 ou xi=x4.

%Projeto 2 - Canal Integral - Sistema Aumentado

```
Aa=[1 -C;           % xi=x1  
    0 A(1,:);  
    0 A(2,:);  
    0 A(3,:)]  
Ba=[0;B];  
Ca=[0 C];
```

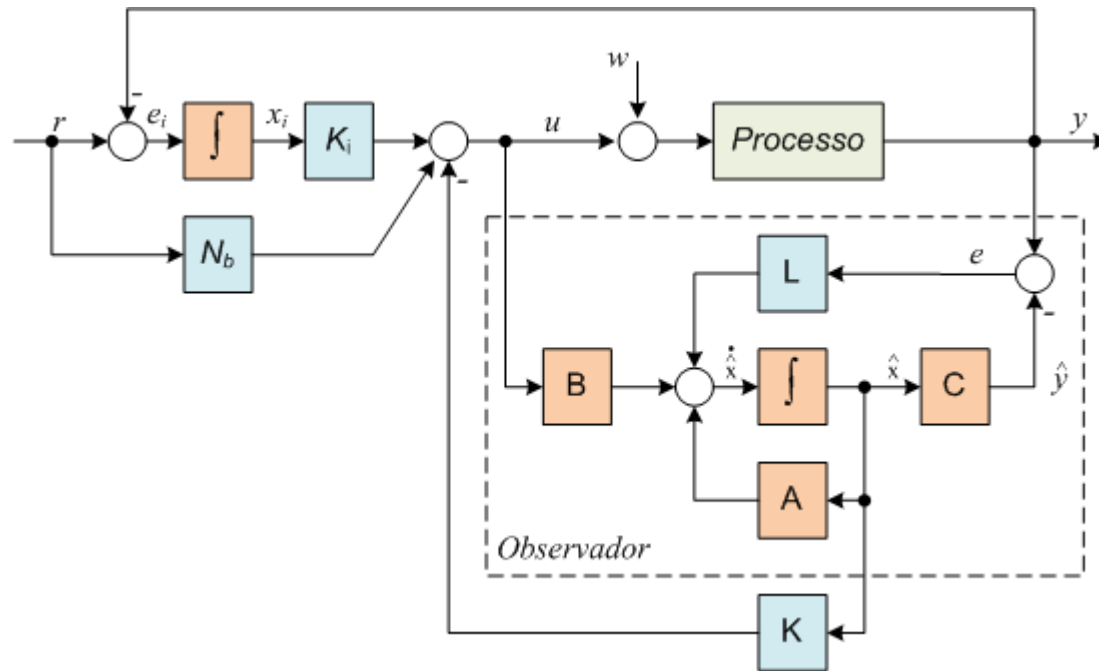
```
Ka=acker(Aa,Ba,[p,p,p,10*p])  
Ki=-Ka(1),K=Ka(2:4),  
L=acker(A',C',[o,o,o])';  
Nb=0;
```

%Projeto 3 - Canal PI - Sistema Aumentado

```
Aa=[A(1,:) 0  
    A(2,:) 0  
    A(3,:) 0  
    -C      0]           %xi= x4  
Ba=[B; 0];  
Ca=[C 0];
```

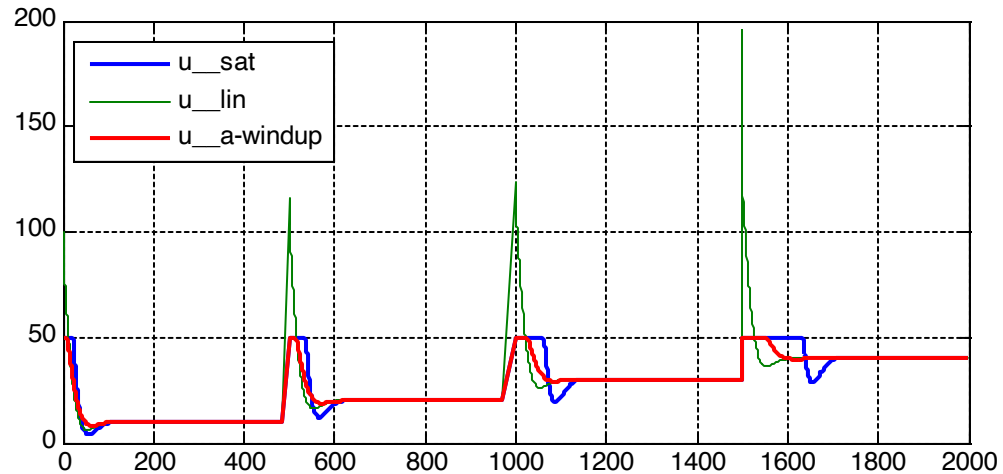
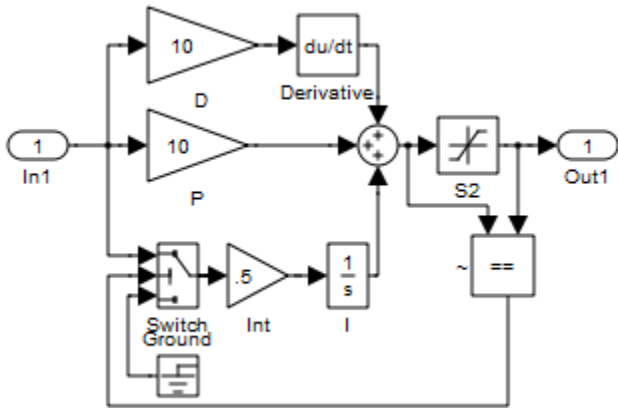
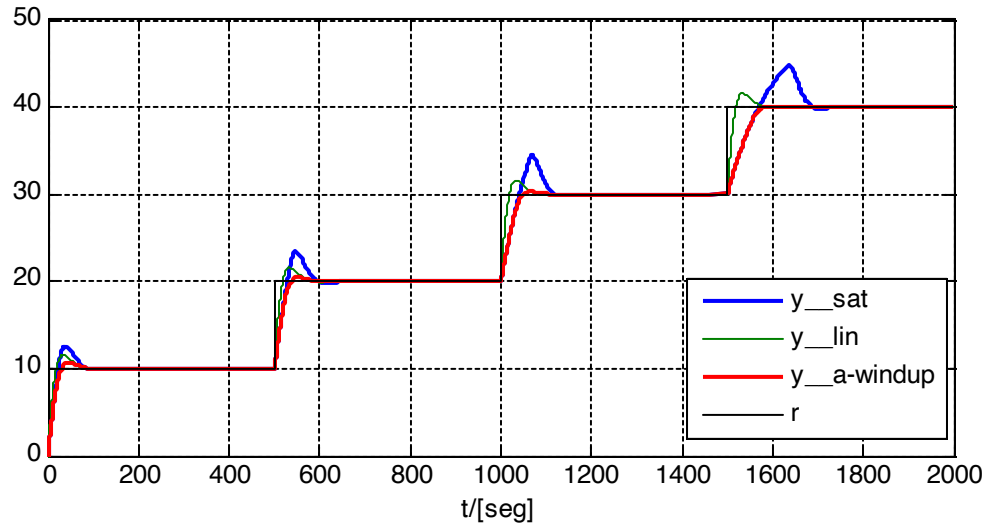
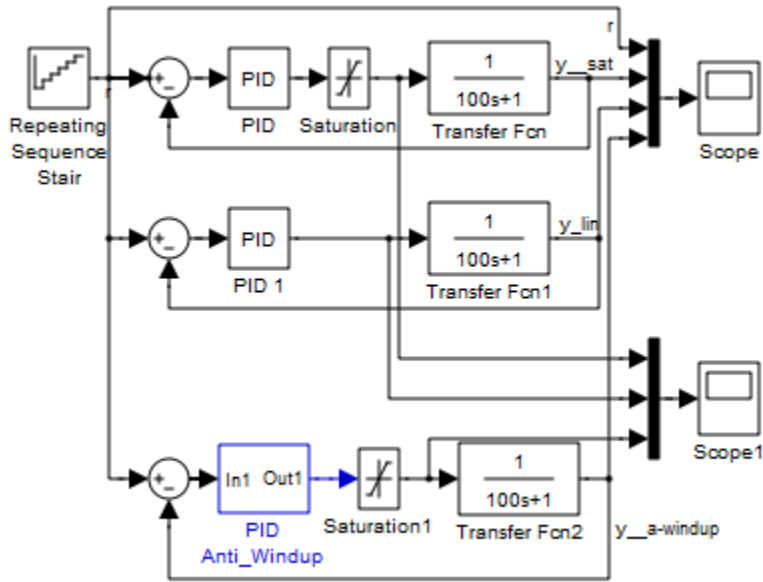
```
Ka=acker(Aa,Ba,[p,-.6-1.0198i,-.6+1.0198i,p])  
Ki=-Ka(4),K=Ka(1:3),  
L=acker(A',C',[o,o,o])';  
Nb=-Ki/p;  % Zero do canal PI cancela um polo
```


Canal Proporcional-Integral do erro

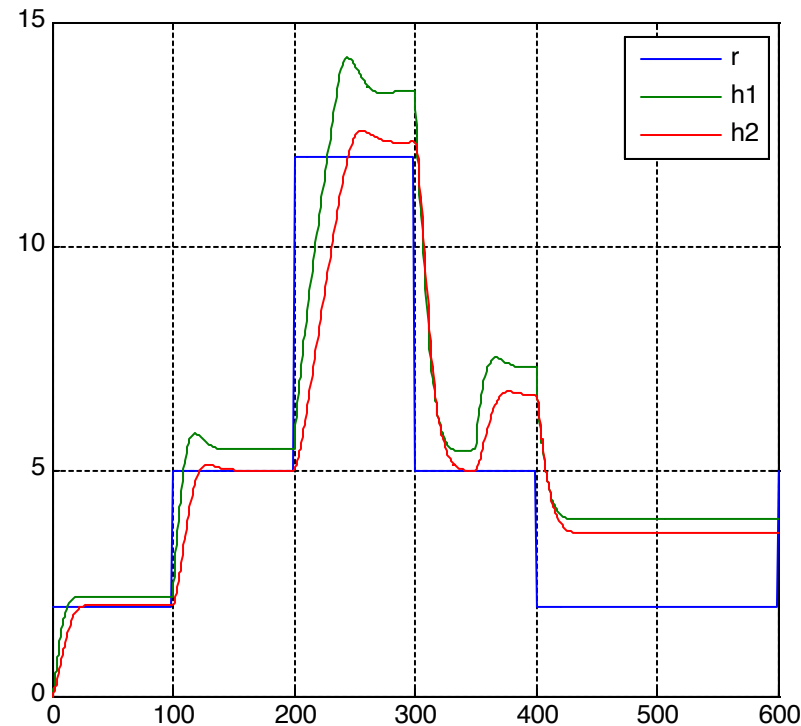
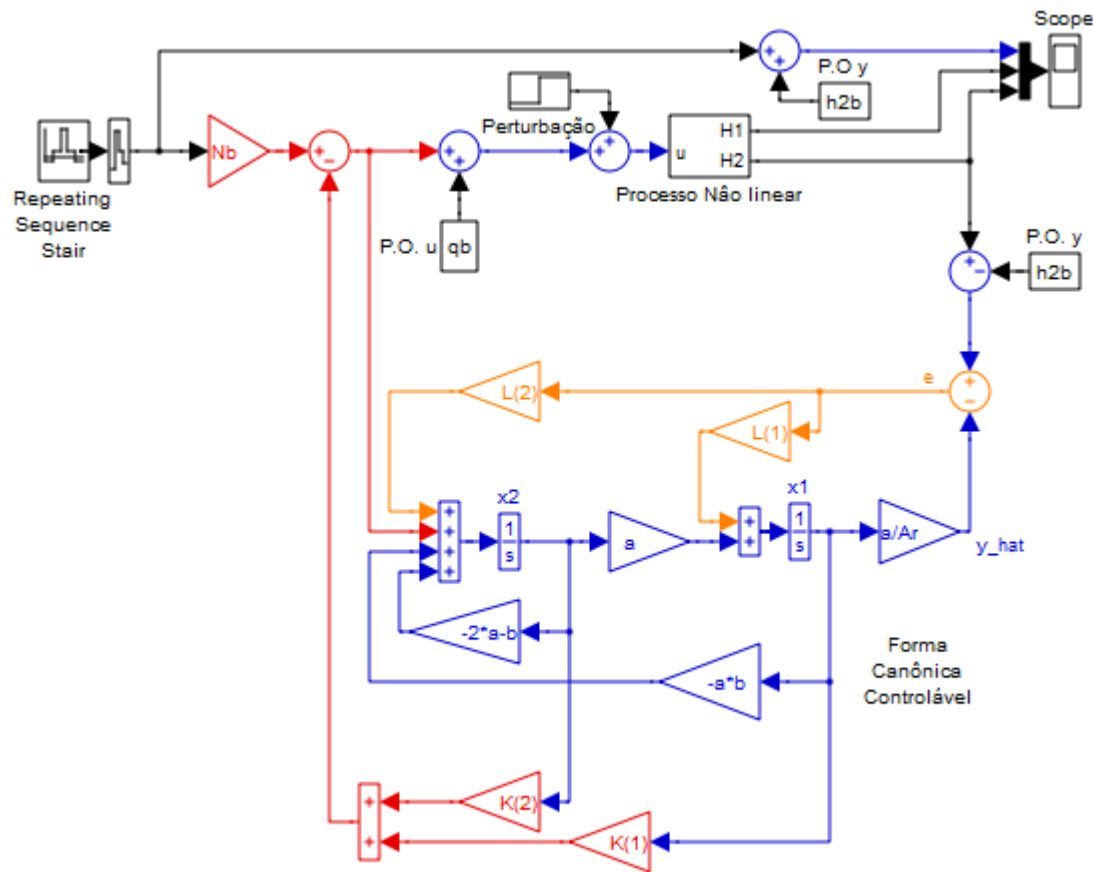


- Canal Proporcional torna o sistema mais rápido
- Cancelamento de um pólo (via zero de $(sN_b + K_i)/s$) permite não aumentar a ordem em M.F.

Anti-Windup do Canal Integral



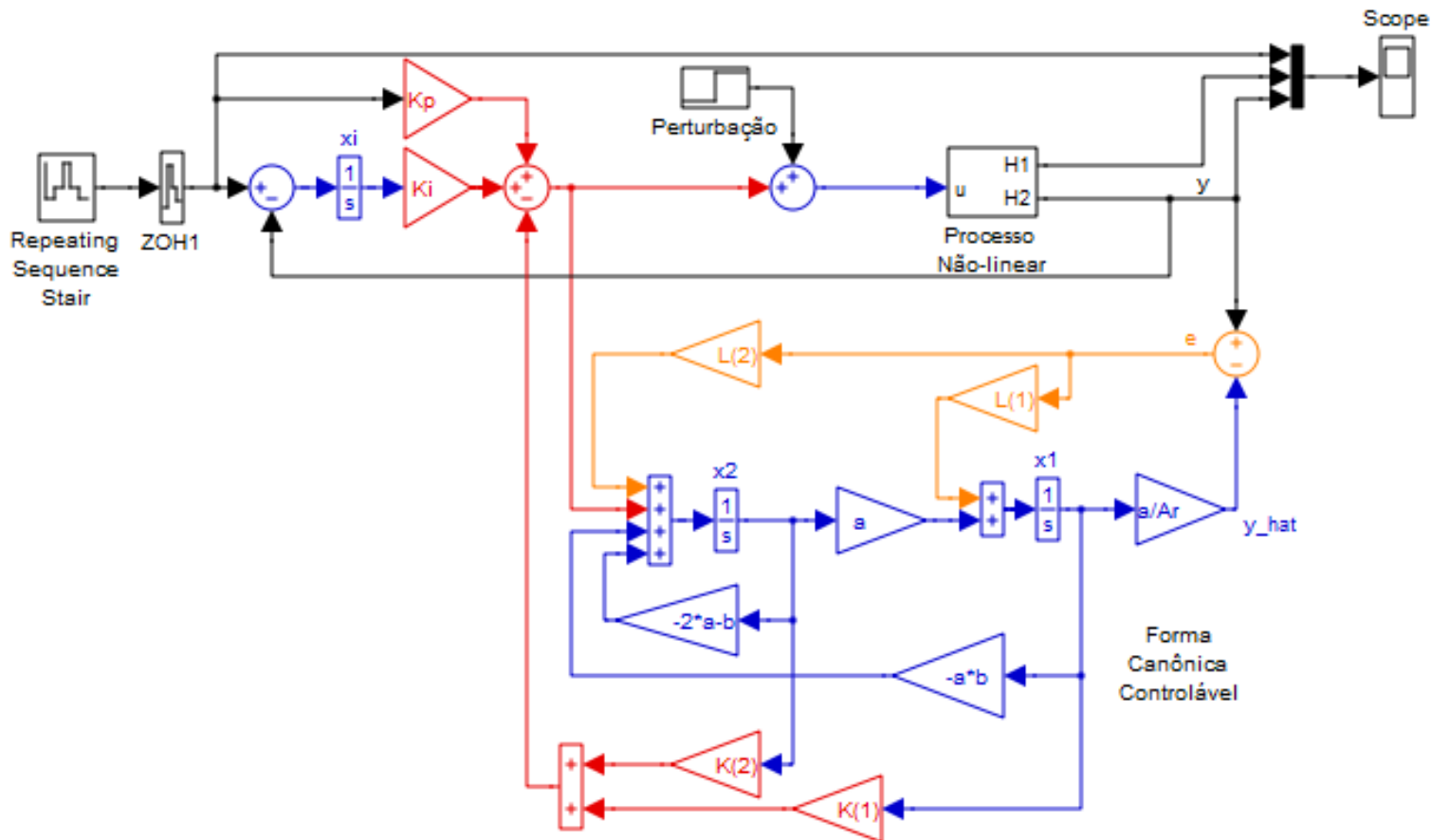
Controle EE – Processo de Nivel 2ª ordem



Ponto de Operação !

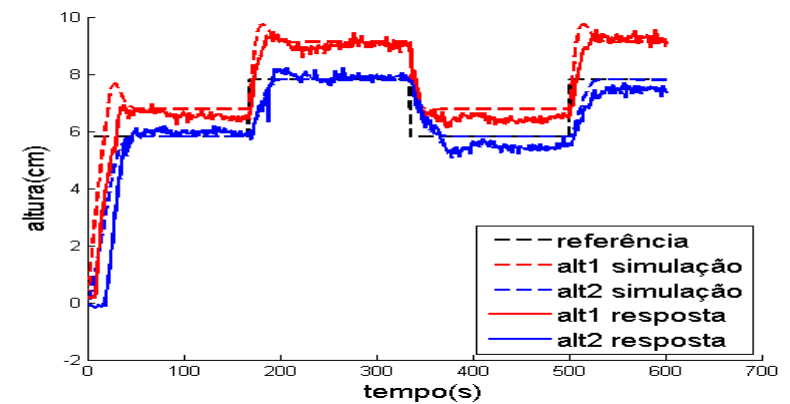
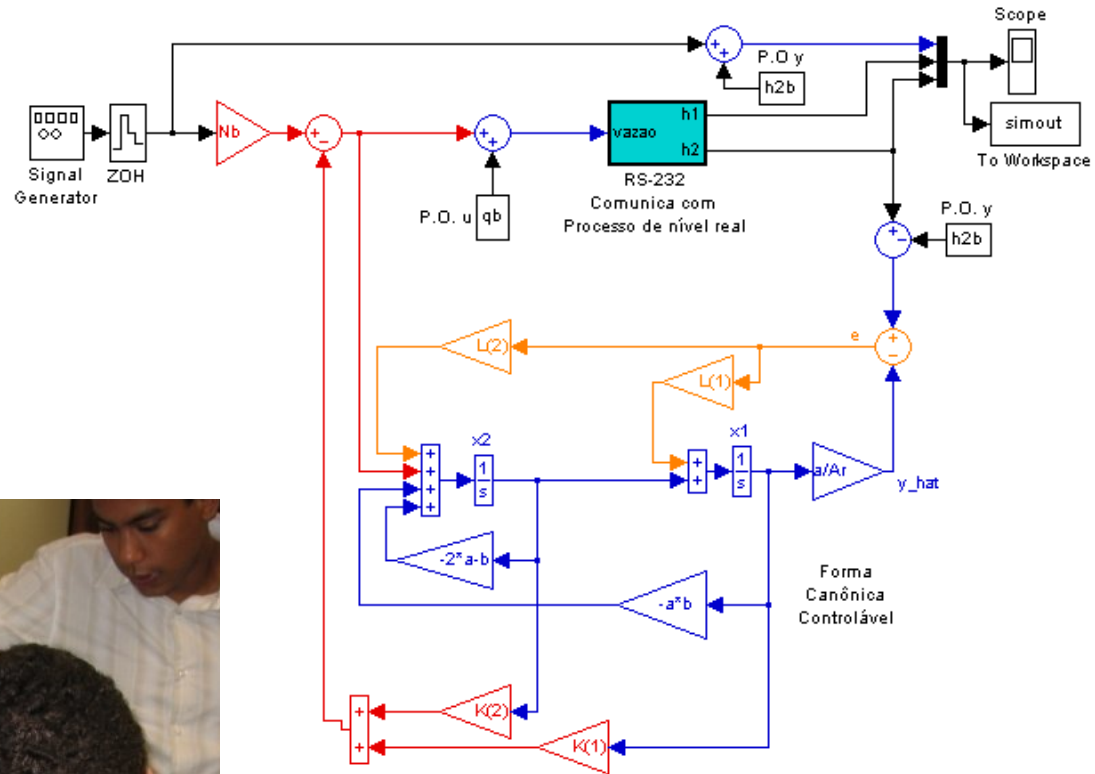
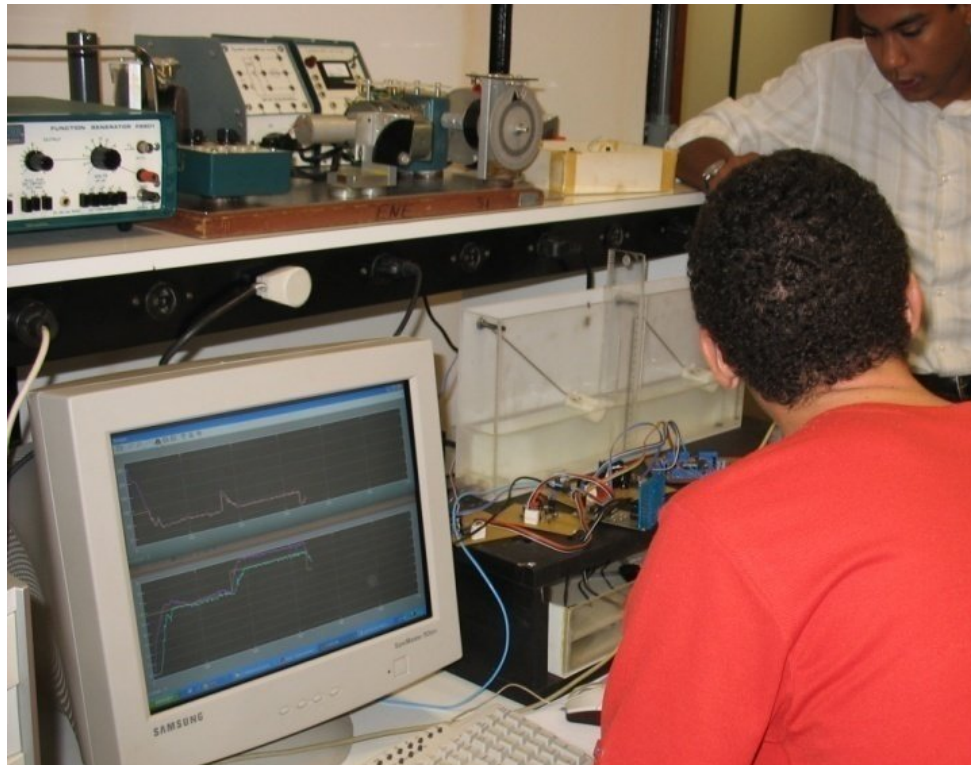
Controle EE – PI

Processo de Nivel 2ª ordem



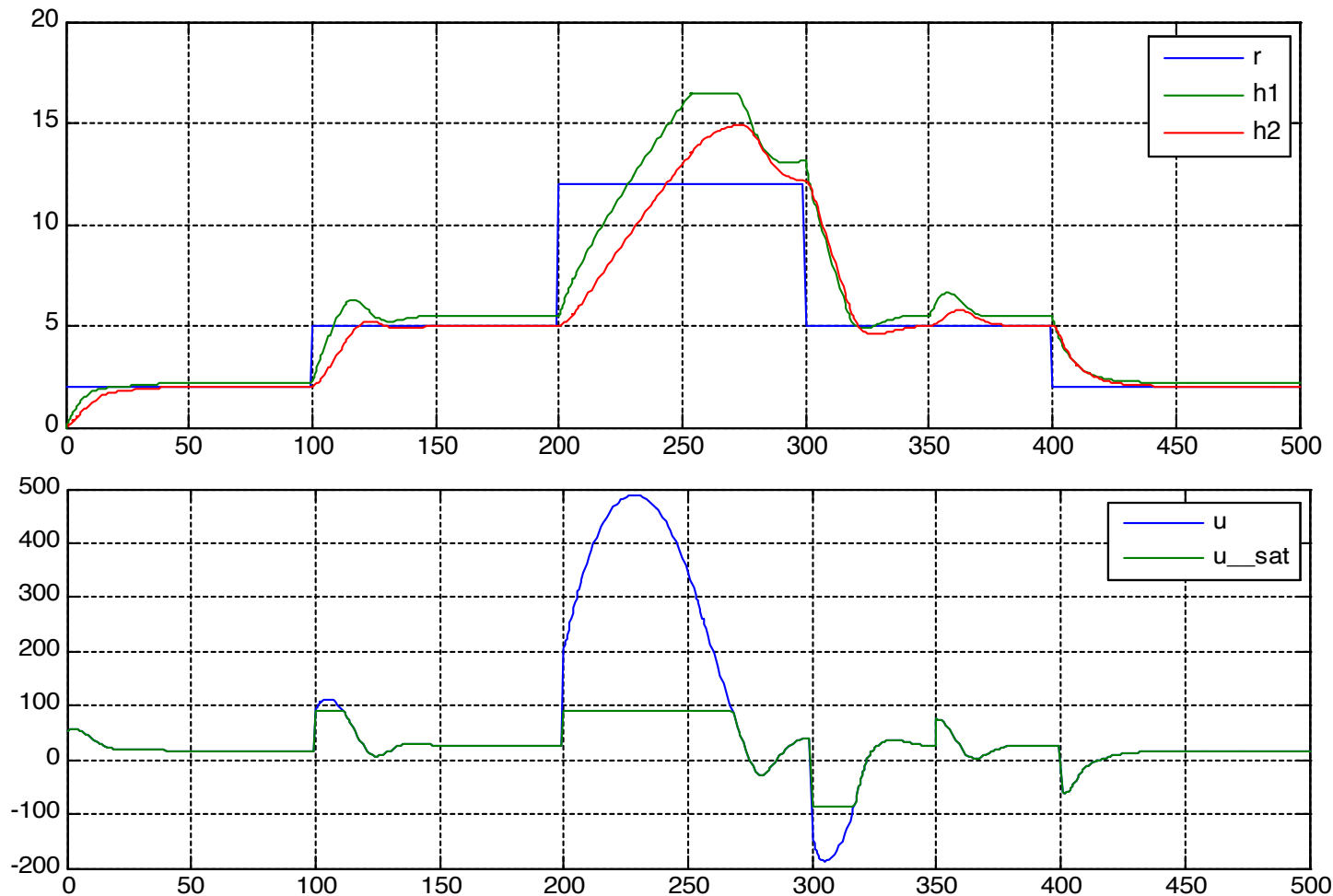
Sistema de nível de líquidos

- LARA/UnB



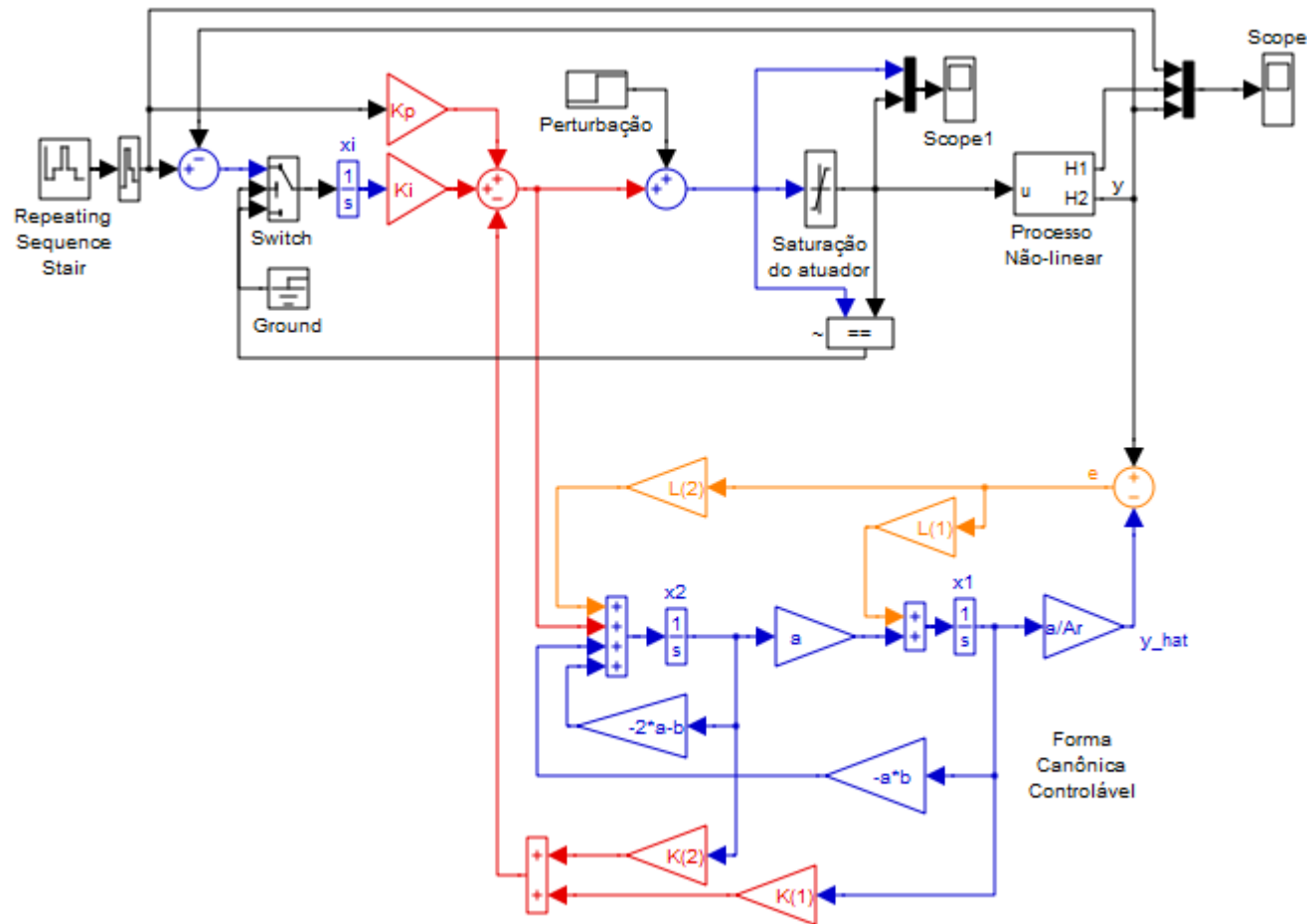
Controle EE – PI

Processo de Nivel 2ª ordem



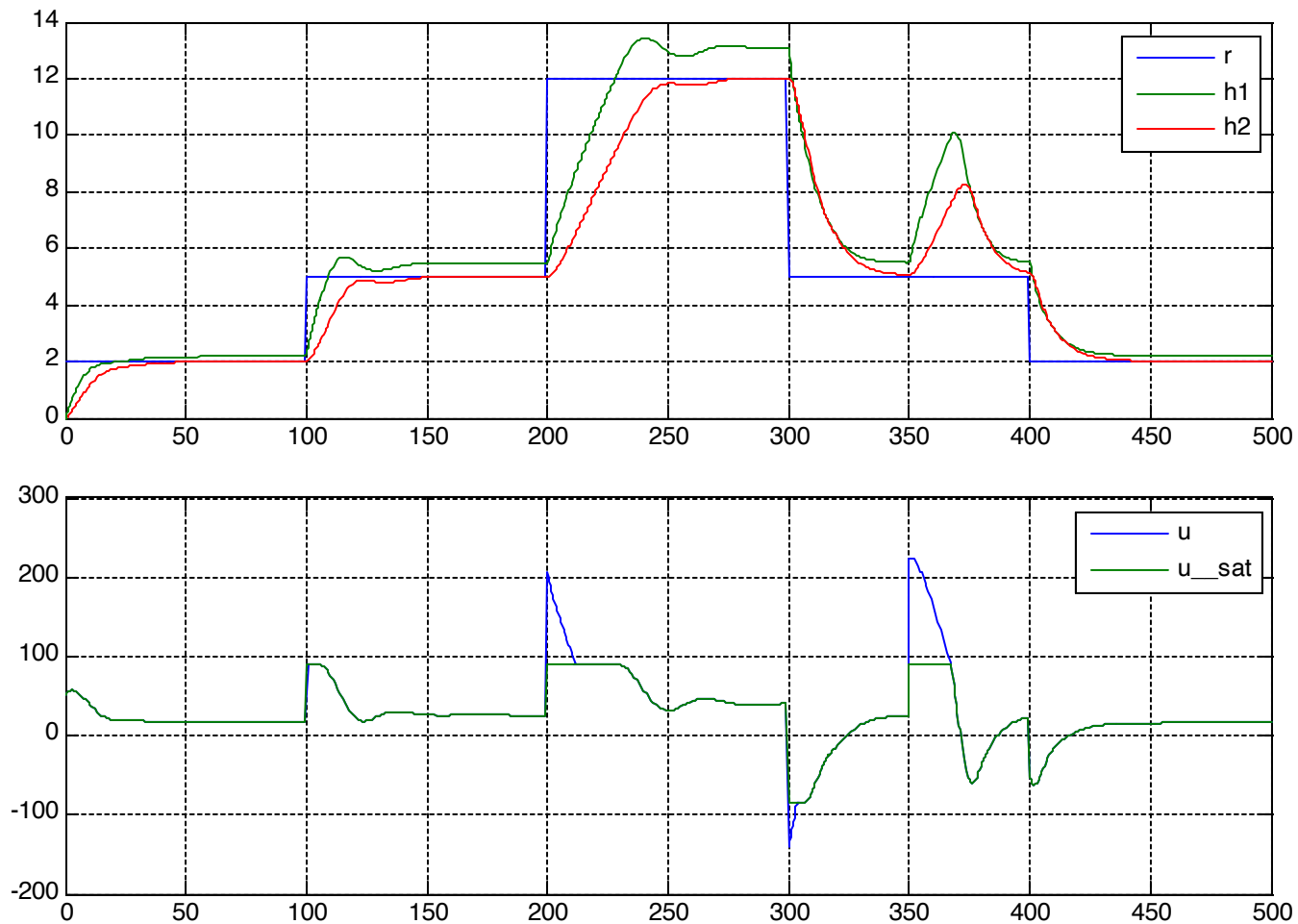
Controle EE – PI – Anti-Windup

Processo de Nivel 2ª ordem

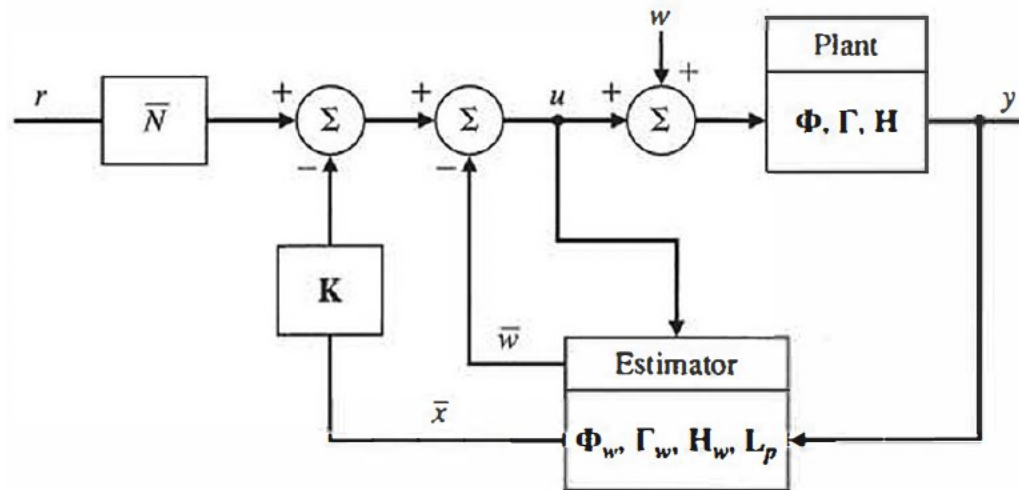


Controle EE – PI – Anti-Windup

Processo de Nivel 2ª ordem



Observador de Perturbações

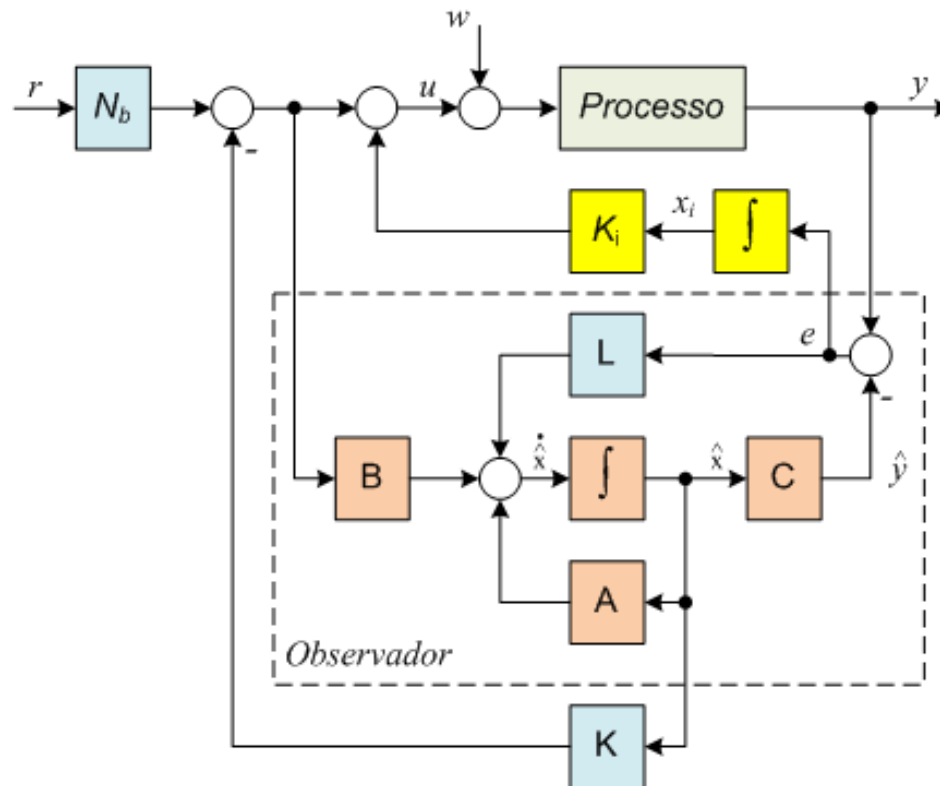


$$\begin{bmatrix} \mathbf{x}(k+1) \\ \mathbf{x}_d(k+1) \end{bmatrix} = \begin{bmatrix} \Phi & \Gamma_d \mathbf{H}_d \\ \mathbf{0} & \Phi_d \end{bmatrix} \begin{bmatrix} \mathbf{x}(k) \\ \mathbf{x}_d(k) \end{bmatrix} + \begin{bmatrix} \Gamma \\ \mathbf{0} \end{bmatrix} u(k)$$

$$y = [\mathbf{H} \quad \mathbf{0}] \begin{bmatrix} \mathbf{x} \\ \mathbf{x}_d \end{bmatrix},$$

Observador de Perturbações Constantes

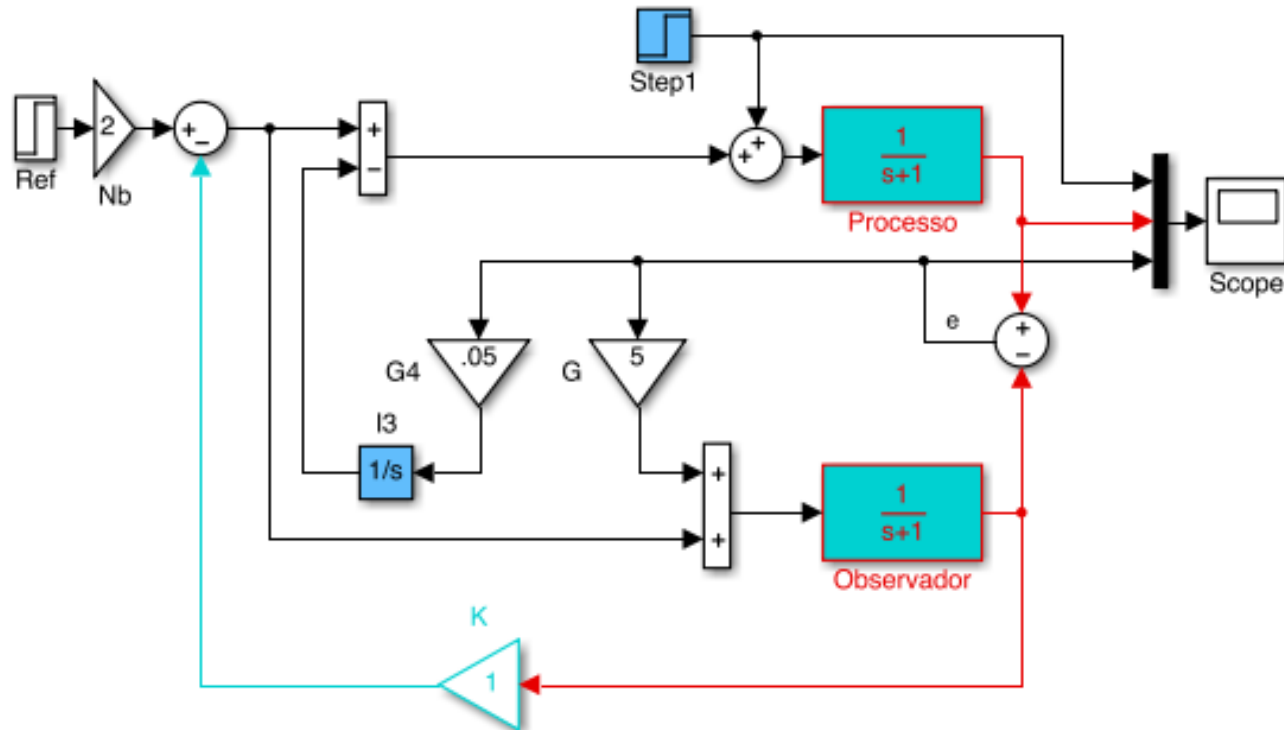
Canal Integral como Observador de Perturbações



- Projeto aumentado do Observador
- Canal integral não aparece na F.T.

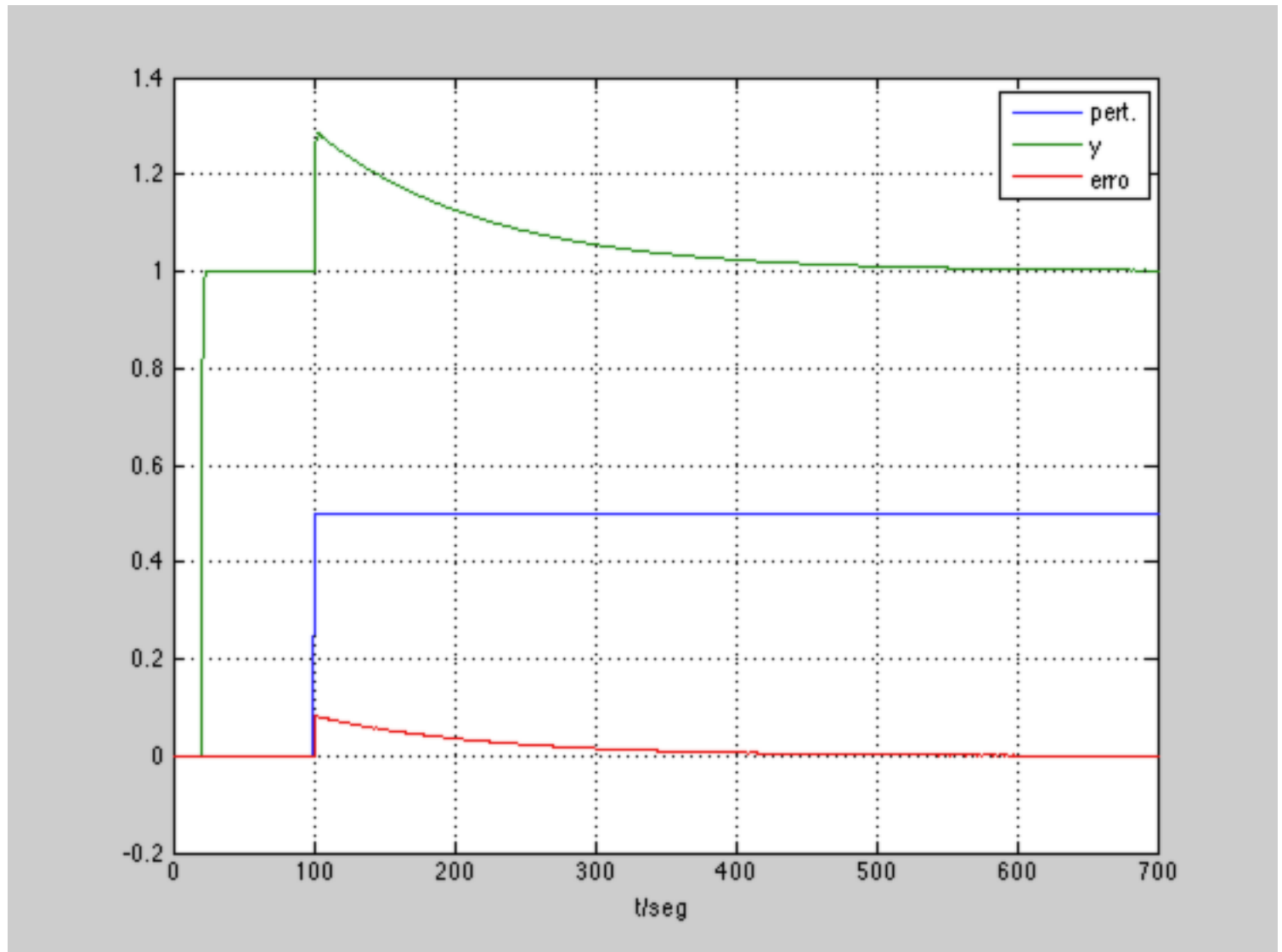
Observador de Perturbações Constantes

- Sistema de 1ª ordem



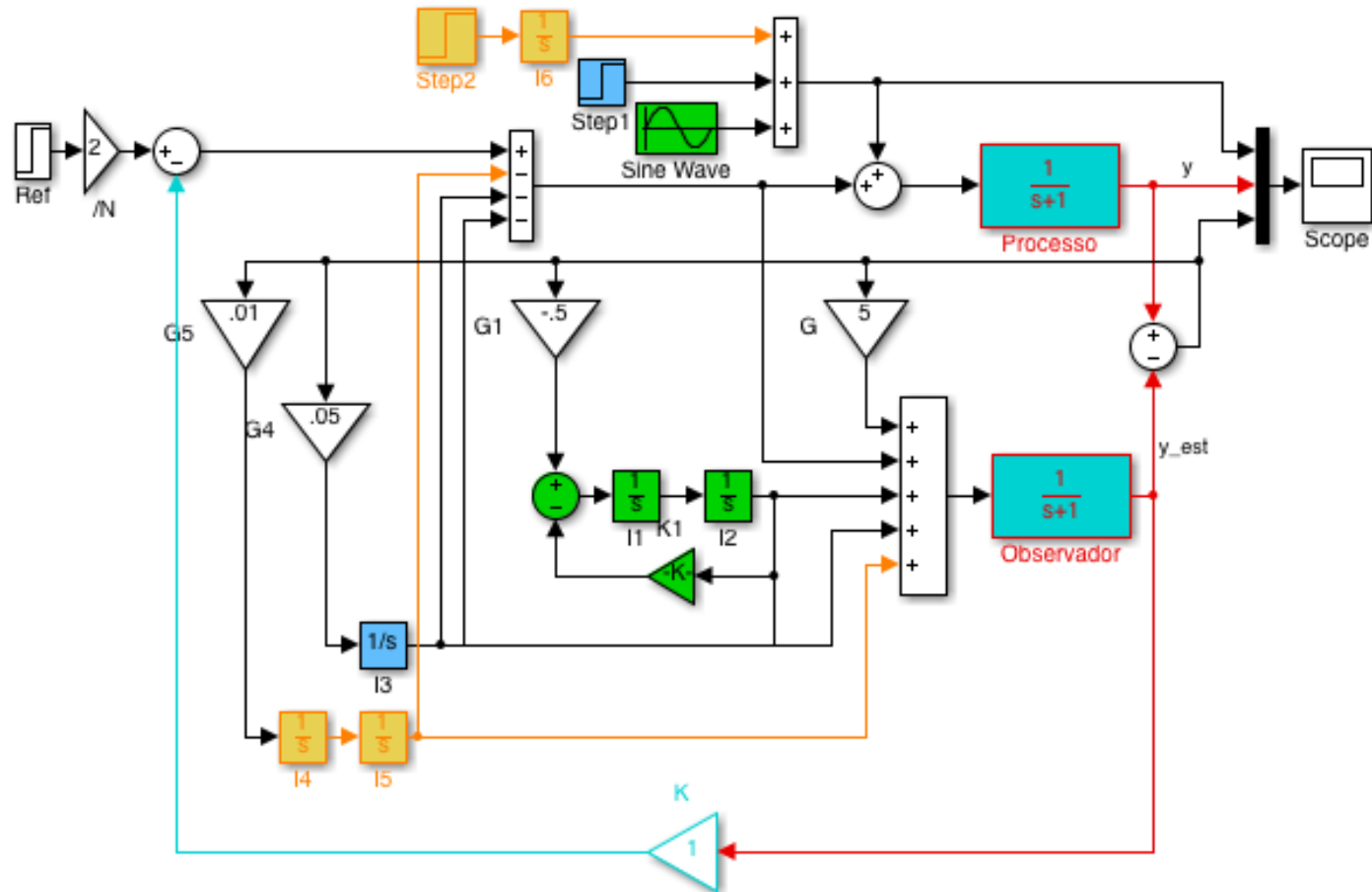
- $Y(s)/R(s) = 2/(s+2)$
- Canal integral não aparece na F.T.

Observador de Perturbações

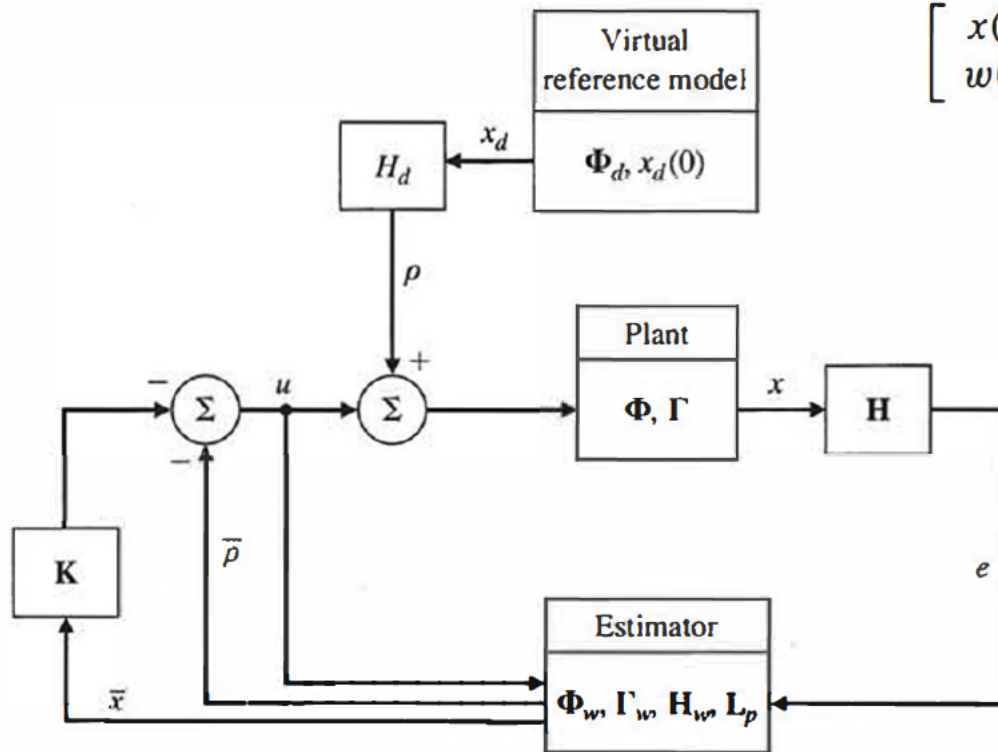


Observador de Perturbações

- Perturbações em degrau, rampa e senoidal



Observador de Perturbações



$$\begin{bmatrix} x(k+1) \\ w(k+1) \end{bmatrix} = \begin{bmatrix} \Phi & \Gamma \\ 0 & 1 \end{bmatrix} \begin{bmatrix} x(k) \\ w(k) \end{bmatrix} + \begin{bmatrix} \Gamma \\ 0 \end{bmatrix} u(k)$$

$$y(k) = \begin{bmatrix} H & 0 \end{bmatrix} \begin{bmatrix} x(k) \\ w(k) \end{bmatrix}.$$

Constantes (por partes) $\dot{w} = 0.$

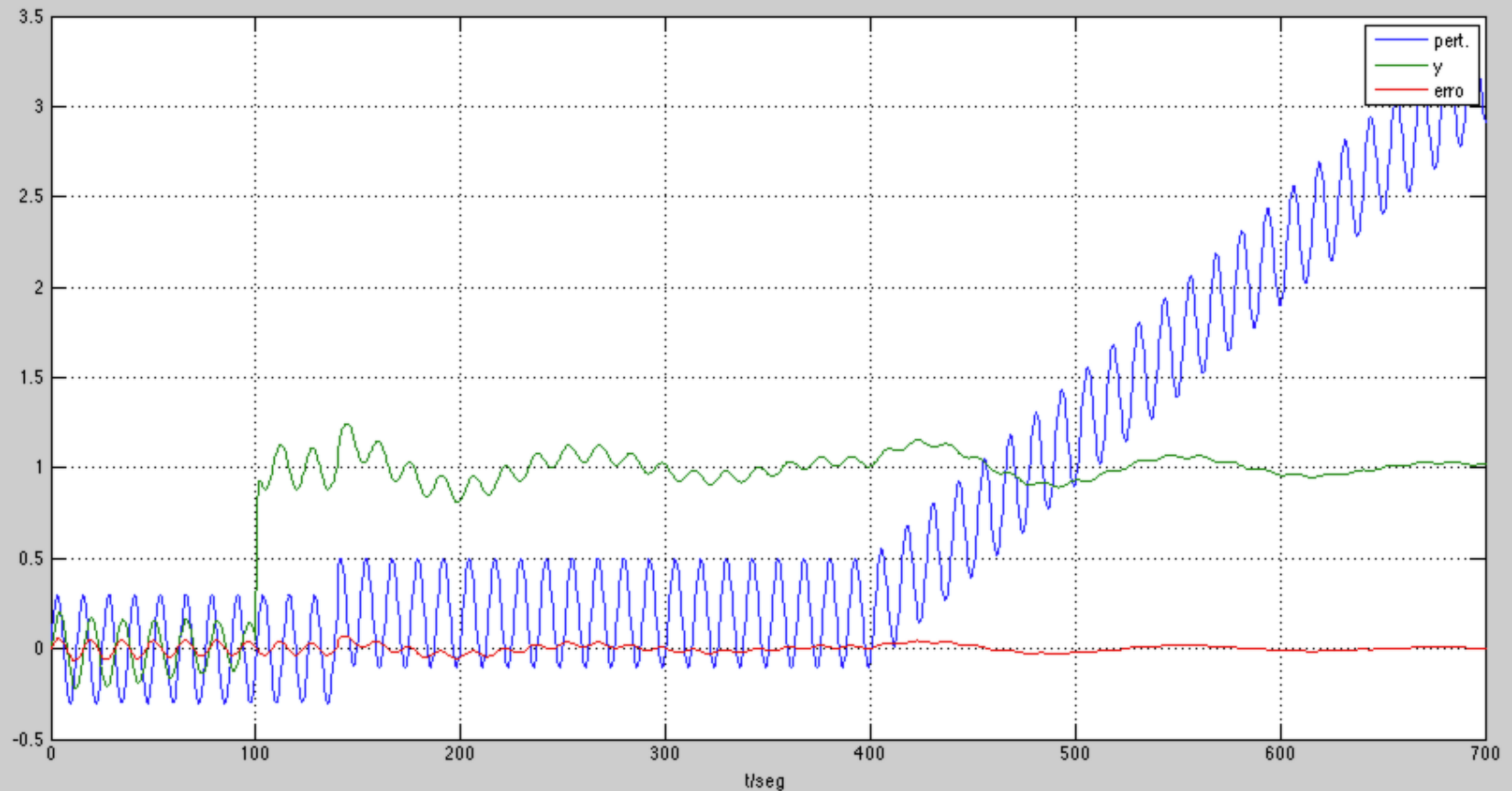
Rampa $\ddot{w} = 0.$

Senóide $\ddot{w} = -\omega_o^2 w,$

- Projeto aumentado do Observador
- Não aparece na F.T.

Observador de Perturbações

- Referência (1,0 em 100 s)
- Perturbação $0.3 \cdot \sin(0.5t)$, degrau (0,2 em 140 s), rampa (0,01 em 400 s)



Ex:Projeto de Controlador EE

```
g=tf([5 6 7],[1 2 3 4])
```

% Função de Transferência do Processo

% $(b_2s^2+b_1s+b_0)/(s^3+a_2s^2+a_1s+a_0)$

% EE na Forma Canônica Controlável (Fórmula de Mason)

```
A=[0 1 0;0 0 1;-4 -3 -2];B=[0;0;1];C=[7 6 5];D=0;
```

% Especificação da dinâmica EE:

% polinômio característico desejado em Malha Fechada $\tilde{a}(s)=(s-p)^3$;

% dinâmica do observador $\Delta(s)=(s-o)^3$

```
p=-1; o=3*p;
```

```
K=acker(A,B,[p,p,p])
```

```
K=acker(A,B,[p,-.6-1.0198i,-.6+1.0198i])
```

% resolve $\det(sI - A + BK) = \tilde{a}(s)$

% cancelamento dos zeros do processo

% (redução de ordem!)

```
L=acker(A',C',[o,o,o])';
```

% resolve $\det(sI - A + LC) = \Delta(s)$

```
ssMF=ss(A-B*K,B,C,D)
```

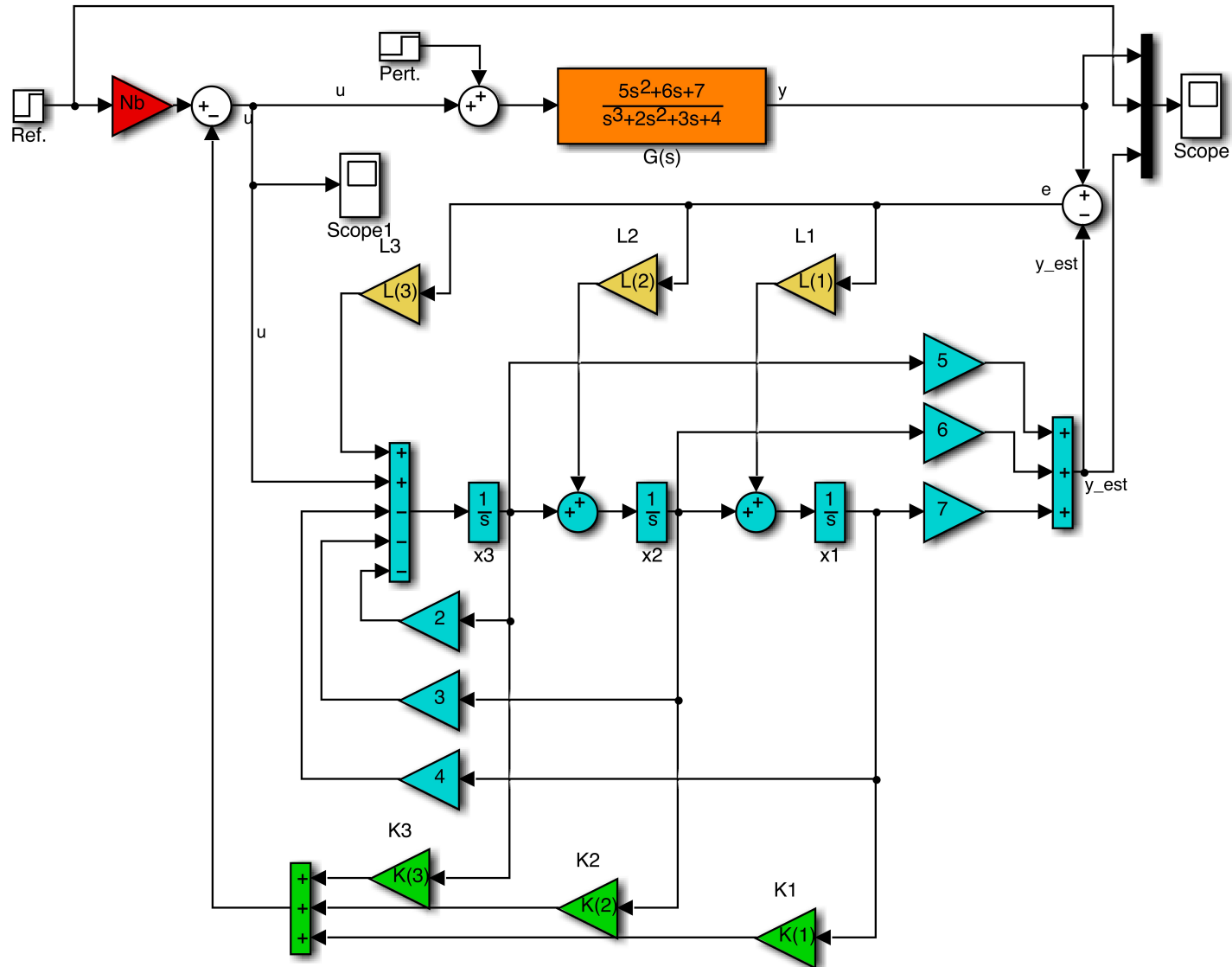
```
gf=tf(ssMF)
```

% função de transferência malha fechada

```
Nb=gf.den{1}(1,4)/gf.num{1}(1,4)
```

% fator de ajuste de ganho $\tilde{a}_0/b_0$

Ex:Projeto de Controlador EE



Ex: Projeto de Controlador EE – canal PI

`g=tf([5 6 7],[1 2 3 4])` % Processo $(b_2s^2+b_1s+b_0)/(s^3+a_2s^2+a_1s+a_0) = b(s)/a(s)$

`A=[0 1 0;0 0 1;-4 -3 -2];B=[0;0;1];C=[7 6 5];D=0;` % FCC

% Sistema aumentado com x_i , nova variável de estado integra o erro (r-y)

`Aa=[A(1,:) 0    %  $x_1$`

`A(2,:) 0    %  $x_2$`

`A(3,:) 0    %  $x_3$`

`-C    0]    %  $x_i$`

`Ba=[B; 0];`

`Ca=[C 0];`

`p=-1; o=3*p;`

`Ka=acker(Aa,Ba,[p,p,p,p])`

`Ki=-Ka(4),K=Ka(1:3),`

% Especificação da dinâmica EE

% resolve $\det(sI - A_a + B_a K_a) = \tilde{a}_a(s)$

% xi não tem realimentação negativa como x_1, x_2, x_3 . Trocar sinal!

`L=acker(A',C',[o,o,o]);L=L'`

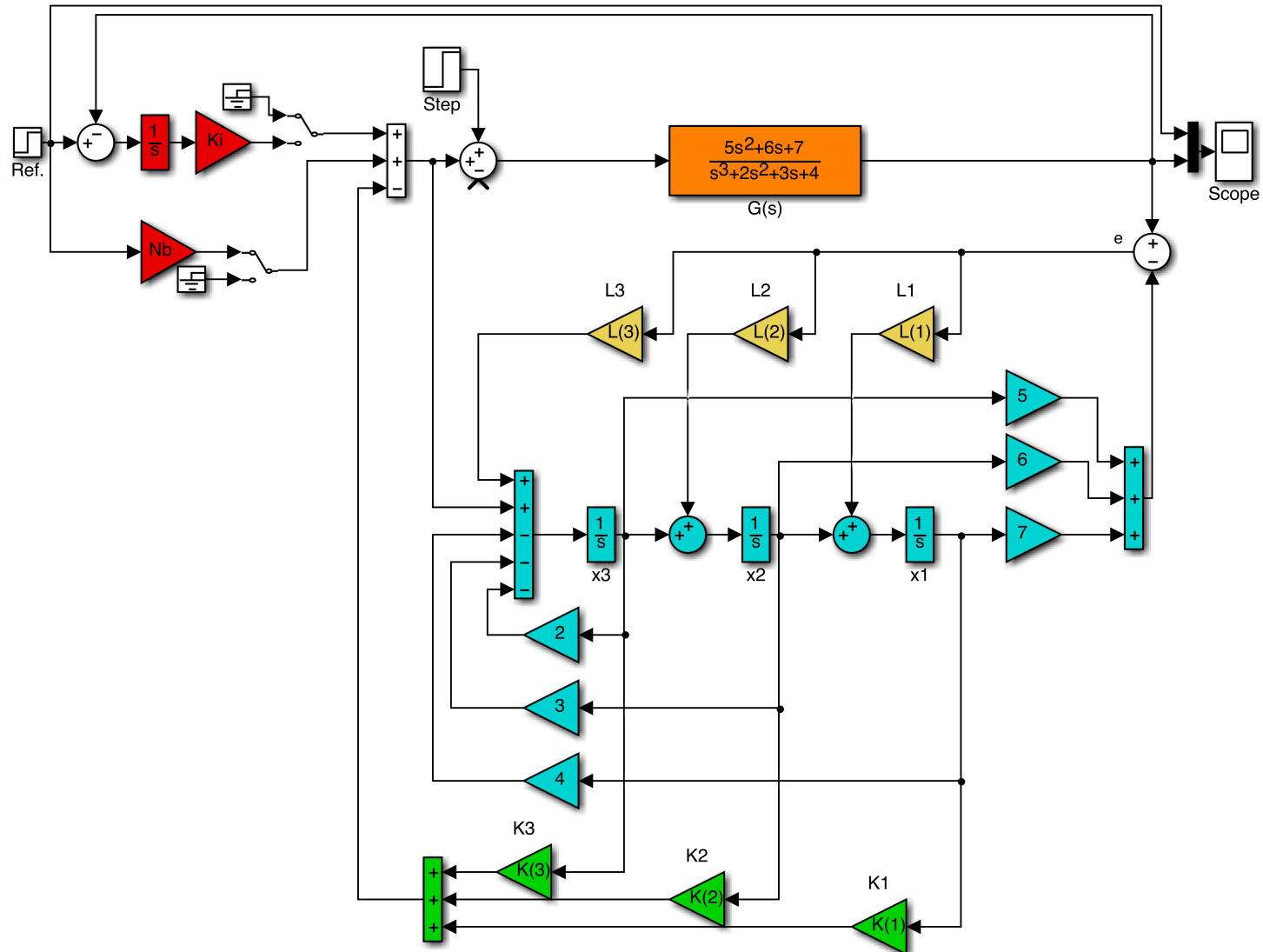
% resolve $\det(sI - A + LC) = \Delta(s)$, não precisa observar xi!

`Nb=-Ki/p;`

% Canal Proporcional à Referência e Integral do Erro

% $(sN_b + K_i)/s$ tem zero em $-K_i/N_b$ que cancela um polo de $\tilde{a}_a(s)$

Ex:Projeto de Controlador EE – canal PI



Ex: Projeto de Controlador EE com Observador de Perturbações Constantes

```
g=tf([5 6 7],[1 2 3 4])
```

% Processo $(b_2s^2+b_1s+b_0)/(s^3+a_2s^2+a_1s+a_0)$

```
A=[0 1 0;0 0 1;-4 -3 -2];B=[0;0;1];  
C=[7 6 5];D=0;
```

% FCC

```
K=acker(A,B,[p,p,p])
```

% Controlador por realimentação de Estados
% $\det(sI - A + BK) = \tilde{a}(s)$

```
ssMF=ss(A-B*K,B,C,D); gf=tf(ssMF);  
Nb=gf.den{1}(1,4)/gf.num{1}(1,4);
```

% Fator de ajuste de Ganhao

% Observador aumentado com x_i , nova variável de estado integra o erro $(y-\hat{y})$

```
Ac=0; Cc=1;
```

% Modelo do Integrador - Obs. Pert. Constantes

```
Aa=[A B;  
0 0 0 Ac];
```

```
Ba=[B;0]; Ca=[C 0];
```

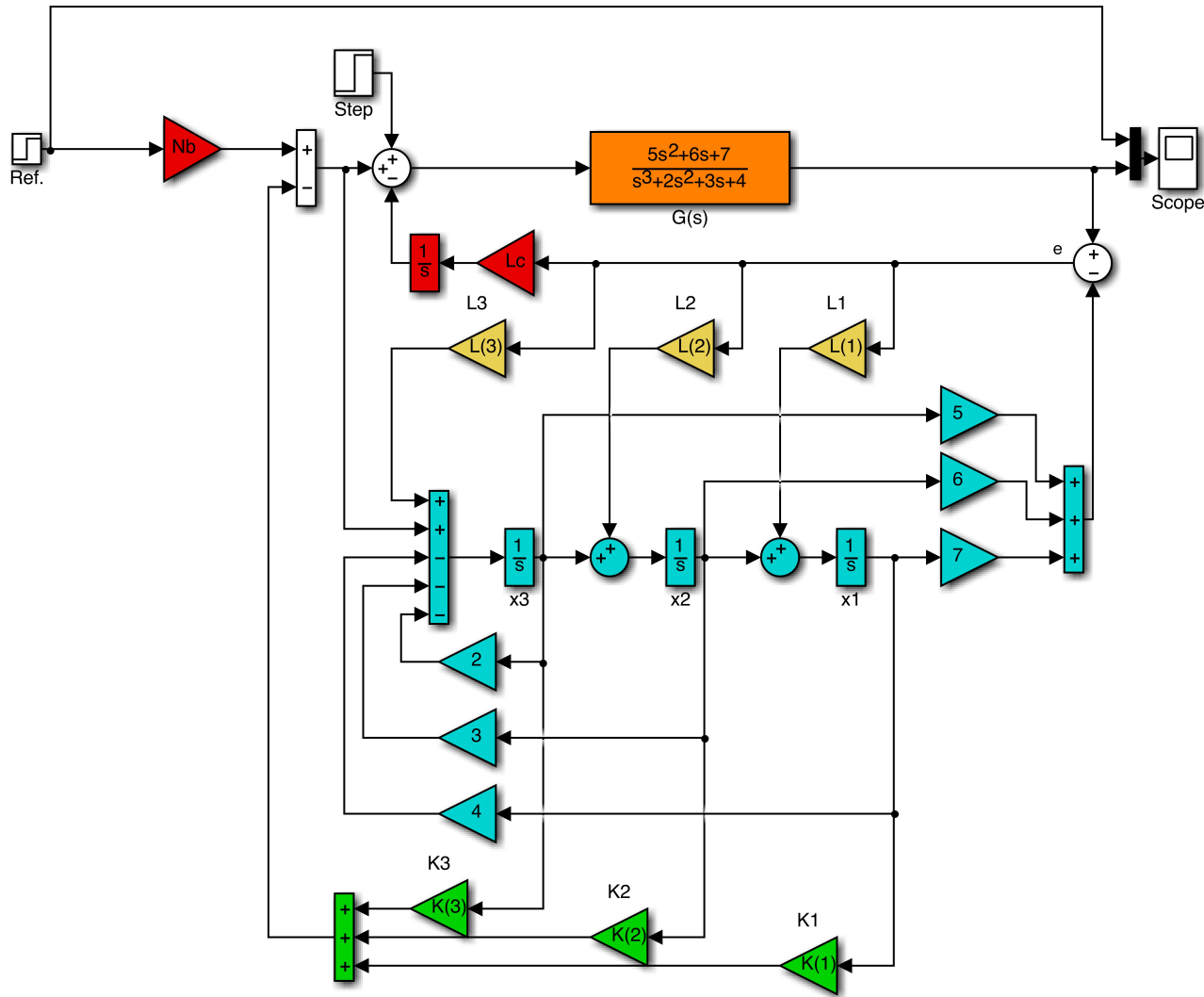
```
L=acker(Aa',Ca',[o,o,o,o])';
```

% Resolve $\det(sI - A_a + L_a C_a) = \Delta_a(s)$

```
Lc=L(4);
```

% Ganho do Observador de Estados e Perturbações

Ex: Projeto de Controlador EE com Observador de Perturbações Constantes



Ex: Projeto de Controlador EE com Obs. de Pert. Constantes e Senoidais

```
g=tf([5 6 7],[1 2 3 4])
```

% Processo $(b_2s^2+b_1s+b_0)/(s^3+a_2s^2+a_1s+a_0)$

```
A=[0 1 0;0 0 1;-4 -3 -2];B=[0;0;1];C=[7 6 5];D=0;
```

```
K=acker(A,B,[p,p,p])
```

% Controlador por realimentação de Estados

```
ssMF=ss(A-B*K,B,C,D); gf=tf(ssMF);
```

```
Nb=gf.den{1}(1,4)/gf.num{1}(1,4);
```

% Fator de ajuste de Ganhao

% Observador aumentado com x_c, x_{s2}, x_{s2} , novas variáveis do observeador integram o erro $(y-\hat{y})$

```
Ac=0; Cc=1;
```

% Modelo do Obs. Pert. Constantes (Integrador)

```
As=[0 1;
```

```
-w0^2 0];
```

```
Cs=[1 0];
```

% Modelo do Obs. Pert. Senoidais

```
Aa=[A B*Cc B*Cs;
```

```
0 0 0 Ac 0 0;
```

```
[0 0 0 0;0 0 0 0] As];
```

```
Ba=[B;0;0;0]; Ca=[C 0 0 0];
```

% $[x_1, x_2, x_3]^T$

% x_c

% $[x_{s1}, x_{s2}]^T$

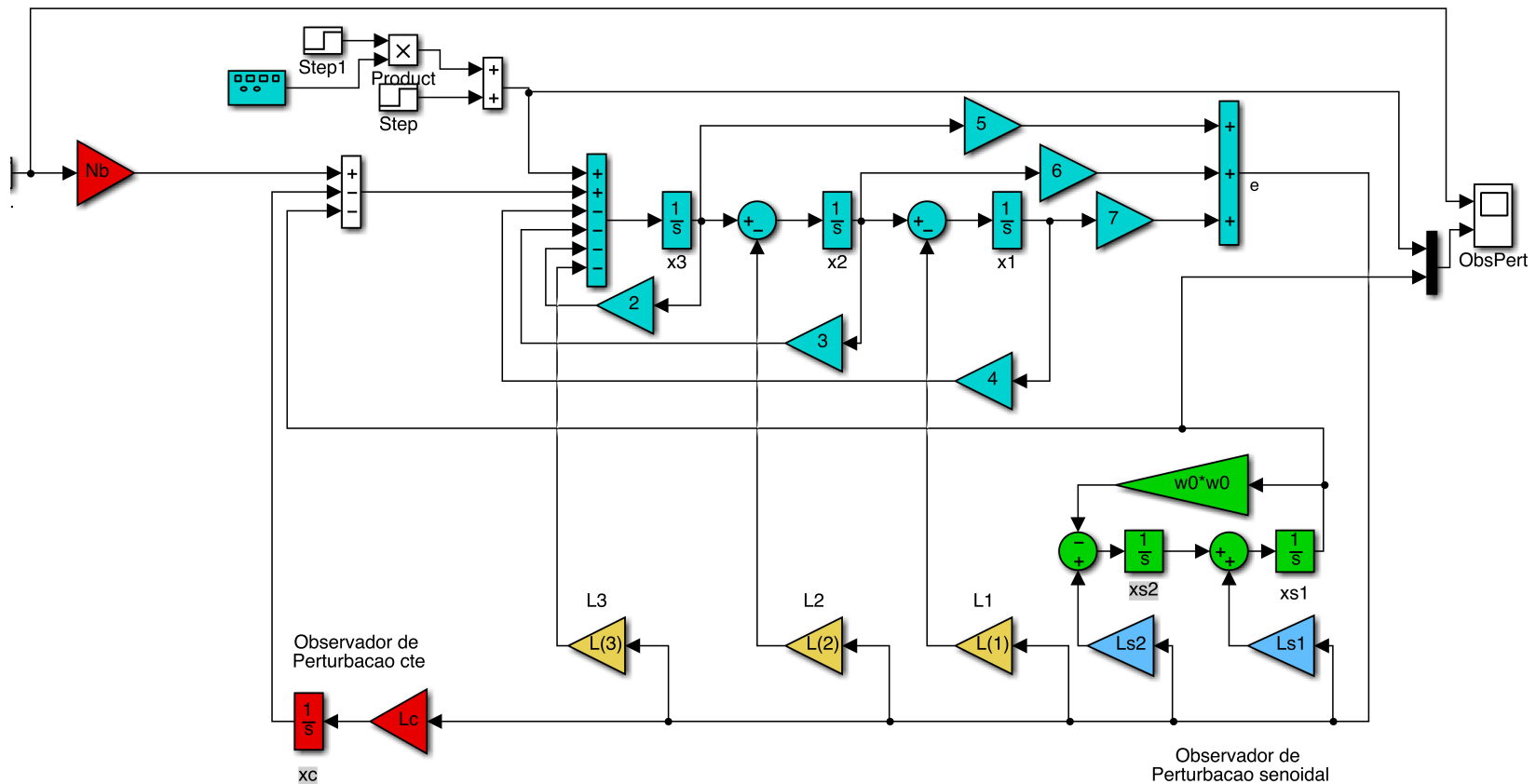
```
L=acker(Aa',Ca',[o,o,o,.3*o,.3*o,.3*o])';
```

% Resolve $\det(sI - A_a + L_a C_a) = \Delta_a(s)$

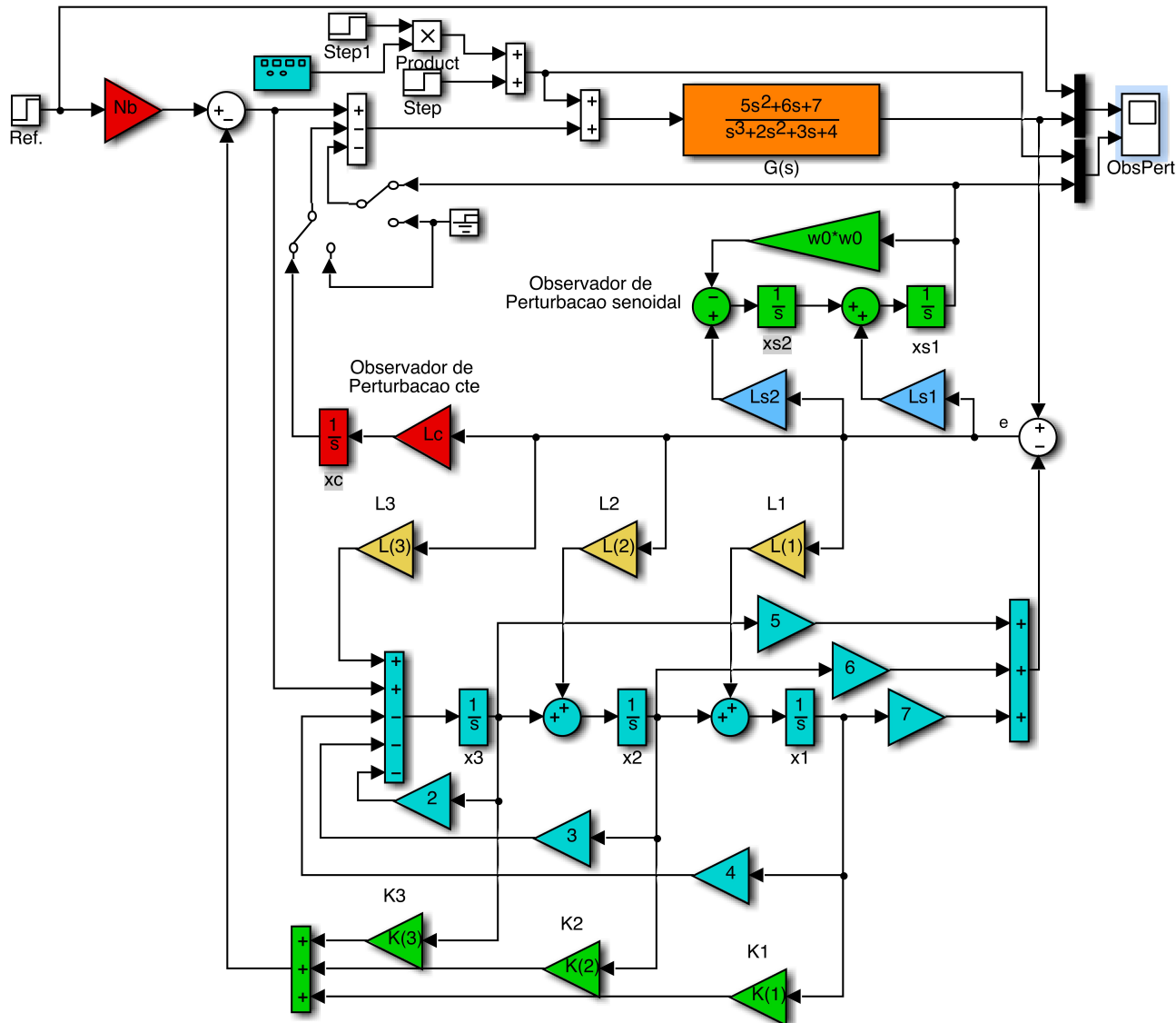
```
Lc=L(4);Ls1=L(5);Ls2=L(6);
```

% Ganhos do Observadores de Perturbações

Ex: Projeto de Controlador EE com Obs. de Pert. Constantes e Senoidais → *Princípio da Separabilidade*

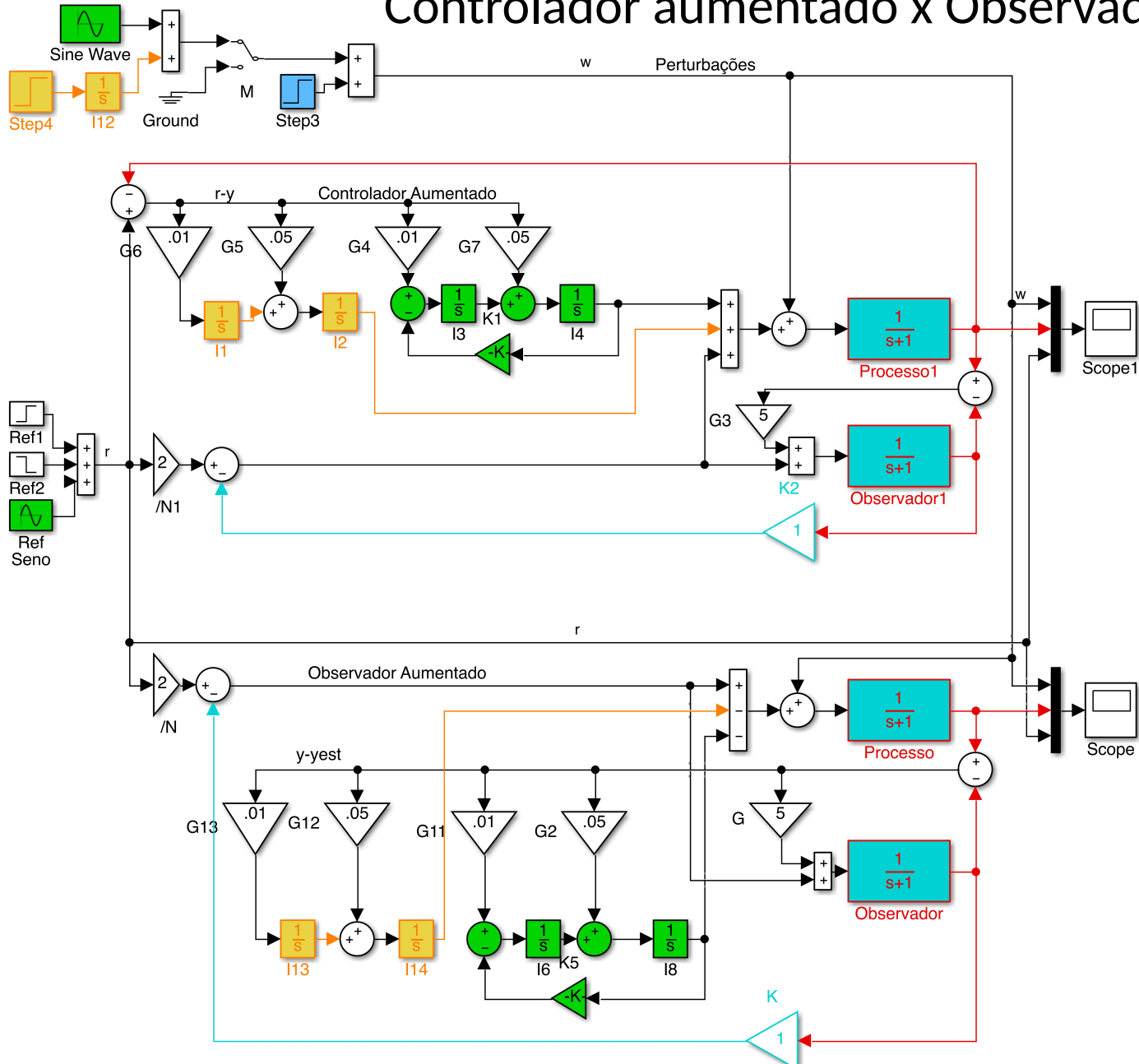


Ex: Projeto de Controlador EE com Obs. de Pert. Constantes e Senoidais



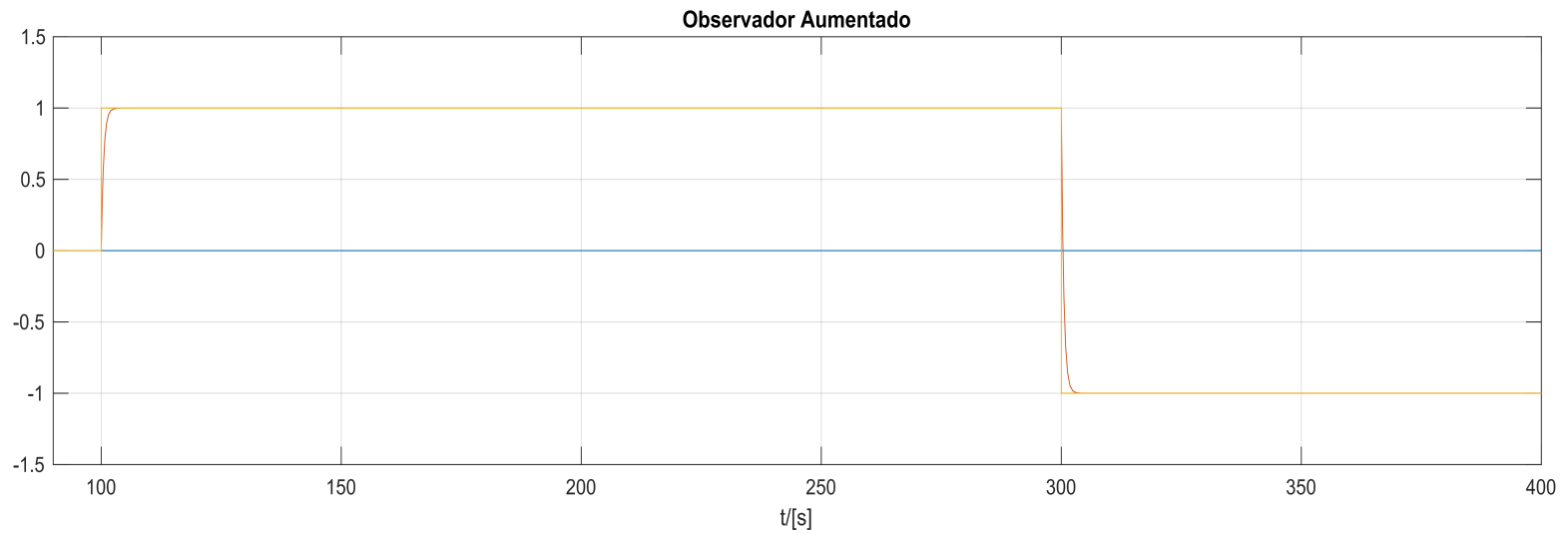
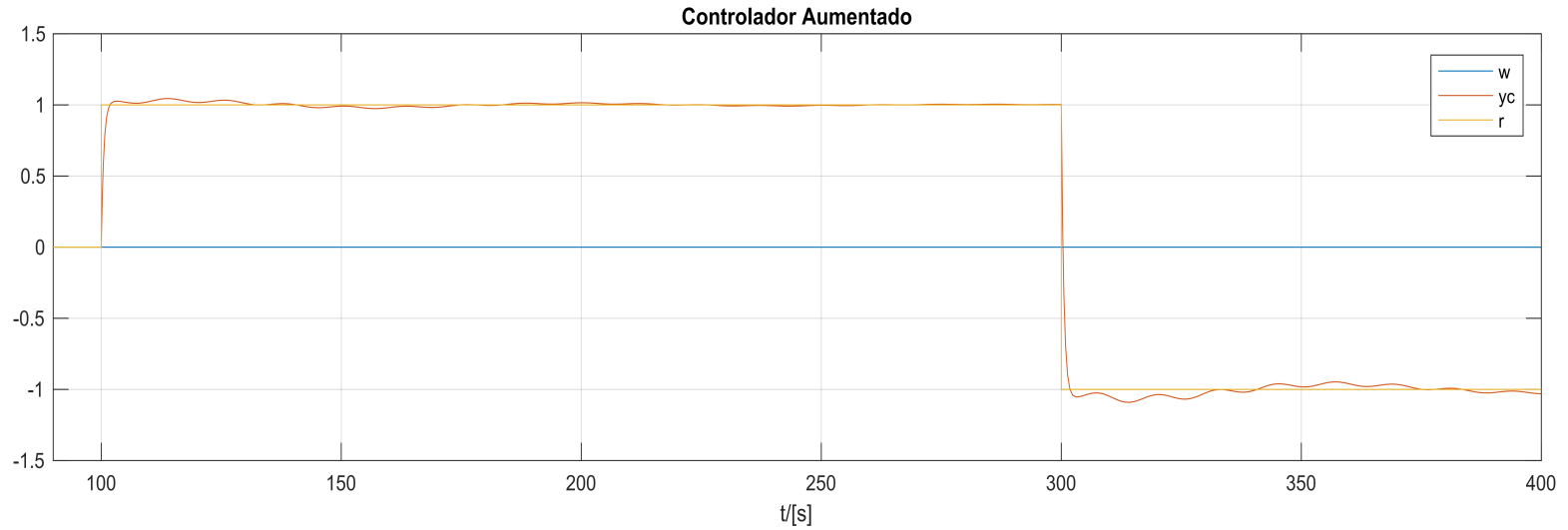
Comparação de Rejeição de Perturbações

Controlador aumentado x Observador aumentado



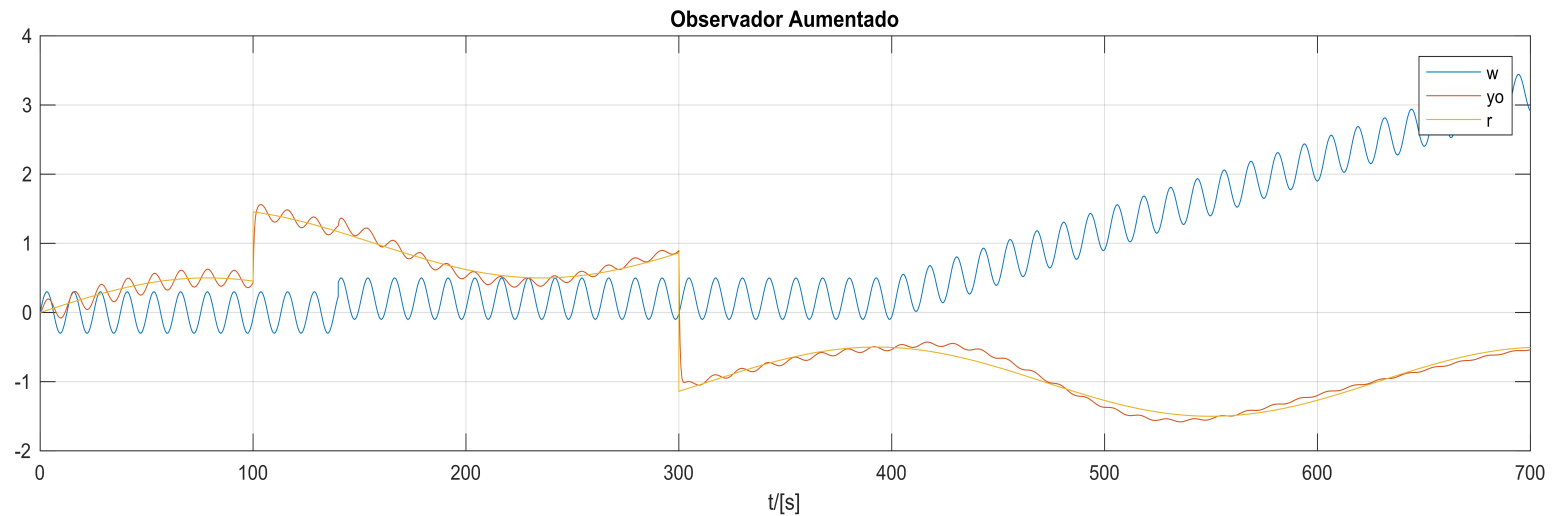
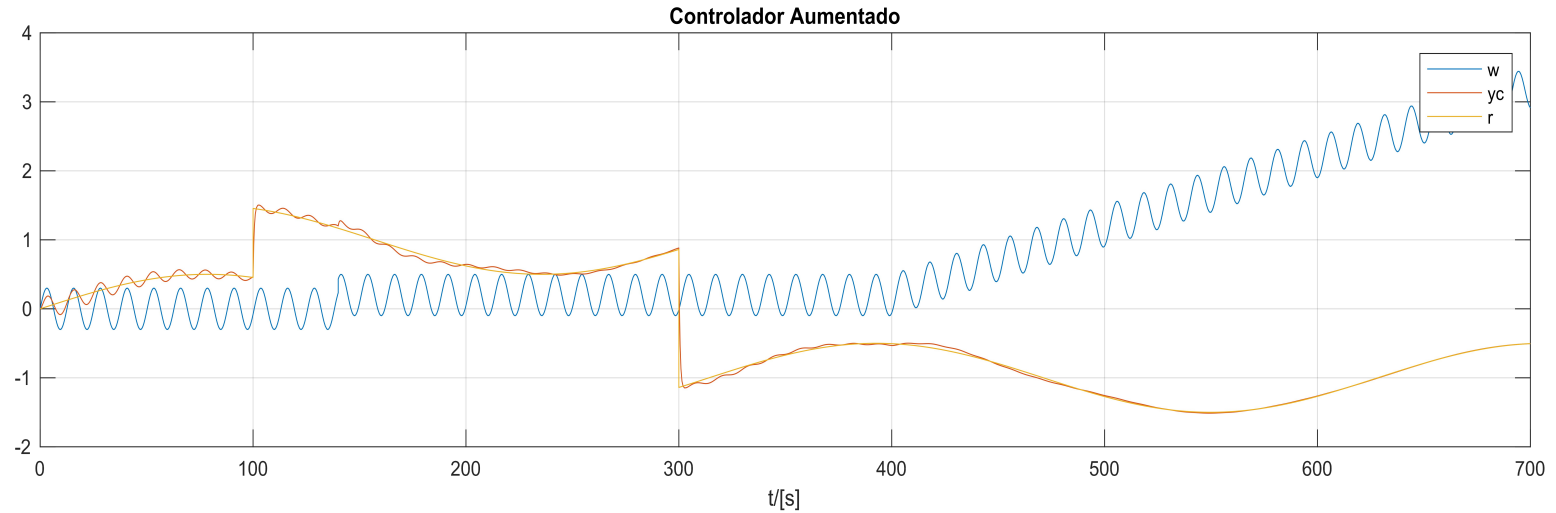
Seguimento de Referência

Controlador aumentado x Observador aumentado

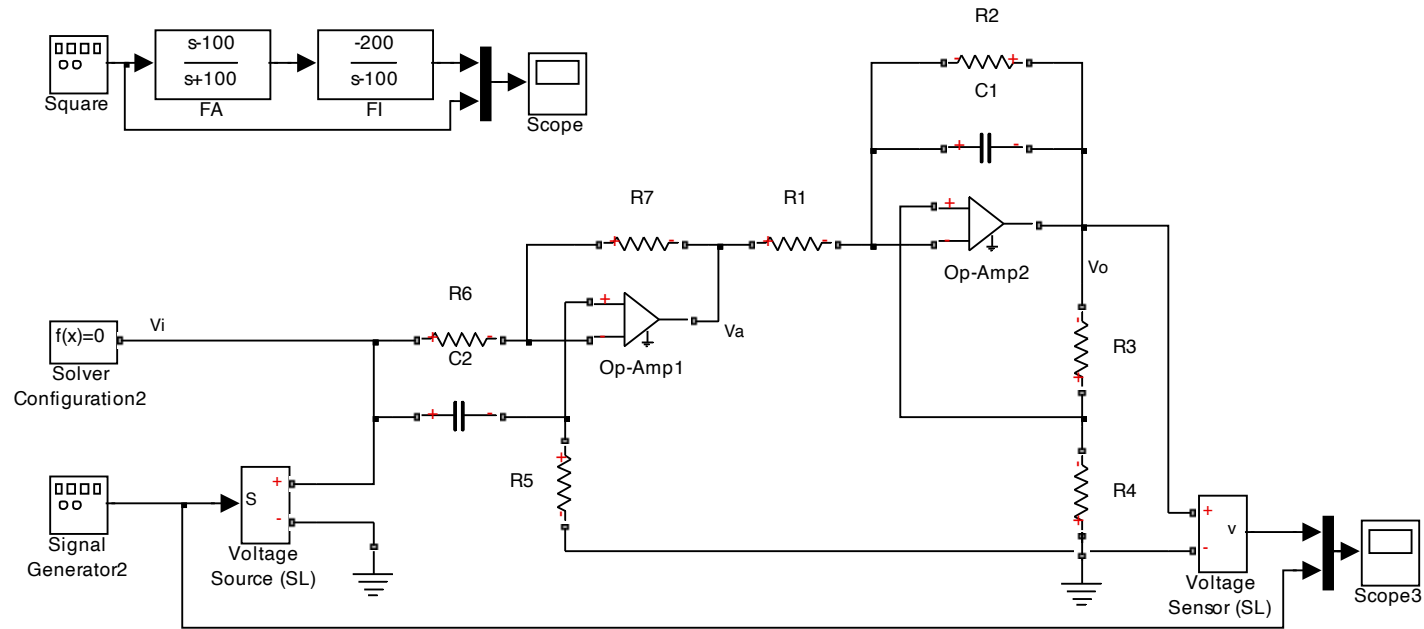


Comparação de Rejeição de Perturbações

Controlador aumentado x Observador aumentado



Exercício Extra 7



Considere o seguinte circuito, com $C1=2 \mu\text{F}$; $C2=1 \mu\text{F}$; $R1=R2=R3=R5=R6=R7=10 \text{ k}\Omega$; $R4=30 \text{ k}\Omega$.

Mostre que

$$\frac{V_a(s)}{V_i(s)} = FA(s) = \frac{s-100}{s+100}$$

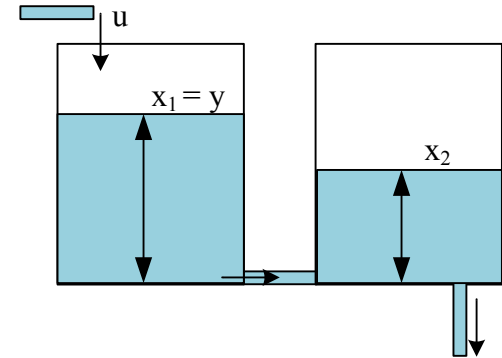
$$\frac{V_o(s)}{V_a(s)} = FI(s) = \frac{-200}{s-100}$$

- Derive uma representação deste sistema no espaço de estados. Variáveis de estado relativas a $V_a(s)$ e $V_o(s)$.
- Verifique se o sistema completo é observável.
- Verifique se o sistema completo é controlável.
- Simule o circuito e comente sobre os sinais obtidos.

Exercício Extra 8

Considere o controle de um processo de nível de líquido de 2ª ordem em um determinado ponto de operação, onde as variáveis de estado x_1 e x_2 correspondem aos níveis do 1º e do 2º tanque. O sinal de entrada u é uma vazão em cm^3/s e a saída é o nível x_1 em cm .

Para lidar com perturbações, erros de modelo e apresentar uma resposta rápida sem oscilação deve ser projetado nesta questão um controlador por realimentação de estados com canal integral. Em malha fechada o sistema deve apresentar a função de transferência do modelo de referência indicado abaixo (bloco azul).



- Obtenha, utilizando x_1 e x_2 indicados, a representação do processo no EE:

$$\begin{cases} \dot{\mathbf{x}} = \mathbf{A}\mathbf{x} + \mathbf{B}u \\ y = \mathbf{C}\mathbf{x} + \mathbf{D}u \end{cases}$$
- Calcule L_1 e L_2 de tal forma que a equação característica do observador seja $s^2 + 20s + 100$.
- Obtenha as matrizes $\mathbf{A}_a$ e $\mathbf{B}_a$ do sistema aumentado que permitem projetar uma realimentação de estados com canal integral $|s\mathbf{I} - \mathbf{A}_a + \mathbf{B}_a\mathbf{K}|$ ($K_p = 0$, por enquanto).
- Calcule $\mathbf{K} = [K_1 \ K_2 \ K_i]$
- Acrescente um canal proporcional (K_p) que reduza o sistema em MF a $3/(s+3)$.
- Simule o processo.

Sistema aumentado:

$$\begin{bmatrix} \dot{\mathbf{x}} \\ \dot{x}_i \end{bmatrix} = \begin{bmatrix} \mathbf{A} & 0 \\ -\mathbf{C} & 0 \end{bmatrix} \begin{bmatrix} \mathbf{x} \\ x_i \end{bmatrix} + \begin{bmatrix} \mathbf{B} \\ 0 \end{bmatrix} u(k) + \begin{bmatrix} 0 \\ 1 \end{bmatrix} r(k)$$

$$y(k) = \begin{bmatrix} \mathbf{C} & 0 \end{bmatrix} \begin{bmatrix} \mathbf{x} \\ x_i \end{bmatrix}$$

