Controle de Sistemas Dinâmicos

CSD Exercicios Cap3-4

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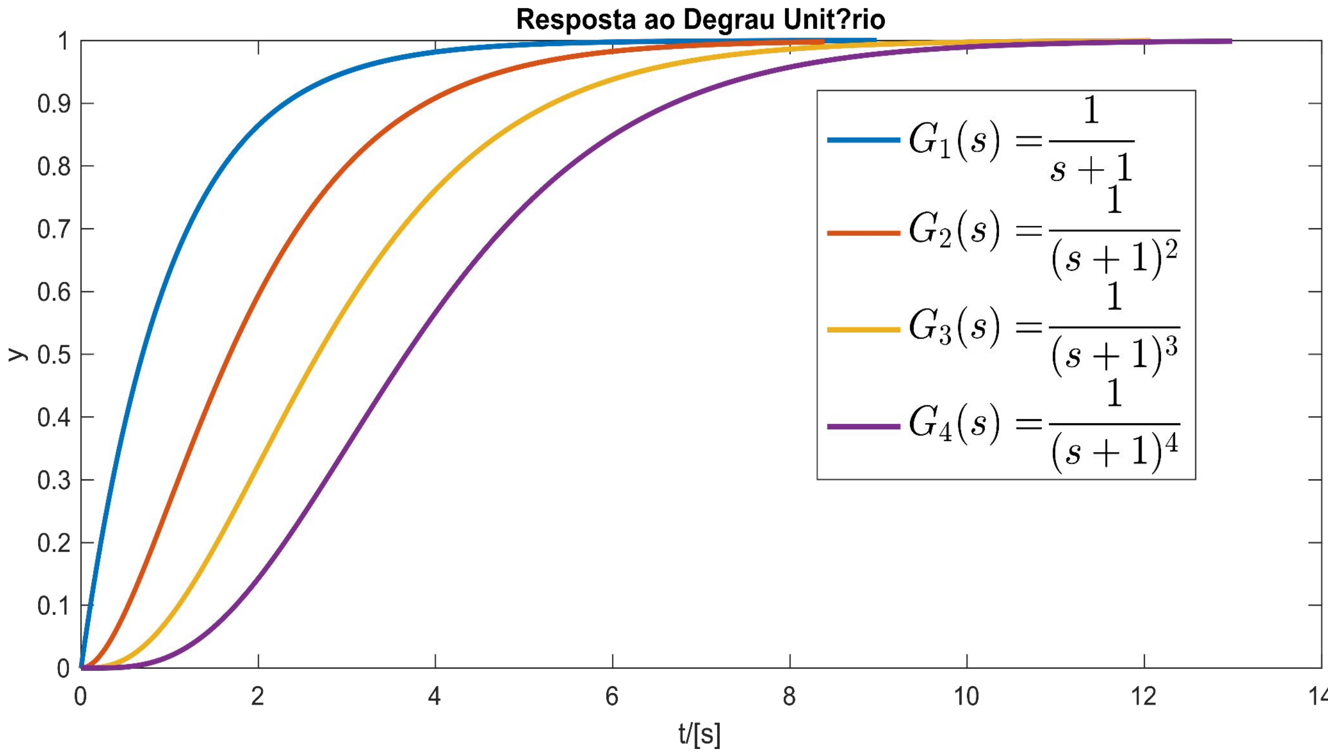
ENE/UnB

(Material de aula *Complementar*)

Respostas no tempo

i) Maior ordem → Maior tempo de resposta



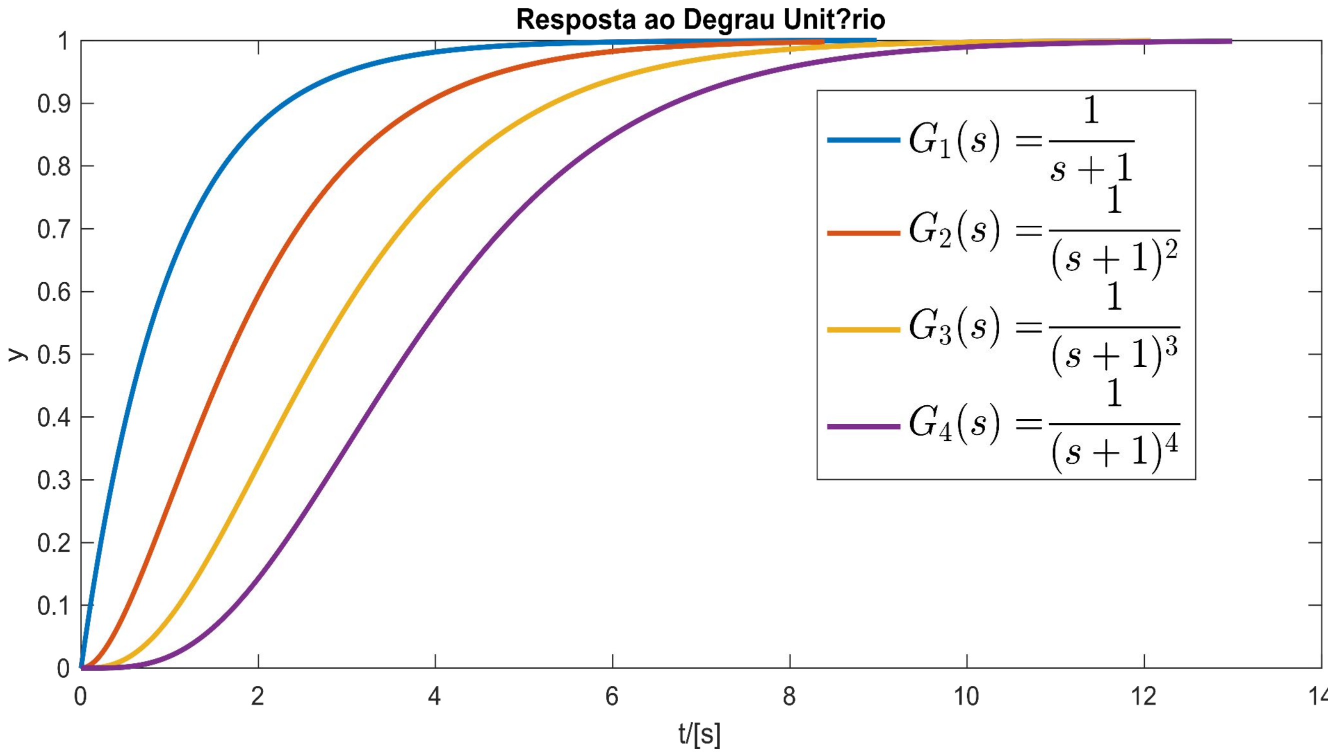
$$G_1(s) = \frac{1}{(s+1)}$$

Teorema do valor inicial

$$R(s) = 1/s$$
$$Y_1(s) = R(s)G_1(s)$$
$$y_1(t) = \lim_{s\to\infty} s\frac{1}{s}\frac{1}{s+1} = 0$$
$$\dot{y}_1(t) = \lim_{s\to\infty} s\frac{1}{s}\frac{s}{s+1} = 1$$

$$y_2(t) = \lim_{s\to\infty} s\frac{1}{s}\frac{1}{(s+1)^2} = 0$$
$$\dot{y}_2(t) = \lim_{s\to\infty} s\frac{1}{s}\frac{s}{(s+1)^2} = 0$$
$$\ddot{y}_2(t) = \lim_{s\to\infty} s\frac{1}{s}\frac{s^2}{(s+1)^2} = 1$$

$$\ldots = 0$$
$$\ldots = 0$$
$$\ldots = 0$$
$$\dddot{y}_3(t) = \lim_{s\to\infty} s\frac{1}{s}\frac{s^3}{(s+1)^3} = 1$$




```
%%
s=sym('s')

g1s=1/(s+1)
g2s=1/(s+1)^2
g3s=1/(s+1)^3
g4s=1/(s+1)^4
```

ilaplace(1/s*g1s) = 1 - exp(-t)

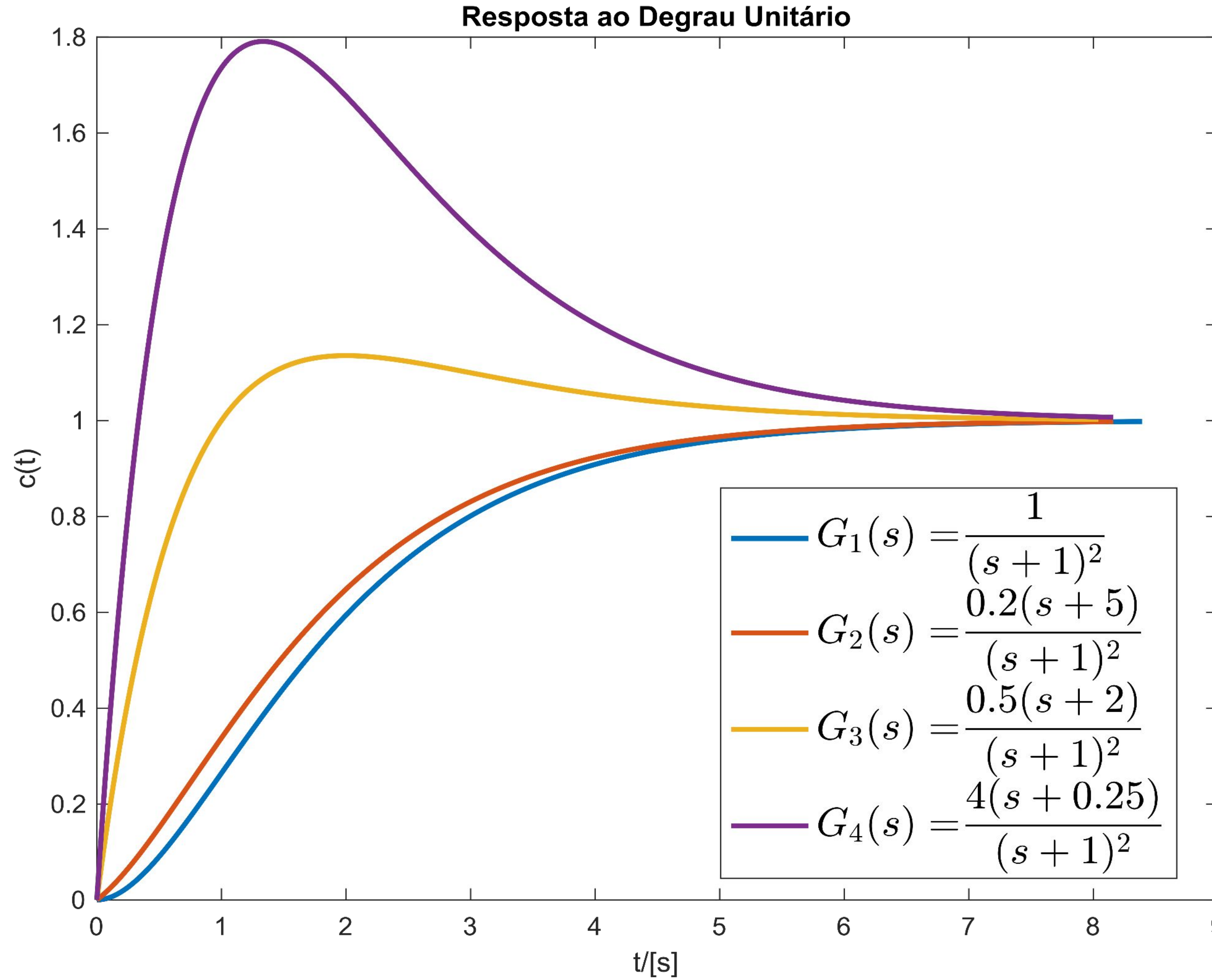
ilaplace(1/s*g2s) = 1 - t*exp(-t) - exp(-t)

ilaplace(1/s*g3s) = 1 - t*exp(-t) - (t^2*exp(-t))/2 - exp(-t)

ilaplace(1/s*g4s) = 1 - t*exp(-t) - (t^2*exp(-t))/2 - (t^3*exp(-t))/6 - exp(-t)

Respostas no tempo

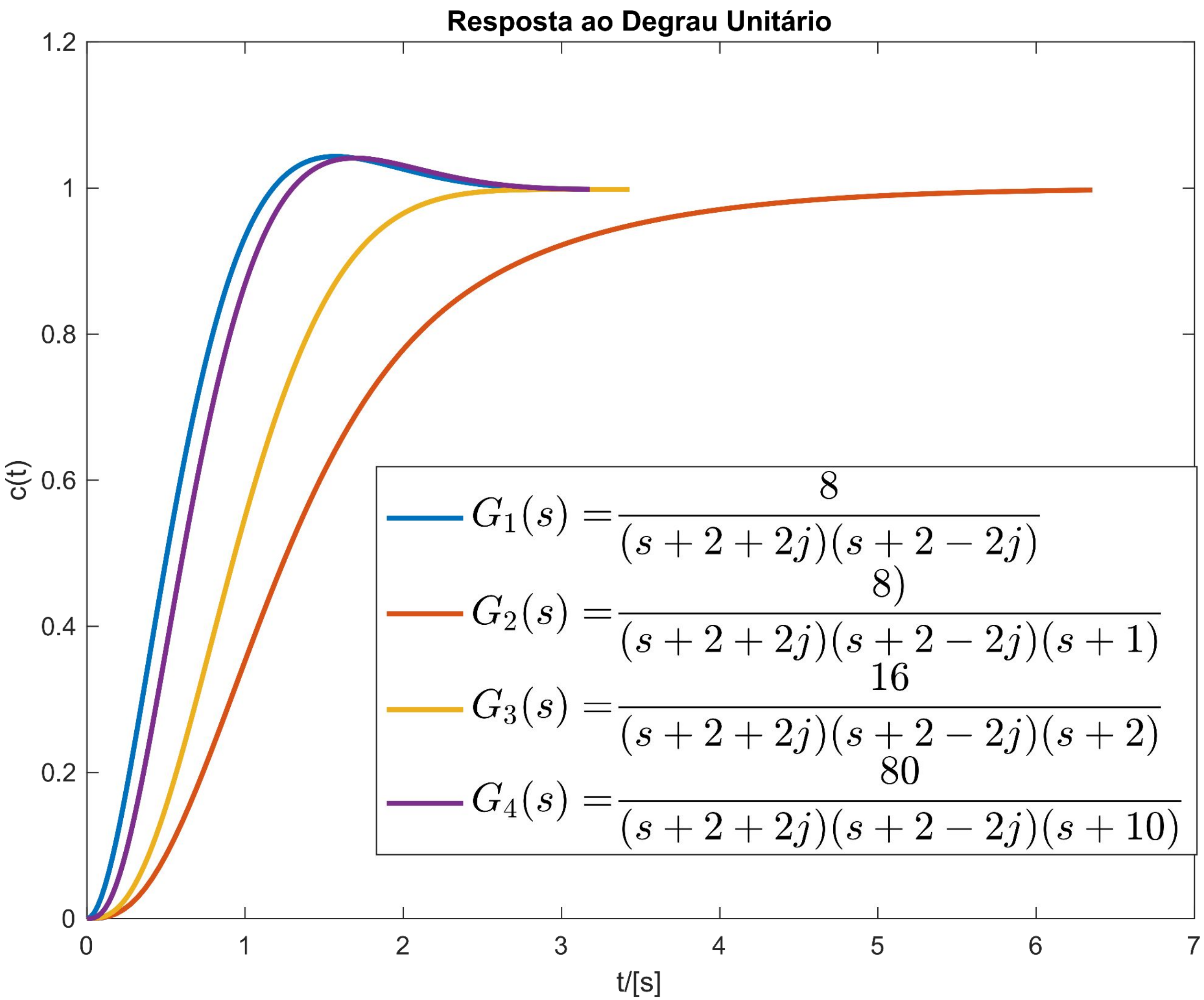
ii) Zeros lentos tornam o sistema subamortecido



ilaplace(1/s*g1s) = 1 - t*exp(-t) - exp(-t)
ilaplace(1/s*g2s) = 1 - (4*t*exp(-t))/5 - exp(-t)
ilaplace(1/s*g3s) = 1 - (t*exp(-t))/2 - exp(-t)
ilaplace(1/s*g4s) = 1+ 3*t*exp(-t) - exp(-t)

Respostas no tempo

iii) Polos Dominantes aproximam a a resposta



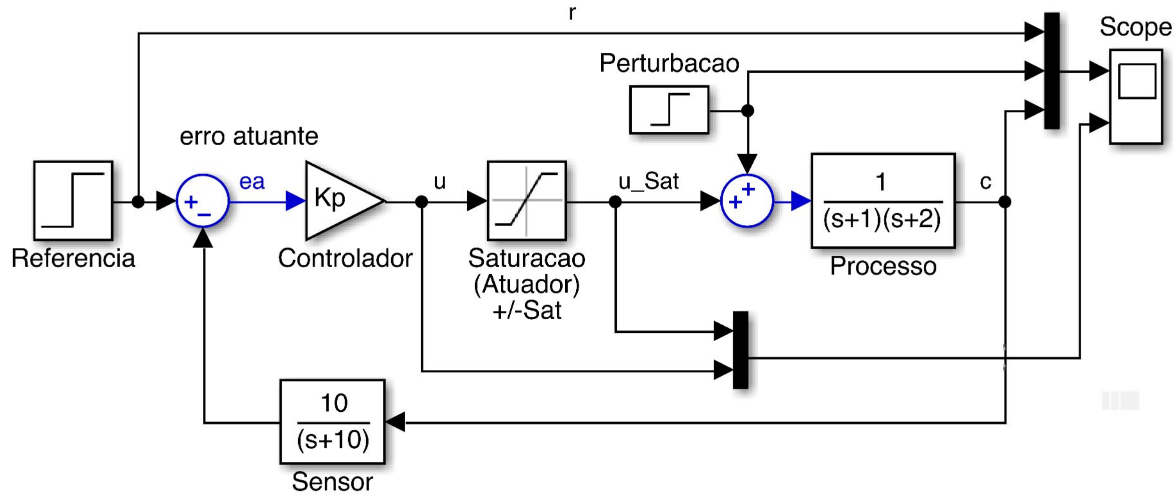
Polos Lentos !!

G_4 aproxima G_1

Dinâmica Dominante de 2a Ordem

iv) Resposta W(s)/R(s)

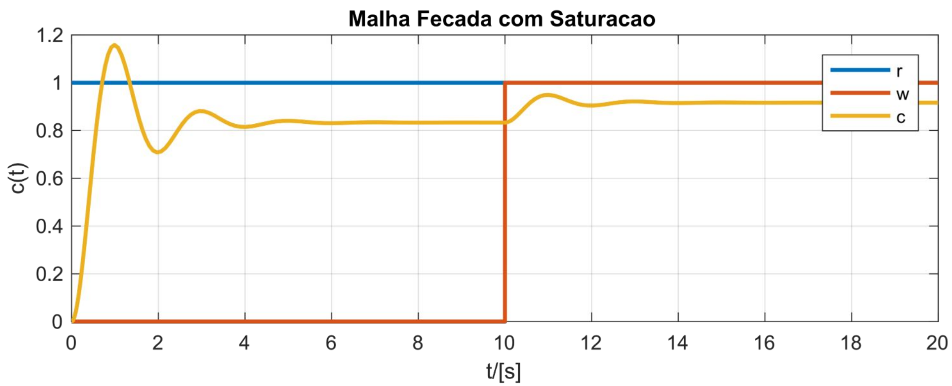
Todo processo físico tem perturbações (modeladas/refletidas na entrada do processo)



K_p=10; Sat = 10;

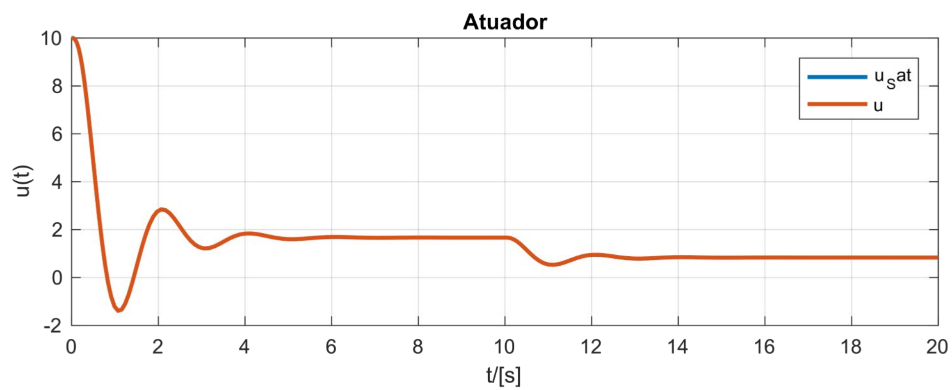
$$\frac{Y(s)}{R(s)} = \frac{\dfrac{K_p}{(s+1)(s+2)}}{1+\dfrac{K_p}{(s+1)(s+2)}\dfrac{10}{(s+10)}}$$

$$\frac{Y(s)}{R(s)} = \frac{K_p(s+10)}{(s+1)(s+2)(s+10)+10K_p}$$



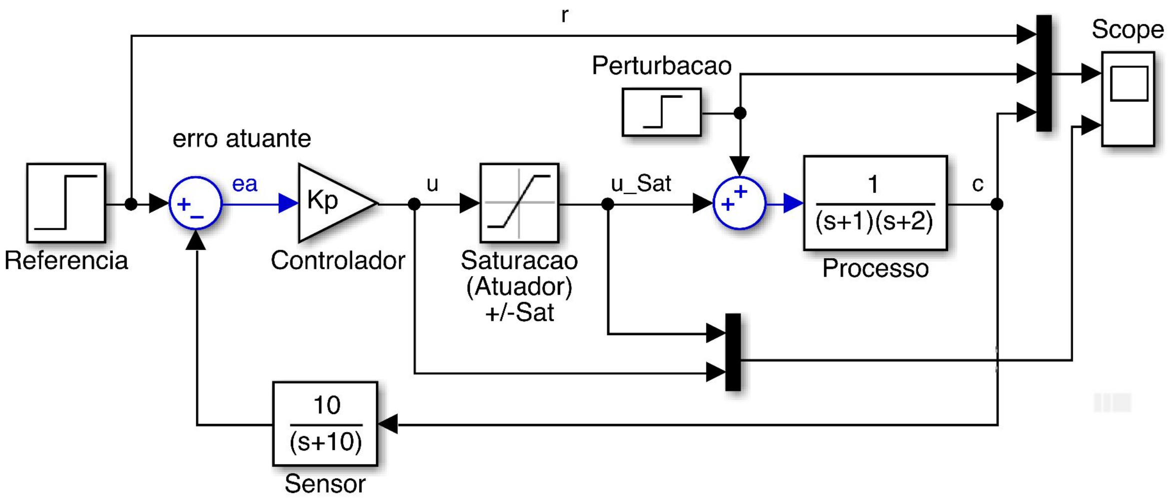
$$\frac{Y\,(s)}{W(s)} = \frac{\dfrac{1}{(s+1)(s+2)}}{1+\dfrac{K_p}{(s+1)(s+2)}\dfrac{10}{(s+10)}}$$

$$\frac{Y(s)}{W(s)} = G_w(s) = \frac{(s+10)}{(s+1)(s+2)(s+10)+10K_p}$$



iv) Resposta U(s)/R(s)

Todo processo físico tem saturação



$$\frac{U(s)}{R(s)} = \frac{K_p}{1 + \frac{K_p}{(s+1)(s+2)} \frac{10}{(s+10)}}$$

$$\frac{U(s)}{R(s)} = G_u(s) = \frac{K_p(s+1)(s+2)(s+10)}{(s+1)(s+2)(s+10) + 10K_p}$$

Teorema do valor inicial

$$\begin{aligned} R(s) &= 1/s \\ U(s) &= R(s)G_u(s) \\ u(0^+) &= \lim_{s\to\infty} s\frac{1}{s}\frac{K_p(s+1)(s+2)(s+10)}{(s+1)(s+2)(s+10)+10K_p} = K_p \end{aligned}$$

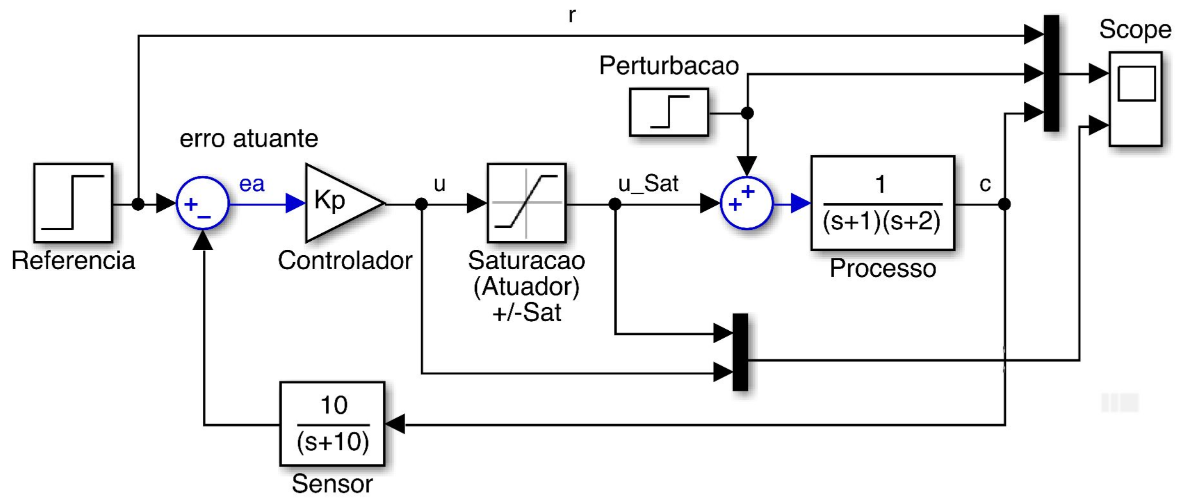
Degraus unitários produzem $u(0^+) = K_p$

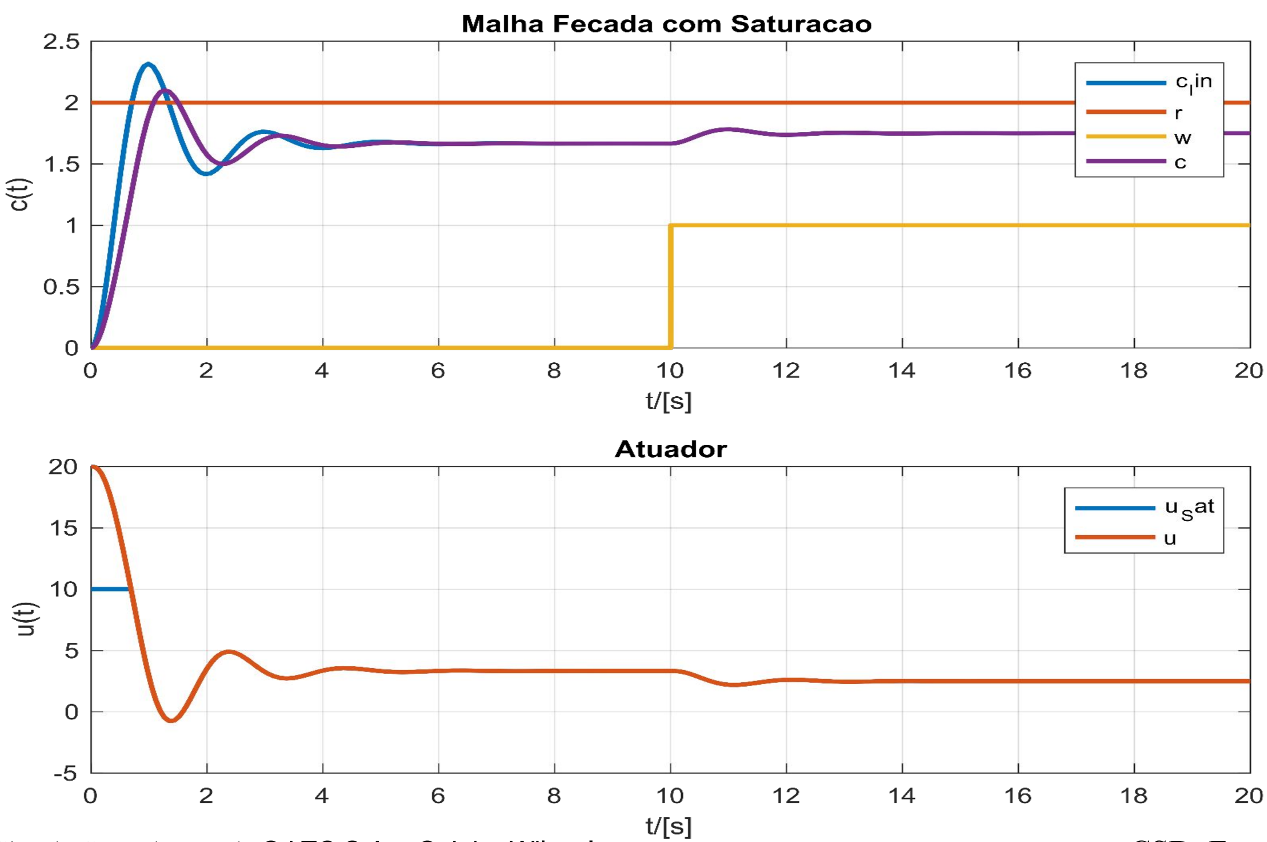
Para não haver saturação de u: $K_p <$ Sat

Se $r \leq R_{max} \Rightarrow K_p \leq \text{Sat}/R_{max}$

iv) Resposta U(s)/R(s) é importante

Todo processo físico tem saturação





Exercício Complementar 1

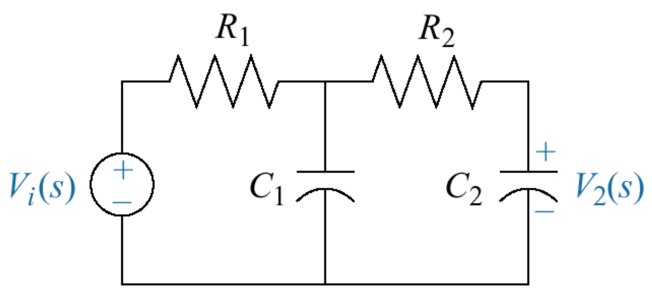
Projete um circuito com resistores e capacitores comerciais que implemente uma função de transferência de 2ª ordem, que apresente a um degrau de entrada, 16 % de sobrepasso e tempo de acomodação de 1 ms.

Quais os valores de M_p e t_s efetivamente obtidos?

Resolução

1) Escolha da topologia.

- Diversas topologias (realizações) são possíveis.
- Um sistema de 2ª ordem precisa de 2 armazenadores independentes de energia.
- Um sistema de controle (c(s) segue r(s)) é um processo passa-baixas.
- Um circuito RC, com tensão no capacitor é um passa baixa de 1ª ordem.
- Cascata de dois filtros RC implementam um passa baixas de 2ª ordem.



Exercício Complementar 1

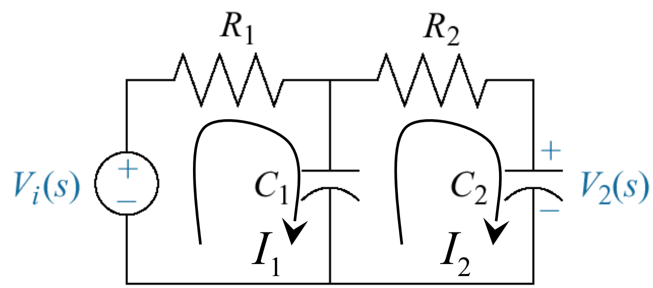
Circuito de 2ª ordem,, M_p = 16% t_s 1 ms.

Resolução...

2) Função de Transferência.

- Lei das malhas:

$$\left(R_1+\frac{1}{sC_1}\right)I_1 \quad - \quad \frac{1}{sC_1}I_2 = V_i$$
$$-\frac{1}{sC_1}I_1 \quad + \left(R_2+\frac{1}{sC_1}+\frac{1}{sC_2}\right)I_2 = 0$$



$$C_s = \frac{C_1C_2}{C_1+C_2}$$
$$V_2 = \frac{I_2}{sC_2}$$

- Regra de Cramer

$$I_2 = \frac{\begin{vmatrix} R_1+\frac{1}{sC_1} & V_i \\ -\frac{1}{sC_1} & 0 \end{vmatrix}}{\begin{vmatrix} R_1+\frac{1}{sC_1} & -\frac{1}{sC_1} \\ -\frac{1}{sC_1} & R_2+\frac{1}{sC_1}+\frac{1}{sC_2} \end{vmatrix}}$$

$$I_2(s) = \frac{\frac{V_i}{sC_1}}{R_1R_2+\frac{1}{s^2C_1C_s}+\frac{R_2}{sC_1}+\frac{R_1}{sC_s}-\frac{1}{s^2C_1^2}}$$

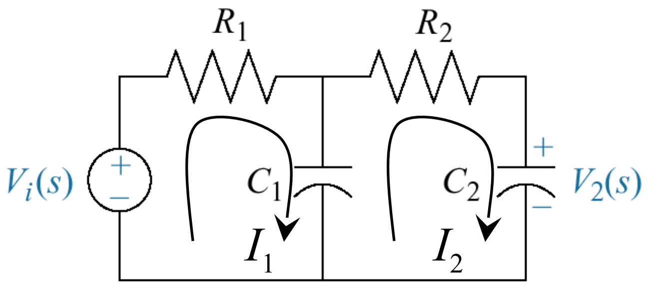
$$\frac{V_2(s)}{V_i(s)} = \frac{1}{s^2R_1R_2C_1C_2+s(R_1C_1+R_2C_2+R_1C_2)+1}$$

Exercício Complementar 1

Circuito de 2ª ordem,, M_p = 16% t_s 1 ms.

Resolução...

3) Função de Transferência na forma padrão.



$$\frac{V_2(s)}{V_i(s)} = \frac{\omega_n^2}{s^2 + 2\zeta\omega_n s + \omega_n^2}$$

$$\frac{V_2(s)}{V_i(s)} = \frac{1}{s^2 R_1 R_2 C_1 C_2 + s(R_1 C_1 + R_2 C_2 + R_1 C_2) + 1}$$

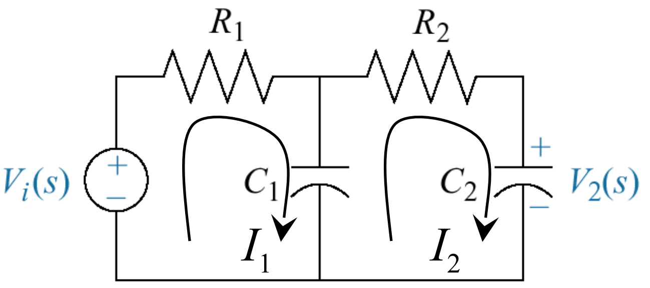
$$\frac{V_2(s)}{V_i(s)} = \frac{1/R_1 R_2 C_1 C_2}{s^2 + s(1/R_1 C_1 + 1/R_2 C_2 + 1/R_2 C_1) + 1/R_1 R_2 C_1 C_2}$$

Exercício Complementar 1

Circuito de 2ª ordem,, M_p = 16% t_s 1 ms.

Resolução...

3) Função de Transferência na forma padrão.



$$\frac{V_2(s)}{V_i(s)} = \frac{\omega_n^2}{s^2 + 2\zeta\omega_n s + \omega_n^2}$$

$$\frac{V_2(s)}{V_i(s)} = \frac{1}{s^2 R_1 R_2 C_1 C_2 + s(R_1 C_1 + R_2 C_2 + R_1 C_2) + 1}$$

$$\frac{V_2(s)}{V_i(s)} = \frac{1/R_1 R_2 C_1 C_2}{s^2 + s(1/R_1 C_1 + 1/R_2 C_2 + 1/R_2 C_1) + 1/R_1 R_2 C_1 C_2}$$

zeta=0.5; % Mp=16%

Exercício Complementar 1

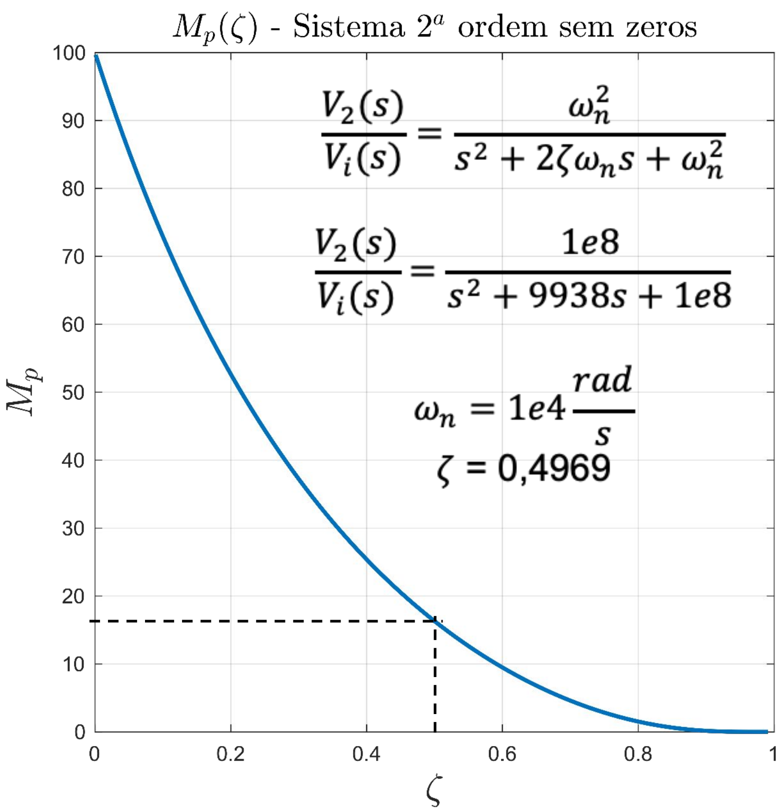
Circuito de 2ª ordem,, M_p = 16% t_s 1 ms.

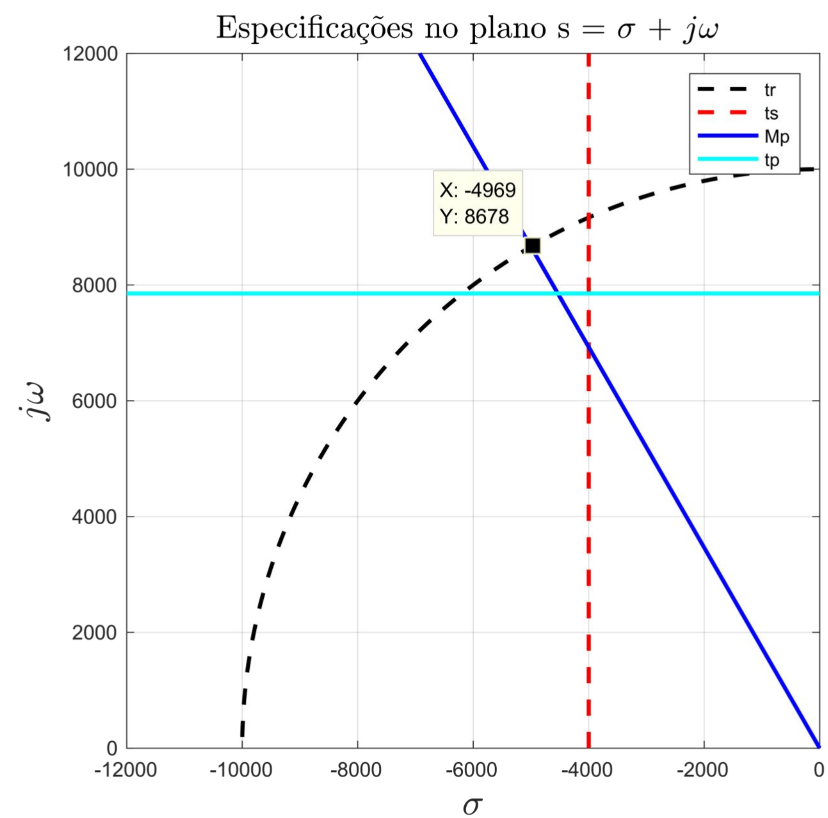
ts=1e-3 *s* ; sigma=4/ts;
tr=0.18e-3;
wn=1.8/tr
tp=.4e-3

Resolução...

s_0 = -4969 ± j8678

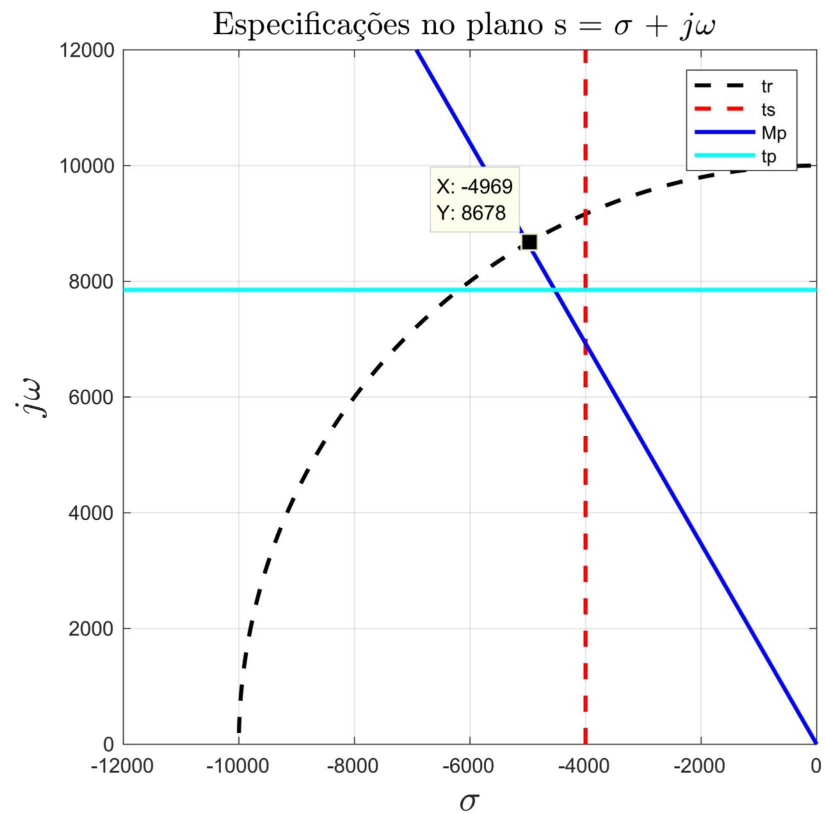
4) Especificações.





Exercício Complementar 1

Circuito de 2ª ordem,, M_p = 16% t_s = 1 ms.



Resolução...

5) Especificações.

$$\frac{V_2(s)}{V_i(s)} = \frac{\omega_n^2}{s^2 + 2\zeta\omega_n s + \omega_n^2}$$

$$\frac{V_2(s)}{V_i(s)} = \frac{1e8}{s^2 + 9938s + 1e8}$$

$$\omega_n = 1e4\frac{rad}{s}$$

$$\zeta = 0{,}4969$$

$$1/R_1R_2C_1C_2 = 1e8$$

$$1/R_1C_1 + 1/R_2C_2 + 1/R_2C_1 = \frac{R_1C_1 + R_2C_2 + R_1C_2}{R_1C_1R_2C_2} = 9938$$

$$\frac{V_2(s)}{V_i(s)} = \frac{1/R_1R_2C_1C_2}{s^2 + s(1/R_1C_1 + 1/R_2C_2 + 1/R_2C_1) + 1/R_1R_2C_1C_2}$$

Exercício Complementar 1

Circuito de 2ª ordem, M_p = 16% t_s = 1 ms.

Resolução...

$$\frac{V_2(s)}{V_i(s)} = \frac{1/R_1R_2C_1C_2}{s^2 + s(1/R_1C_1 + 1/R_2C_2 + 1/R_2C_1) + 1/R_1R_2C_1C_2}$$

4) Escolha dos Componentes.

$$\frac{V_2(s)}{V_i(s)} = \frac{\omega_n^2}{s^2 + 2\zeta\omega_n s + \omega_n^2}$$

$$\frac{V_2(s)}{V_i(s)} = \frac{1e8}{s^2 + 9938s + 1e8}$$

$$\omega_n = 1e4\frac{rad}{s}$$

$$\zeta = 0{,}4969$$

$$1/R_1R_2C_1C_2 = 1e8$$

$$1/R_1C_1 + 1/R_2C_2 + \frac{1}{R_2C_1} = \frac{R_1C_1 + R_2C_2 + R_1C_2}{R_1C_1R_2C_2} = 9938$$

Arbitrando $C_1 = C_2 = 1e\text{-}6$

$R_1R_2 = 1e4$

$(2R_1 + R_2) = 9{,}938e9$

$(2e4/R_2 + R_2) = 9{,}938e9$

$R_2^2 - 9{,}938R_2 + 2e4 = 0$

Raízes: $R_{2,1} = 2e-6$; $R_{2,2} = 9{,}938e9$;

Exercício Complementar 1

Circuito de 2ª ordem,, M_p = 16% t_s = 1 ms.

Resolução...

4) Escolha dos Componentes.

$$\frac{V_2(s)}{V_i(s)} = \frac{\omega_n^2}{s^2 + 2\zeta\omega_n s + \omega_n^2}$$

$$\frac{V_2(s)}{V_i(s)} = \frac{1e8}{s^2 + 9938s + 1e8}$$

$R_1R_2C_1C_2$=1e-8

$R_1C_1 + R_2C_2 + R_1C_2 = 9938e-8$

$$\frac{V_2(s)}{V_i(s)} = \frac{1/R_1R_2C_1C_2}{s^2 + s(1/R_1C_1 + 1/R_2C_2 + 1/R_2C_1) + 1/R_1R_2C_1C_2}$$

Arbitrando $C_1 = C_2 = 10e\text{-}6$

$R_1R_2C_1C_2$=1e−8 $\qquad R_1R_2 = 1e4$

$(2R_1 + R_2)$ *1e-5=9938*1e-8

$(2R_1 + R_2)$ =9,938

$(2e4/R_2 + R_2)$ =9,938

$R_2^2 - 9{,}938R_2 + 2e4 = 0$

Raízes: $R_{2,1:2} = (0{,}04969 \pm j1{,}413) * 1e2;$

Arbitrando $C_2 = 10e\text{-}6;\ R_2 = 1e3$

$R_1R_2C_1C_2$=1e−8 $\qquad R_1R_2 = 1e4$

$(2R_1 + R_2)$ *1e-5=9938*1e-8

$(2R_1 + R_2)$ =9,938

$(2e4/R_2 + R_2)$ =9,938

$R_2^2 - 9{,}938R_2 + 2e4 = 0$

Raízes: $R_{2,1:2} = (0{,}04969 \pm j1{,}413) * 1e2;$

Exercício Complementar 1

Circuito de 2ª ordem,, M_p = 16% t_s = 1 ms.

Resolução...

4) Escolha dos Componentes.

$$\frac{V_2(s)}{V_i(s)} = \frac{\omega_n^2}{s^2 + 2\zeta\omega_n s + \omega_n^2}$$

$$\frac{V_2(s)}{V_i(s)} = \frac{1e8}{s^2 + 9938s + 1e8}$$

$$\omega_n = 1e4\,\frac{rad}{s}$$
$$\zeta = 0{,}4969$$

$R_1R_2C_1C_2$=1e-8
$1/R_1C_1 + 1/R_2C_2 + 1/R_2C_1$ =9938

$R_1C_1 + R_2C_2 + R_1C_2 = 9938e-8$

Arbitrando $C_1 = C_2$ = 1e-6
R_1R_2=1e4
$(2R_1 + R_2)$ =9938e-2
$(2e4/R_2 + R_2)$ =99,38

$R_2^2 - 99{,}38R_2 + 2e4 = 0$

Raízes: $R_2 = (0{,}4969 \pm j1.324) * 1e2$

Arbitrando $C_1 = C_2$ = 1e-9
$R_1R_2C_1C_2$=1e−8 R_1R_2=1e10

$(2R_1 + R_2)$ *1e-9=9938*1e-8
$(2R_1 + R_2)$ =99380

$(2e10/R_2 + R_2)$ =99380

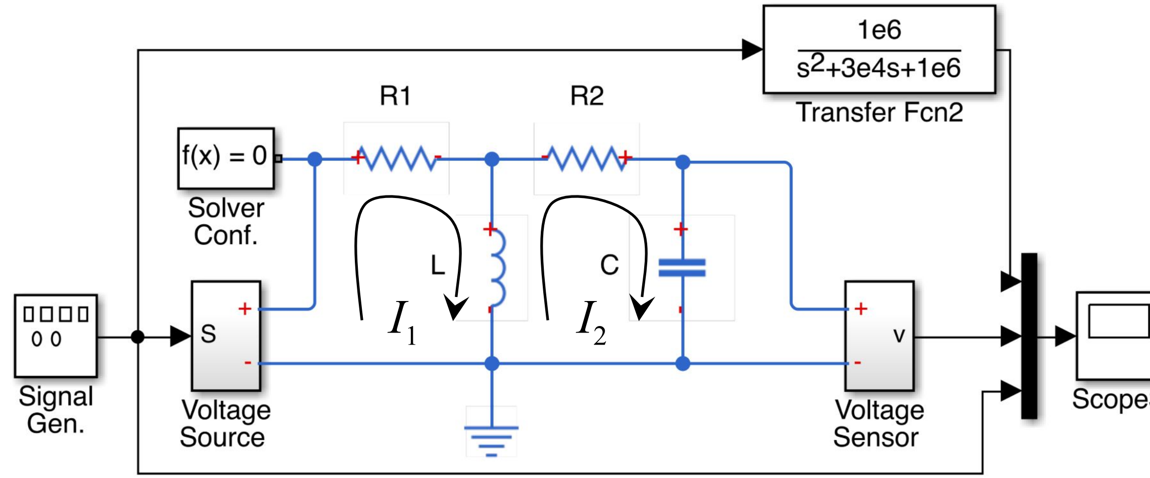
$R_2^2 - 99380R_2 + 2e10 = 0$

Raízes: $R_{2,1:2} = (0{,}4969 \pm j1{,}324\,) * 1e5;$

Arbitrando $R_1 = 1e3$ e C_1 = 1e-6
$R_1R_2C_1C_2$=1e−8 R_2C_2=1e-11

(1e-3 +1e-11+$1e3C_2$) =9938*1e-8
C_2 =-9,0062e-7

Exercício Complementar 1



M_p = 16% t_s 1 ms.

Resolução...

2) Função de Transferência.

- Lei das malhas:

$$\begin{aligned}(R_1 + sL)I_1 \quad & - sLI_2 = V_i \\ -sLI_1 \quad & + \left(R_2 + sL + \frac{1}{sC}\right) I_2 = 0\end{aligned}$$

$$I_2(s) = \frac{sLV_i}{s^2L^2 + sL(R_1 + R_2) + R_1R_2 + \frac{L}{C} + \frac{R_1}{sC} - s^2L^2}$$

- Regra de Cramer

$$I_2 = \frac{\begin{vmatrix} R_1 + sL & V_i \\ -sL & 0 \end{vmatrix}}{\begin{vmatrix} R_1 + sL & -sL \\ -sL & R_2 + sL + \frac{1}{sC} \end{vmatrix}}$$

$$V_2(s) = \frac{\frac{L}{C} V_i}{sL(R_1 + R_2) + R_1R_2 + \frac{L}{C} + \frac{R_1}{sC}}$$

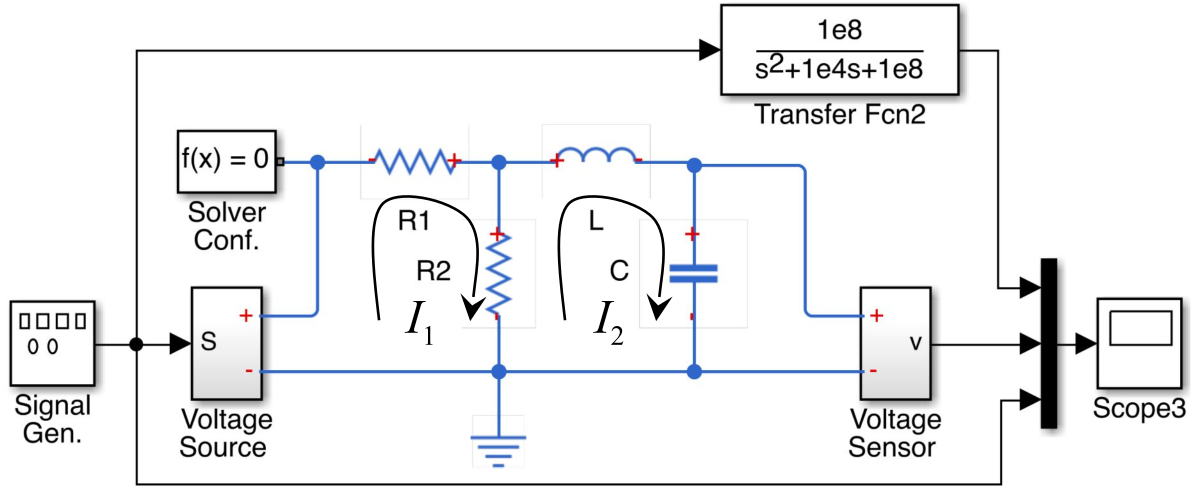
$$\frac{V_2(s)}{V_1(s)} = \frac{s\frac{L}{C}}{s^2L(R_1 + R_2) + s(R_1R_2 + \frac{L}{C}) + \frac{R_1}{C}}$$

$$V_2 = \frac{I_2}{sC}$$

Exercício Complementar 1

M_p = 16% t_s 1 ms.

Resolução...



2) Função de Transferência.

- Lei das malhas:

$$(R_1 + R_2)I_1 \qquad - R_2 I_2 = V_i$$
$$-R_2 I_1 \ + \left(R_2 + sL + \frac{1}{sC}\right) I_2 = 0$$

- Regra de Cramer

$$I_2 = \frac{\begin{vmatrix} R_1 + R_2 & V_i \\ -R_2 & 0 \end{vmatrix}}{\begin{vmatrix} R_1 + R_2 & -sL \\ -R_2 & R_2 + sL + \frac{1}{sC} \end{vmatrix}}$$

$$V_2 = \frac{I_2}{sC}$$

$$I_2(s) = \frac{R_2 V_i}{R_2(R_1 + R_2) + sL(R_1 + R_2) \ + \frac{R_1 + R_2}{sC} - sLR_2}$$

$$V_2(s) = \frac{\frac{R_2}{sC} V_i}{sLR_1 + R_1 R_2 + R_2\, R_2 + \frac{R_1 + R_2}{sC}}$$

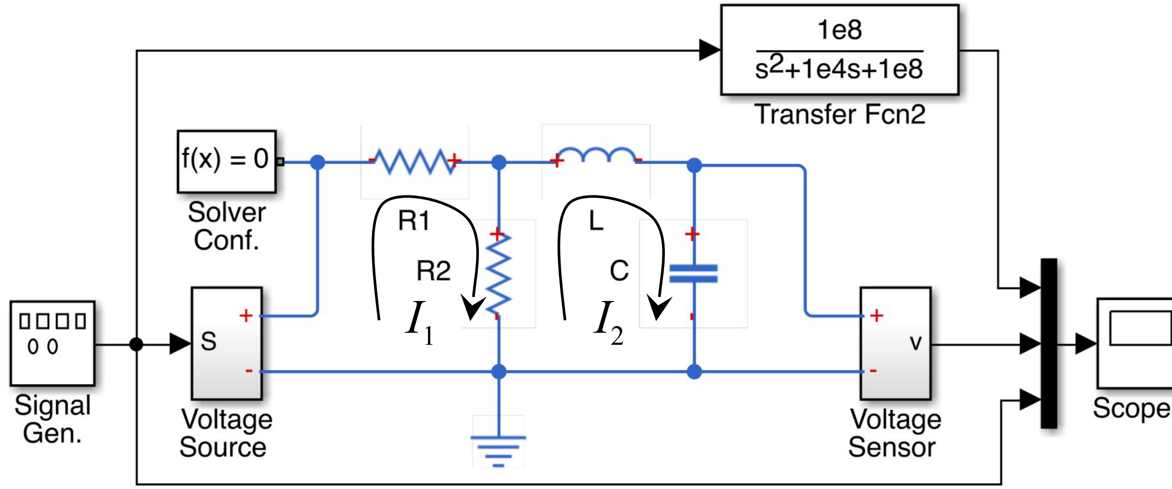
$$\frac{V_2(s)}{V_1(s)} = \frac{\frac{R_2}{C} V_i}{s^2 LR_1 + s(R_1 R_2 + R_2\, R_2) + \frac{R_1 + R_2}{C}}$$

$$\frac{V_2(s)}{V_1(s)} = \frac{\frac{R_2}{CLR_1} V_i}{s^2 + s(R_1 R_2 + R_2\, R_2)/LR_1 + \frac{R_1 + R_2}{CLR_1}}$$

Exercício Complementar 1

M_p = 16% t_s 1 ms.

Resolução...



2) Função de Transferência.

- Lei das malhas:

$$\frac{V_2(s)}{V_1(s)} = \frac{\frac{R_2}{CLR_1} V_i}{s^2 + s(R_1R_2 + R_2\,R_2)/LR_1 + \frac{R_1 + R_2}{CLR_1}}$$

$$\frac{V_2(s)}{V_i(s)} = \frac{A\omega_n^2}{s^2 + 2\zeta\omega_n s + \omega_n^2}$$

$$\frac{V_2(s)}{V_i(s)} = \frac{1e8}{s^2 + 9938s + 1e8}$$

$\frac{R_1+R_2}{CLR_1}$=1e8

$(R_1R_2 + R_2\,R_2)/LR_1 \;= 9938$

Arbitrando C = 1e-4; L = 1e-2

$\frac{R_1+R_2}{R_1}$=1e2 $\quad (R_1R_2 + R_2\,R_2)/R_1 \;= 99{,}38$

$\frac{R_2}{R_1}$=99 $\quad R_2 + R_2\;99 \;= 99{,}38$

100R2= 99,38

R2= 0,9938 R1 = 0,01

Arbitrando C = 1e-3; L = 1e-3

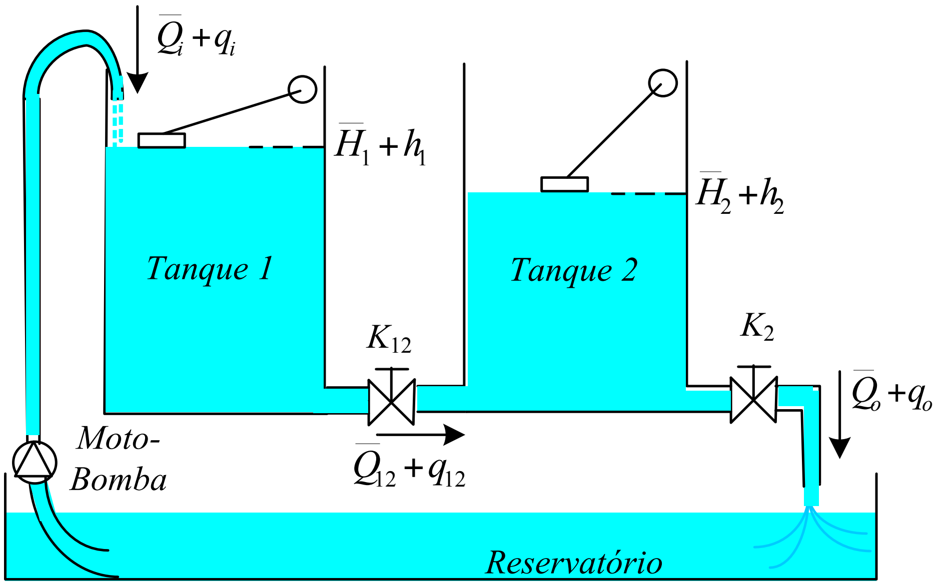
$\frac{R_1+R_2}{R_1}$=1e2 $\quad (R_1R_2 + R_2\,R_2)/R_1 \;= 9{,}938$

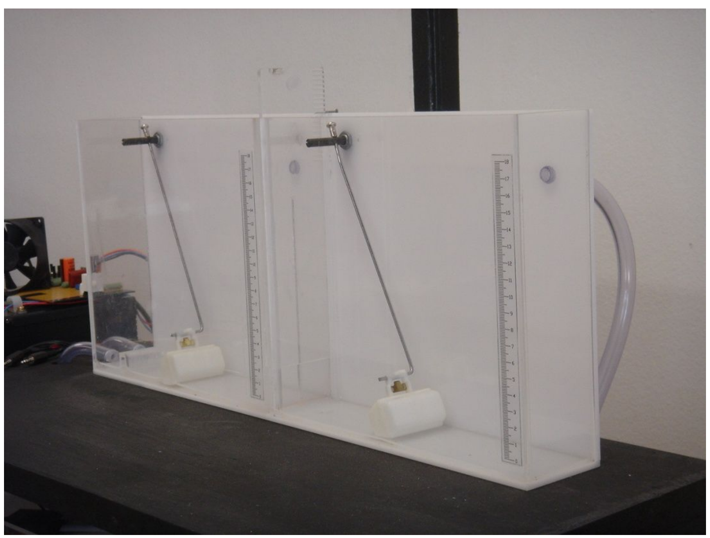
$\frac{R_2}{R_1}$=99 $\quad R_2 + R_2\;99 \;= 9{,}938$

R2= 0,09938

R1 = 0,0009947

SISTEMAS DE NÍVEL DE LÍQUIDO





Balanço de Massa p/ Fluxo Turbulento

Equações Dif. Não Lineares ($Q = K\sqrt{H}$)

$$A\frac{dH_1}{dt} = Q_i - K_{12}\sqrt{H_1 - H_2}$$

$$A\frac{dH_2}{dt} = K_{12}\sqrt{H_1 - H_2} - K_2\sqrt{H_2}$$

$Q_i = \bar{Q}_i + q_i$; $Q_{12} = \bar{Q}_{12} + q_{12}$; $Q_o = \bar{Q}_o + q_o$;
$H_1 = \bar{H}_1 + h_1$; $H_2 = \bar{H}_2 + h_2$;

Ponto de operação: $\bar{Q}_i = \bar{Q}_{12} = \bar{Q}_o = \bar{Q}$
Ponto de operação: $\bar{H}_1$; $\bar{H}_2$

"X" − grande sinal, "x" − pequeno sinal

$$\bar{Q} = K_{12}\sqrt{\bar{H}_1 - \bar{H}_2} = K_2\sqrt{\bar{H}_2}$$

SISTEMAS DE NÍVEL DE LÍQUIDO

$$A\frac{dh_1}{dt} = q_i - ah_1 + ah_2$$

$$A\frac{dh_2}{dt} = ah_1 - (a+b)h_2$$

$$a = \frac{K_{12}}{2\sqrt{\overline{H}_1 - \overline{H}_2}}; b = \frac{K_2}{2\sqrt{\overline{H}_2}}$$

$$AsH_1(s) = Q_i(s) - aH_1(s) + aH_2(s)$$

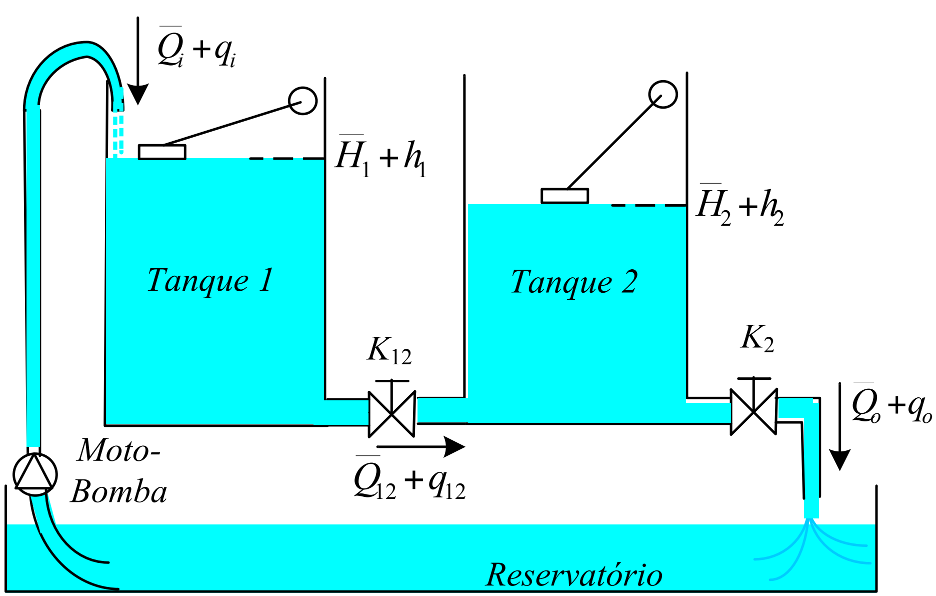
$$AsH_2(s) = aH_1(s) - (a+b)H_2(s)$$

$$(As + a)H_1(s) - aH_2(s) = Q_i(s)$$

$$- aH_1(s) + (As + a + b)H_2(s) = 0$$

$$H_2(s) = \frac{\begin{vmatrix} As+a & Q_i(s) \\ -a & 0 \end{vmatrix}}{\begin{vmatrix} As+a & -a \\ -a & As+a+b \end{vmatrix}}$$

$$\frac{H_{2(s)}}{Q_i(s)} = \frac{a}{A^2s^2 + As(2a+b) + ab}$$



Modelo Linearizado
(para pequena excursão em torno
de um Ponto de Operação)

SISTEMAS DE NÍVEL DE LÍQUIDO

$$A\frac{dh_1}{dt} = q_i - ah_1 + ah_2$$

$$A\frac{dh_2}{dt} = ah_1 - (a+b)h_2$$

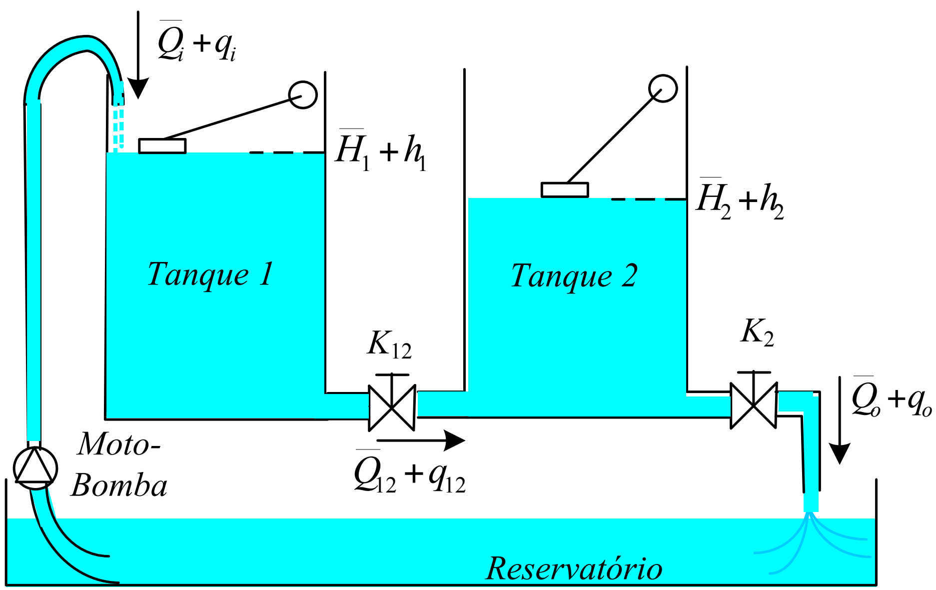
$$a = \frac{K_{12}}{2\sqrt{\overline{H}_1 - \overline{H}_2}}; b = \frac{K_2}{2\sqrt{\overline{H}_2}}$$

$$AsH_1(s) = Q_i(s) - aH_1(s) + aH_2(s)$$

$$AsH_2(s) = aH_1(s) - (a+b)H_2(s)$$

$$(As + a)H_1(s) - aH_2(s) = Q_i(s)$$

$$-aH_1(s) + (As + a + b)H_2(s) = 0$$



$$H_1(s) = \frac{\begin{vmatrix} Q_i(s) & -a \\ 0 & As+a+b \end{vmatrix}}{\begin{vmatrix} As+a & -a \\ -a & As+a+b \end{vmatrix}}$$

$$\frac{H_1(s)}{Q_i(s)} = \frac{As+a+b}{A^2s^2+As(2a+b)+ab}$$

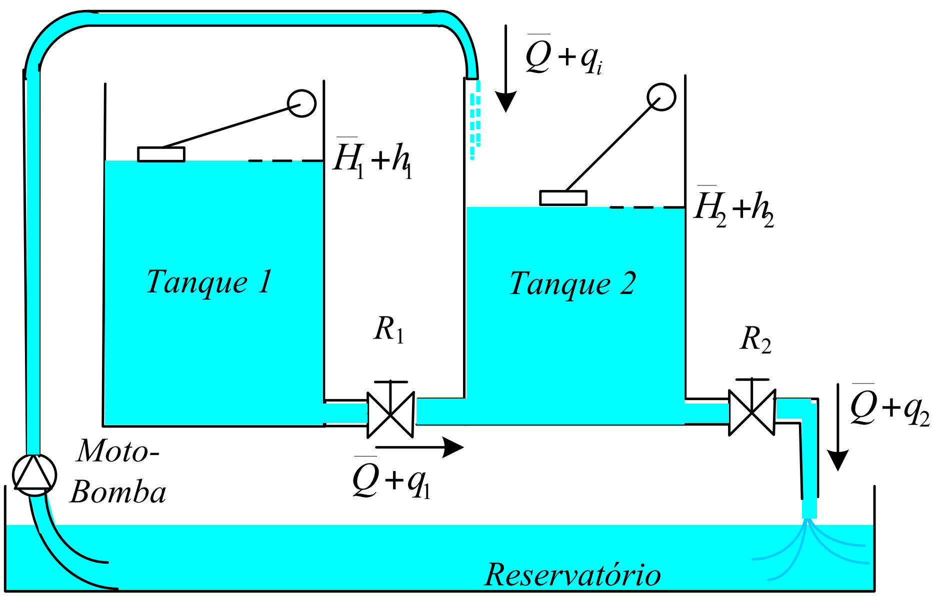
Modelo Linearizado
(para pequena excursão em torno
de um Ponto de Operação)

SISTEMAS DE NÍVEL DE LÍQUIDO

$$A\frac{dh_1}{dt} = -ah_1 + ah_2$$

$$A\frac{dh_2}{dt} = q_i + ah_1 - (a+b)h_2$$

$$a = \frac{K_{12}}{2\sqrt{\bar{\bar{H}}_1 - \bar{\bar{H}}_2}};\, b = \frac{K_2}{2\sqrt{\bar{\bar{H}}_2}}$$



$$AsH_1(s) = -aH_1(s) + aH_2(s)$$

$$AsH_2(s) = Q_i(s) + aH_1(s) - (a+b)H_2(s)$$

$$(As + a)H_1(s) - aH_2(s) = 0$$
$$- aH_1(s) + (As + a + b)H_2(s) = Q_i(s)$$

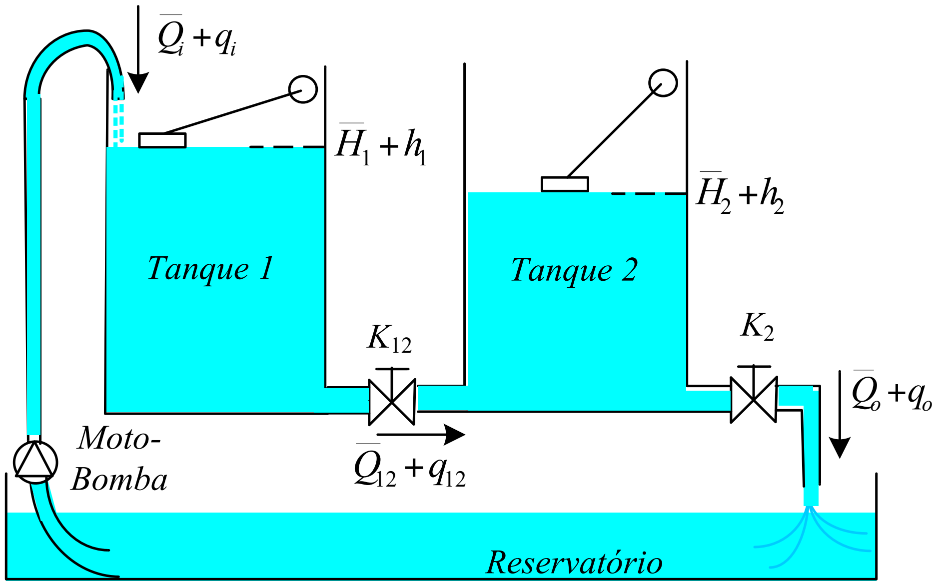
$$H_1(s) = \frac{\begin{vmatrix} 0 & -a \\ Q_i(s) & As+a+b \end{vmatrix}}{\begin{vmatrix} As+a & -a \\ -a & As+a+b \end{vmatrix}}$$

$$\frac{H_{1(s)}}{Q_i(s)} = \frac{a}{A^2s^2+As(2a+b)+ab}$$

$$H_2(s) = \frac{\begin{vmatrix} As+a & 0 \\ -a & Q_i(s) \end{vmatrix}}{\begin{vmatrix} As+a & -a \\ -a & As+a+b \end{vmatrix}}$$

$$\frac{H_{2(s)}}{Q_i(s)} = \frac{As+a}{A^2s^2+As(2a+b)+ab}$$

SISTEMAS DE NÍVEL DE LÍQUIDO



Balanço de Massa p/ Fluxo Turbulento

Equações Dif. Não Lineares ($Q = K\sqrt{H}$)

$$A_1 \frac{dH_1}{dt} = Q_i - K_{12}\sqrt{H_1 - H_2}$$

$$A_2 \frac{dH_2}{dt} = K_{12}\sqrt{H_1 - H_2} - K_2\sqrt{H_2}$$

$$q_o = Q_o - K_2\sqrt{\bar{H}_2} = \frac{K_2}{2\sqrt{\bar{H}_2}} h_2$$

$$A_1 \frac{dh_1}{dt} = q_i - ah_1 + ah_2$$

$$A_2 \frac{dh_2}{dt} = ah_1 - (a+b)h_2$$

$$q_o = bh_2$$

$$a = \frac{K_{12}}{2\sqrt{\bar{H}_1 - \bar{H}_2}};\ b = \frac{K_2}{2\sqrt{\bar{H}_2}}$$

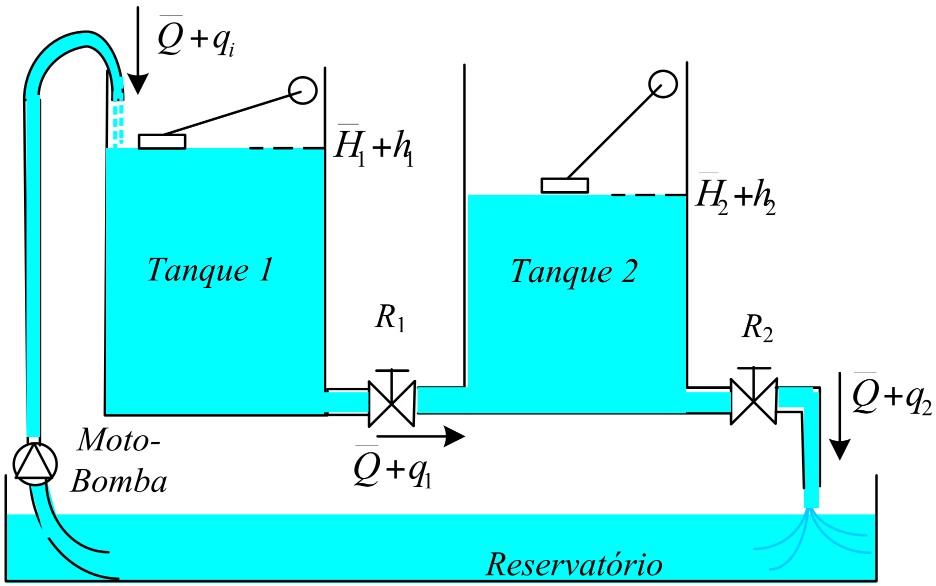
$$A_1 s H_1(s) = Q_i(s) - aH_1(s) + aH_2(s)$$
$$A_2 s H_2(s) = aH_1(s) - (a+b)H_2(s)$$

$$(A_1 s + a)H_1(s) - aH_2(s) = Q_i(s)$$
$$-aH_1(s) + (A_2 s + a + b)H_2(s) = 0$$

$$H_2(s) = \frac{\begin{vmatrix} A_1 s + a & Q_i(s) \\ -a & 0 \end{vmatrix}}{\begin{vmatrix} A_1 s + a & -a \\ -a & A_2 s + a + b \end{vmatrix}}$$

$$\frac{H_{2(s)}}{Q_i(s)} = \frac{a}{A_1 A_2 s^2 + s(A_1(a+b) + A_2 a) + a^2 + ab - a^2}$$

SISTEMAS DE NÍVEL DE LÍQUIDO



$$\frac{H_{2(s)}}{Q_i(s)} = \frac{a}{A_1A_2s^2+s(A_1(a+b)+A_2a)+a^2+ab-a^2}$$

$$\frac{Q_0(s)}{Q_i(s)} = \frac{ab}{A_1A_2s^2+s(A_1(a+b)+A_2a)+ab}$$

$$\frac{Q_0(s)}{Q_i(s)} = \frac{ab/A_1A_2}{A_1A_2s^2+s((a+b)/A_2+a/A_1)+ab/A_1A_2}$$

$$A_1\frac{dh_1}{dt} = q_i - ah_1 + ah_2$$

$$A_2\frac{dh_2}{dt} = ah_1 - (a+b)h_2$$

$$q_o = bh_2$$

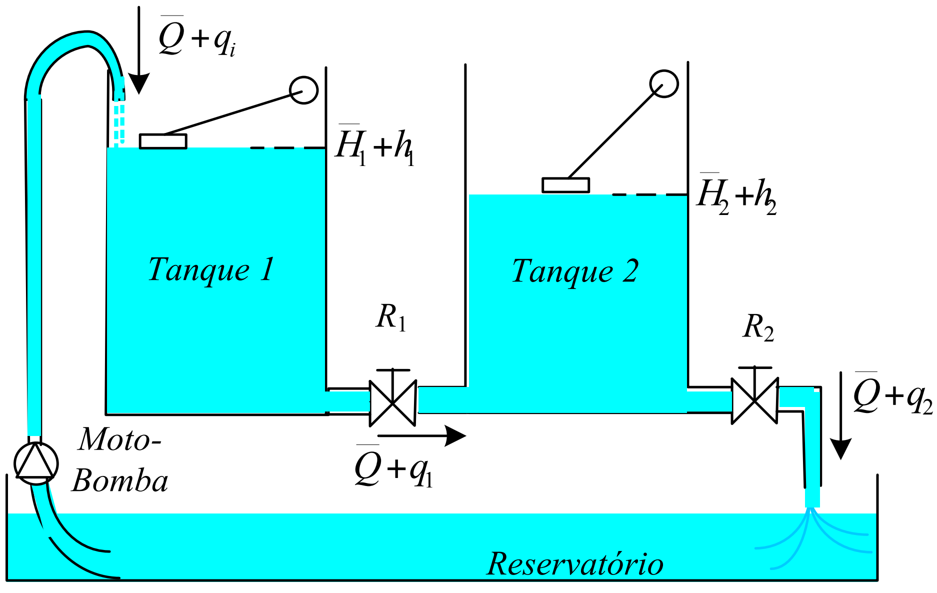
$$a = \frac{K_{12}}{2\sqrt{\overline{H}_1 - \overline{H}_2}} = \frac{1}{R_1}; b = \frac{K_2}{2\sqrt{\overline{H}_2}} = \frac{1}{R_2}; A_i = C_i;$$

$$\frac{Q_0(s)}{Q_i(s)} = \frac{\frac{1}{R_1R_2C_1C_2}}{s^2+s\left(\frac{1}{C_2R_1}+\frac{1}{C_2R_2}+\frac{1}{C_1R_1}\right)+\frac{1}{R_1R_1C_1C_2}}$$

$$\frac{Q_0(s)}{Q_i(s)} = \frac{1}{R_1R_1C_1C_2s^2+s(C_1R_2+C_1R_1+C_2R_2)+1}$$

SISTEMAS DE NÍVEL DE LÍQUIDO COM INTERAÇÃO

$$\frac{h_1 - h_2}{R_1} = q_1 \qquad \frac{h_2}{R_2} = q_2 \qquad C_1\frac{dh_1}{dt} = q - q_1 \qquad C_2\frac{dh_2}{dt} = q_1 - q_2$$



$$\frac{Q_2(s)}{Q(s)} = \frac{1}{R_1C_1R_2C_2s^2 + (R_1C_1 + R_2C_2 + R_2C_1)s + 1}$$