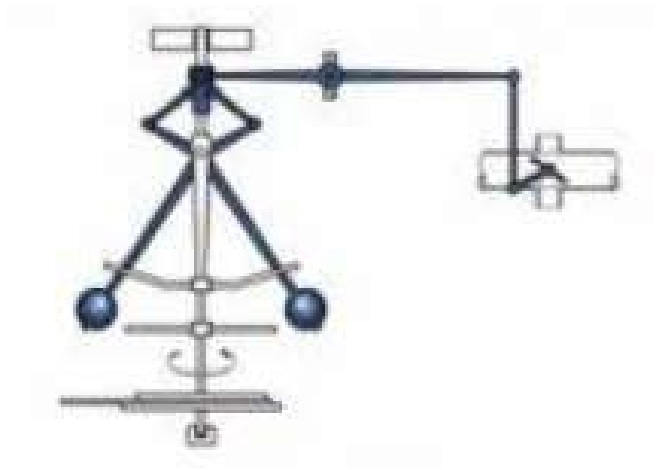
Controle de Sistemas Dinâmicos

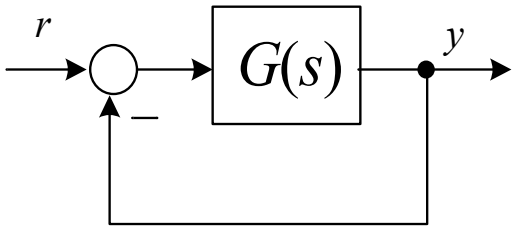
CSD11-Projeto em ω



Especificações no domínio-ω

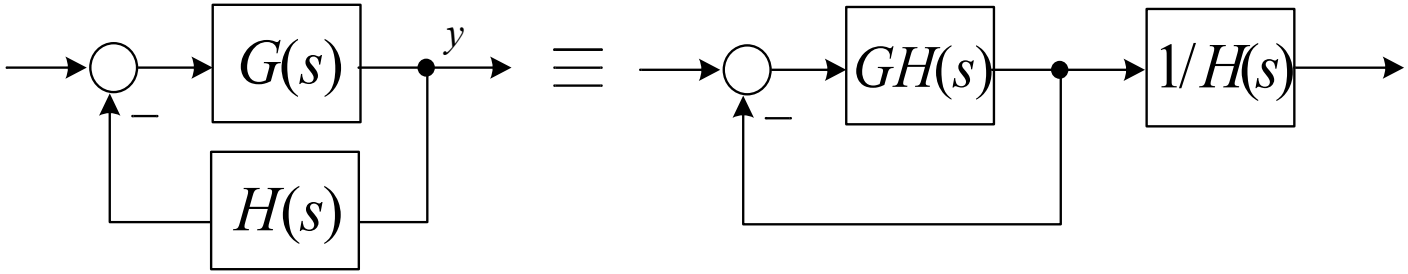
Avanço, Atraso, Avanço-Atraso, PID

Resposta em frequência de malha fechada – a partir de G(jω) em MA

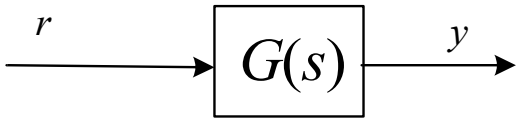


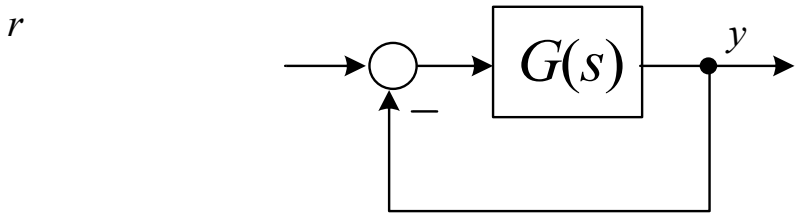
Realimentação Unitária

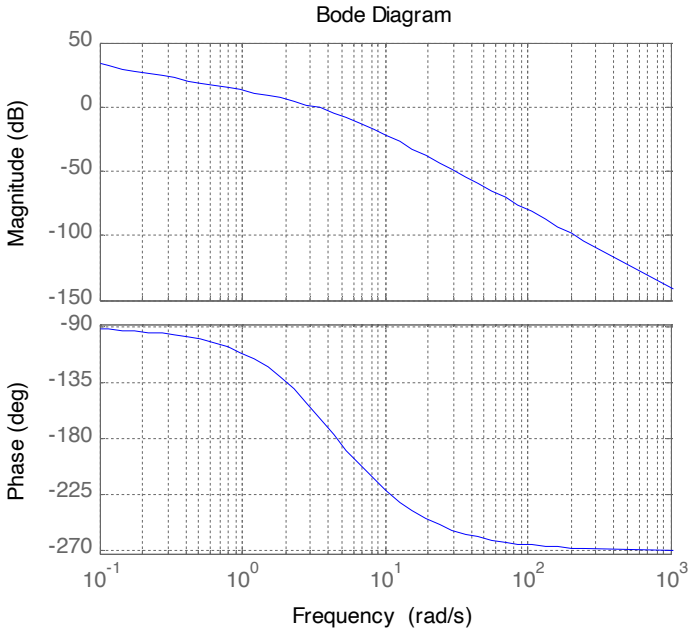
Obs:

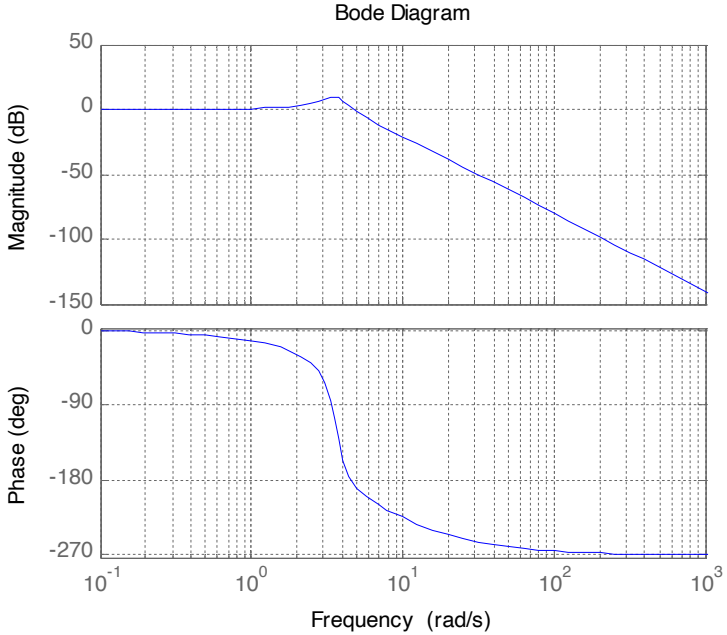


Resposta em frequência de malha fechada – a partir de G(jω) em MA





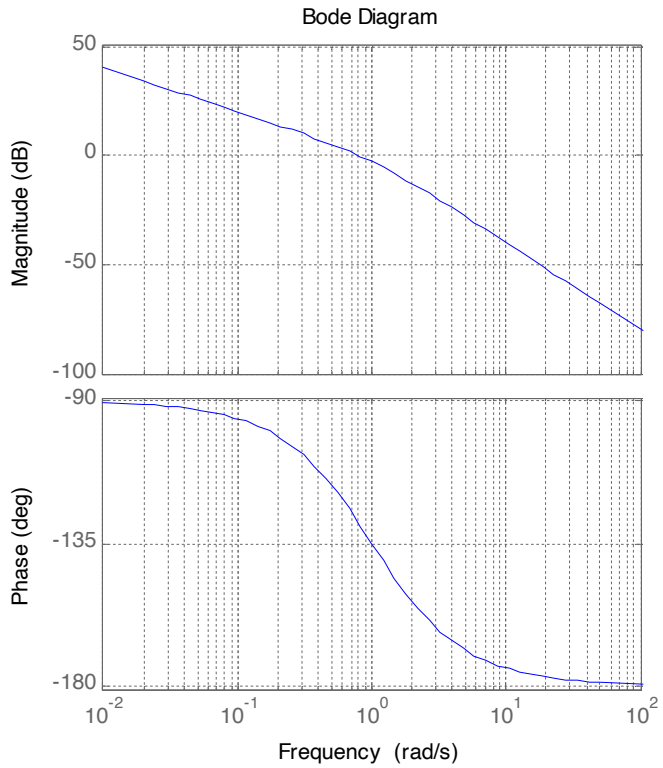


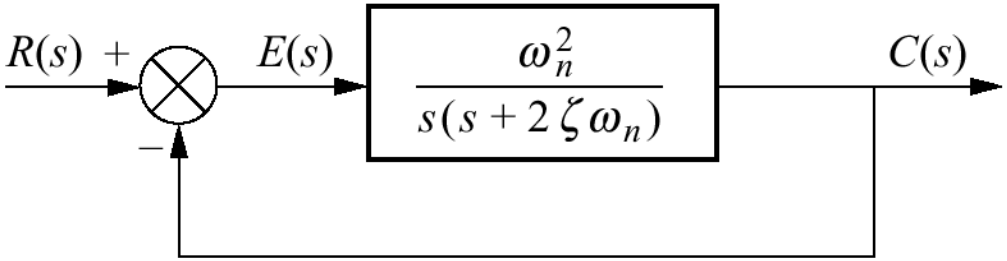


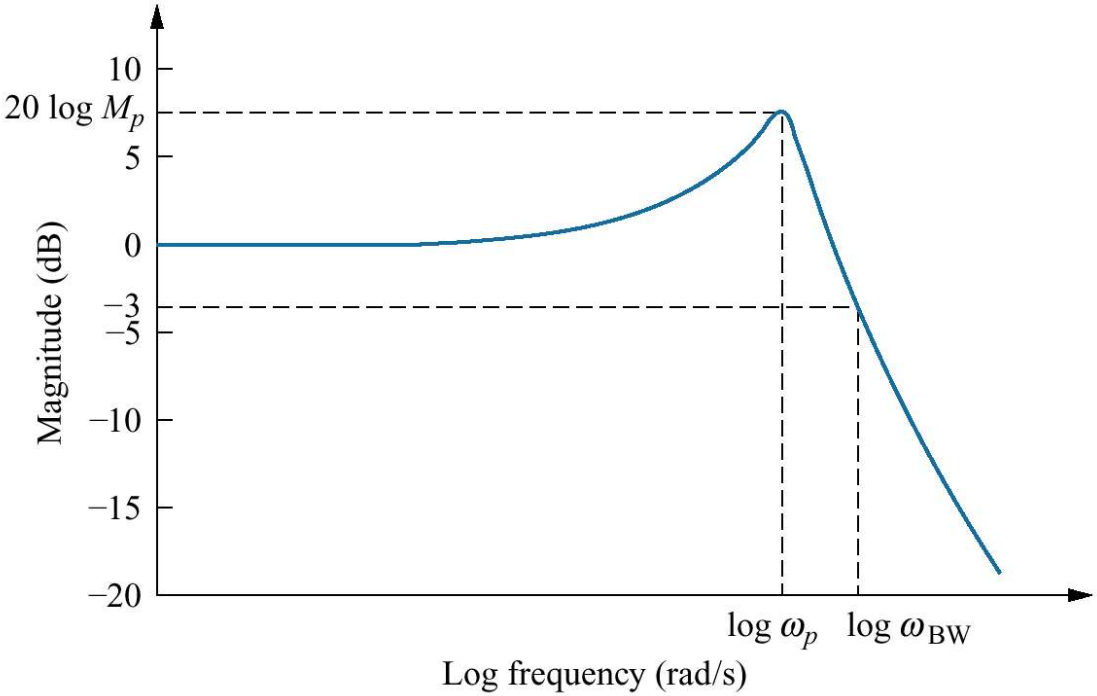
Especificações no domínio- ω

Banda Passante: -3dB

↑ ω_{BP} => ↓ tempos t_r, t_s, t_p





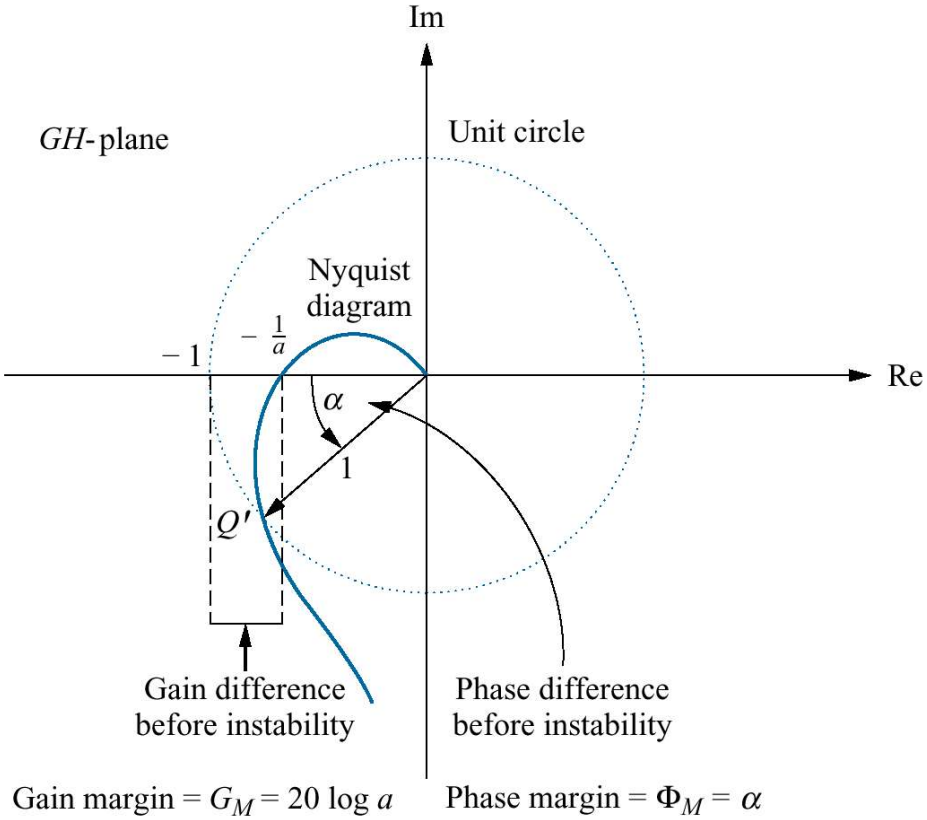


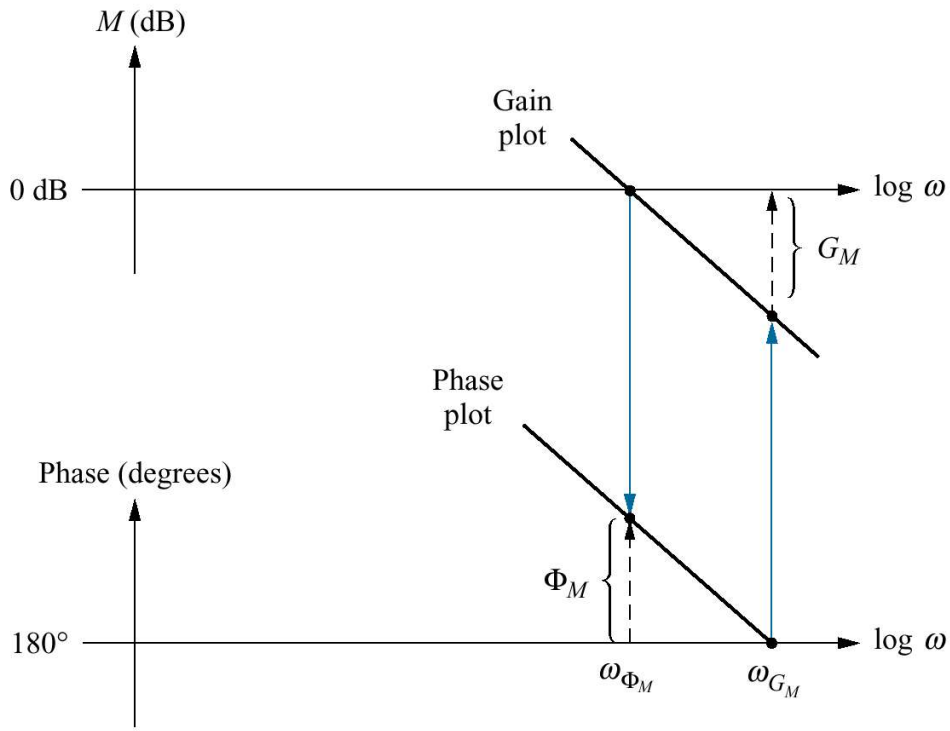
Especificações no domínio- ω

Margem de Ganho MG e Margem de Fase MF

O que "falta" para em o sistema fique instável

$$a|G(j\omega_F)|=1 \qquad \underline{/G(j\omega_G)} + \Phi_M$$

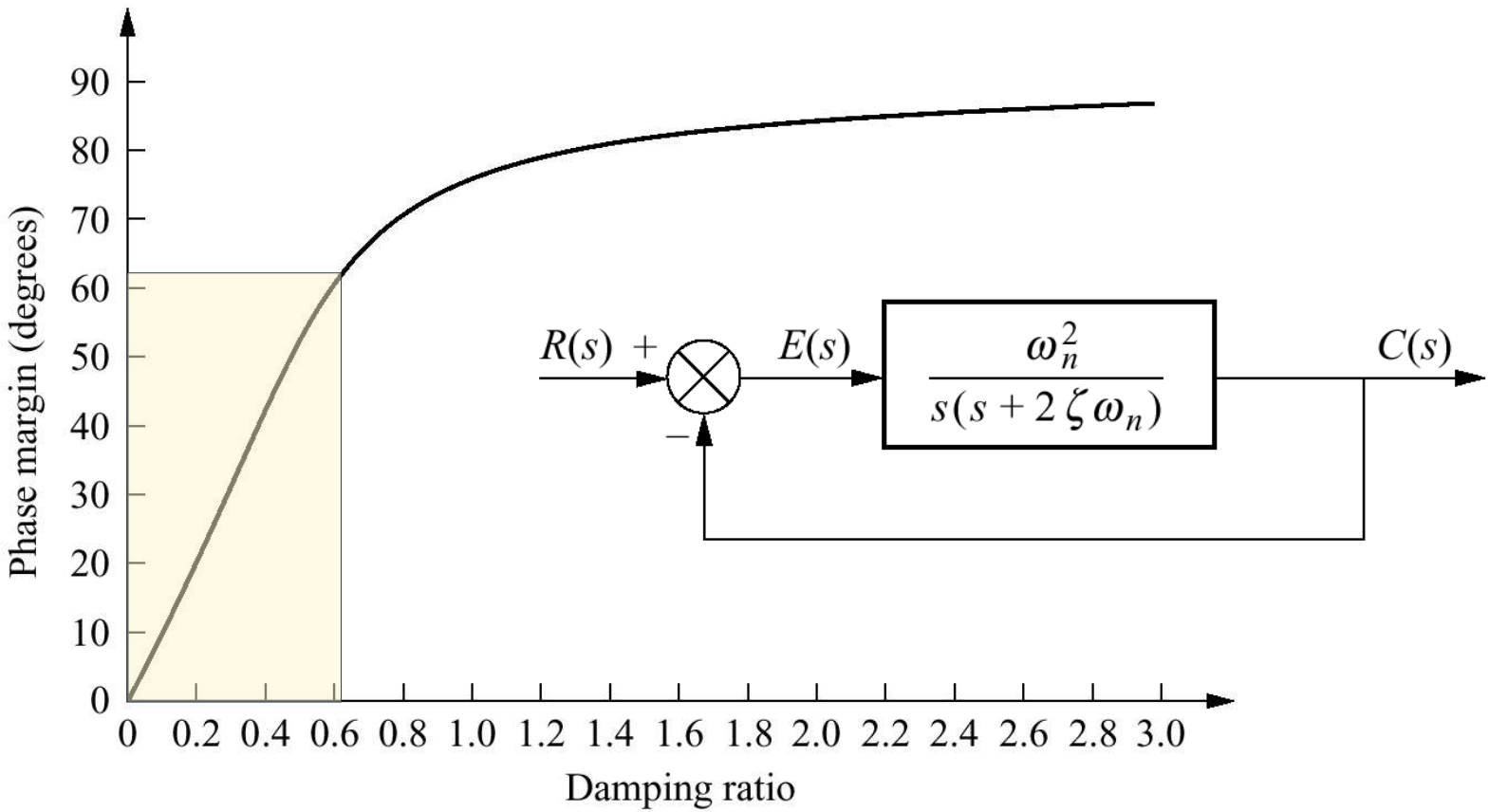




Especificações no domínio- ω

Margem de Fase x Fator de amortecimento

$$MF \sim 100\zeta$$

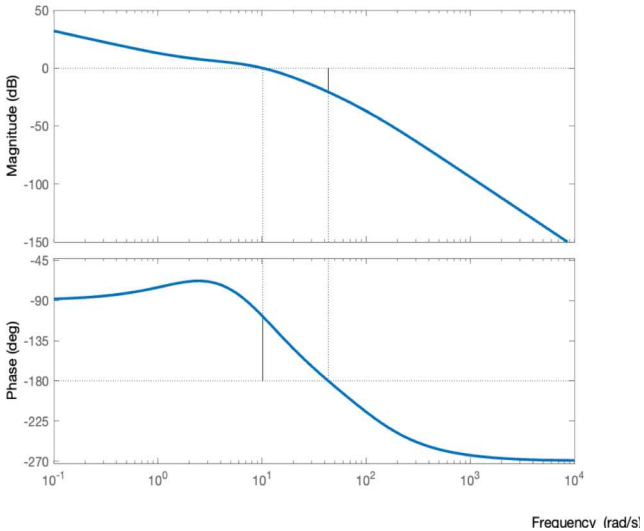


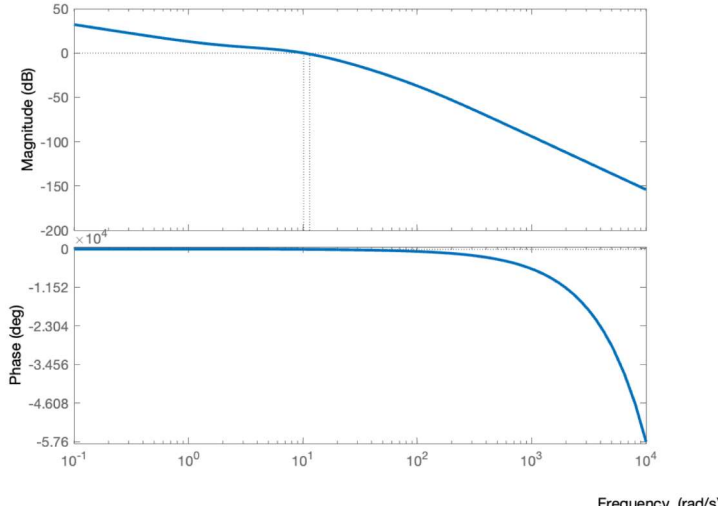
Sistemas com atraso de transporte no domínio- ω

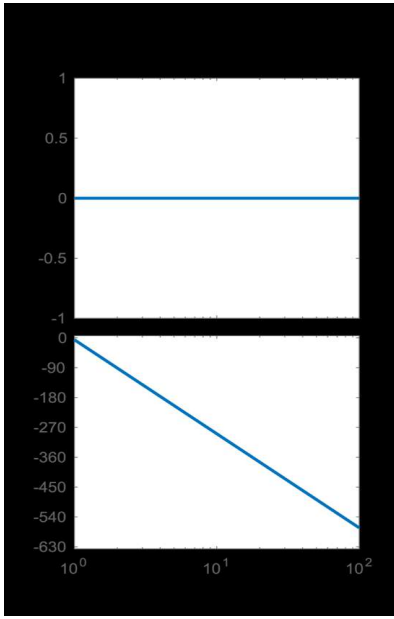
g1=zpk([-2],[0 -10 -10 -100],20000);

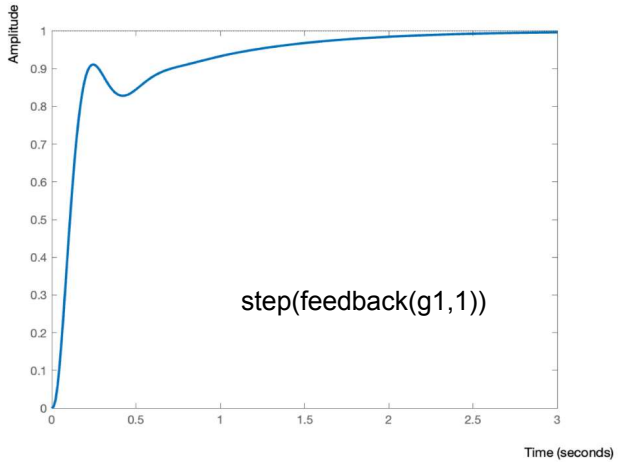
g2=g1; g2.InputDelay=.1

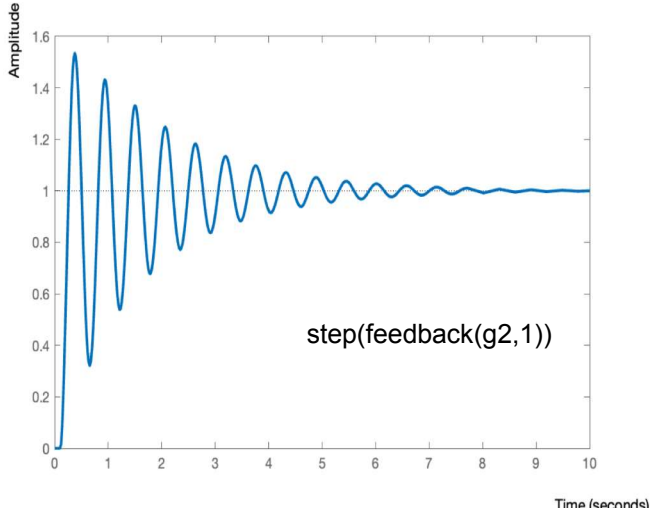
$G_3(s)e^{-0.1s}$











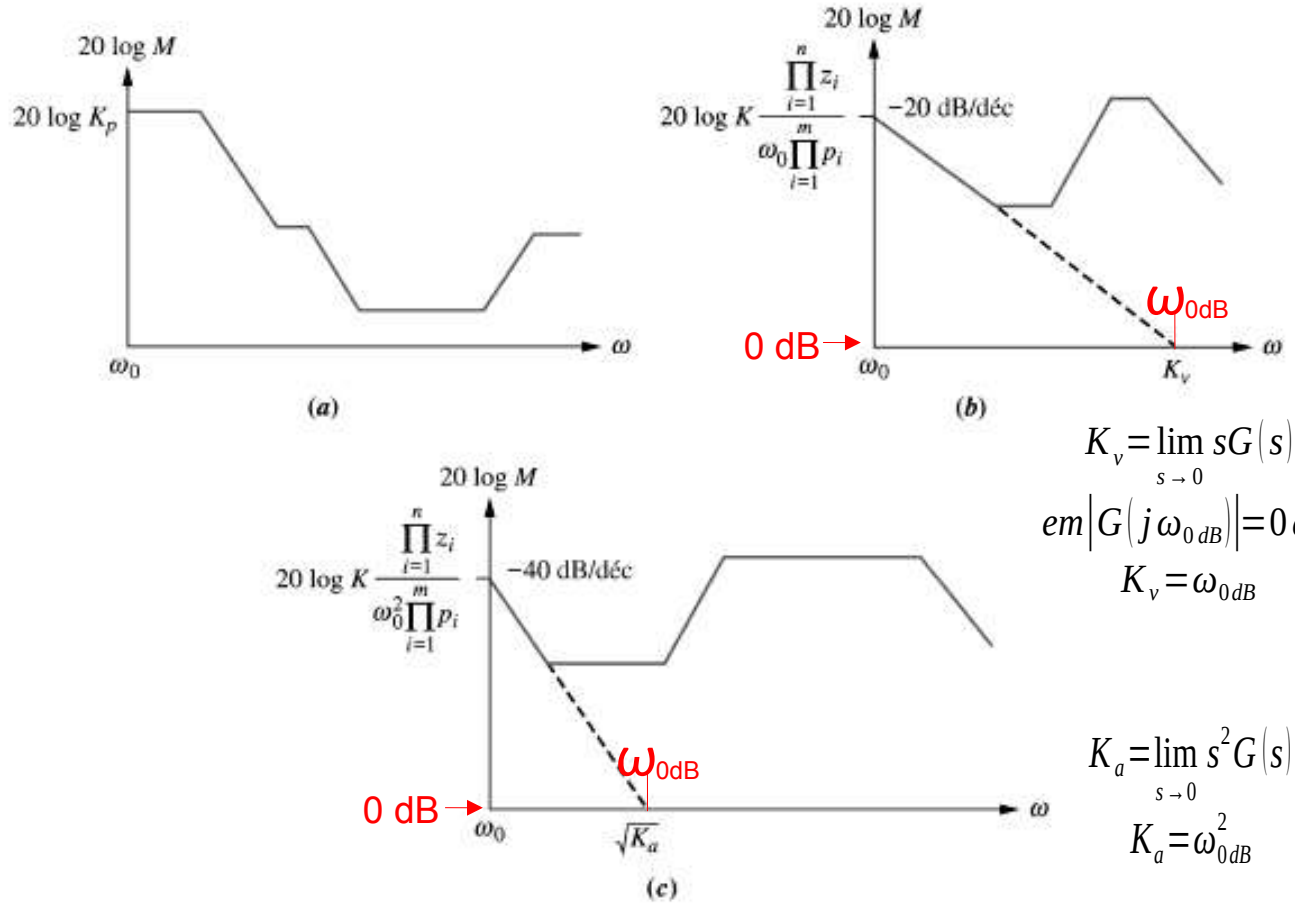
Especificações no domínio- ω

Erro em Regime Permanentes: K_p, K_v, K_a

Método 1 (Nise) encontrar ω_{0dB}

$$a.\,Tipo\,0: Kp = \lim_{s\to 0} G(s)$$
$$b.\,Tipo\,1: Kv = \lim_{s\to 0} sG(s)$$
$$c.\,Tipo\,2: Ka = \lim_{s\to 0} s^2 G(s)$$

¿



$$K_v = \lim_{s\to 0} sG(s)$$
$$em\left|G(j\omega_{0\,dB})\right| = 0\,dB$$
$$K_v = \omega_{0\,dB}$$

$$K_a = \lim_{s\to 0} s^2 G(s)$$
$$K_a = \omega_{0\,dB}^2$$

Especificações no domínio- ω

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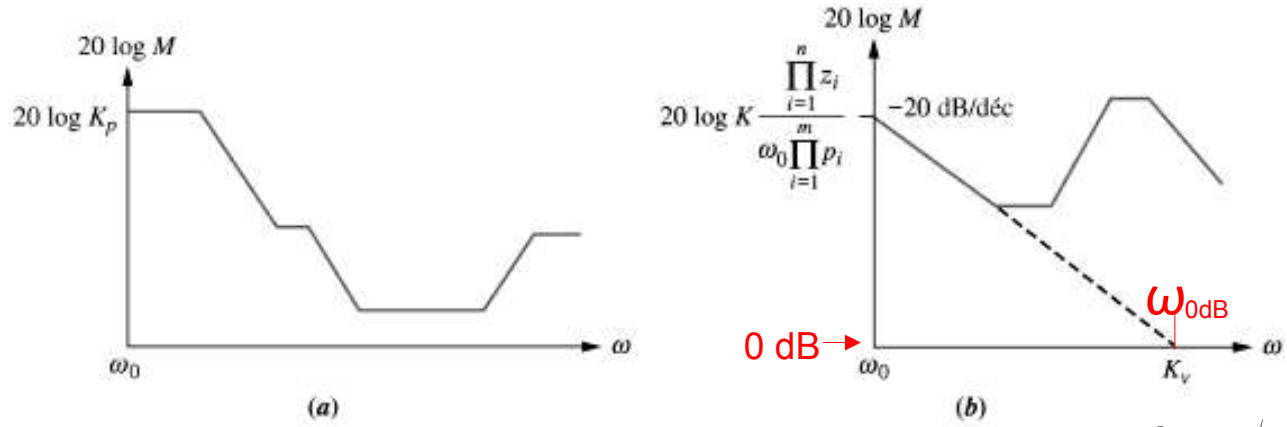

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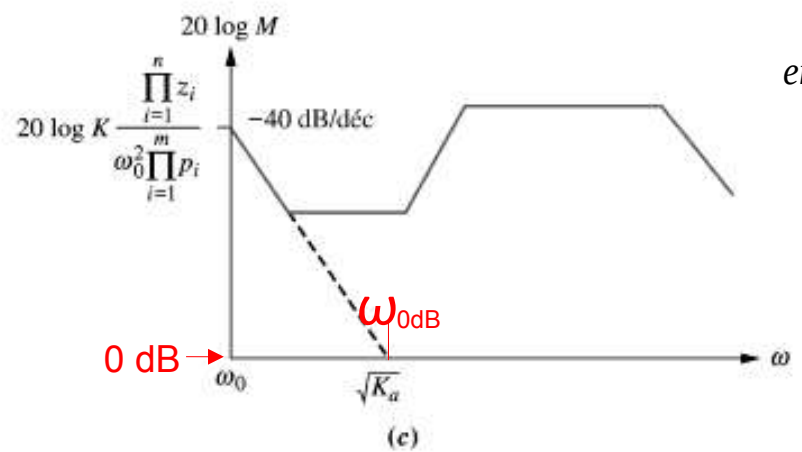


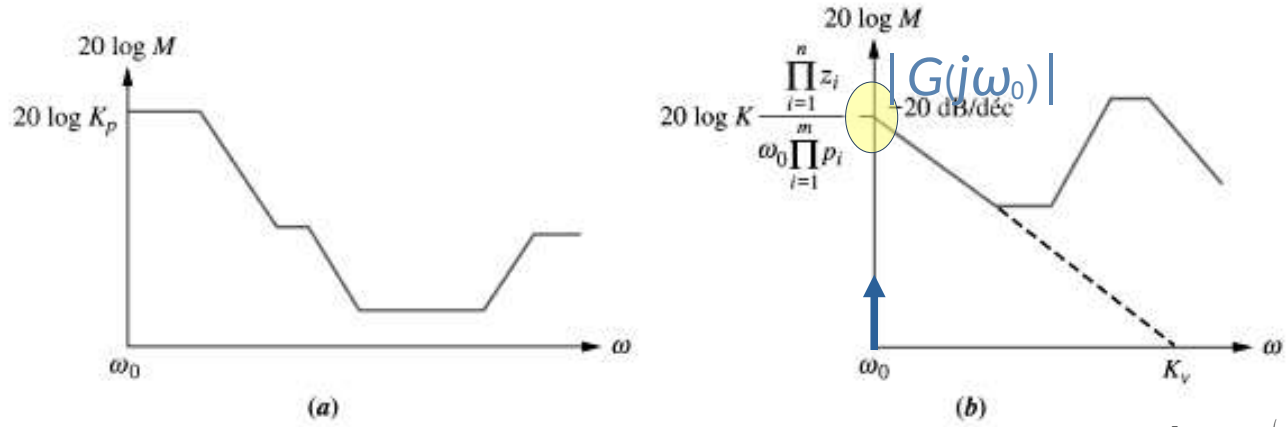
$$K_a = \lim_{s\to 0} s^2 G(s)$$
$$K_a = \omega_{0\,dB}^2$$

Especificações no domínio- ω

Erro em Regime Permanentes: K_p, K_v, K_a

Método 2 => encontrar $|G(j\omega_0)|$

onde ω_0 é a menor freq. disponível no diagrama de Bode

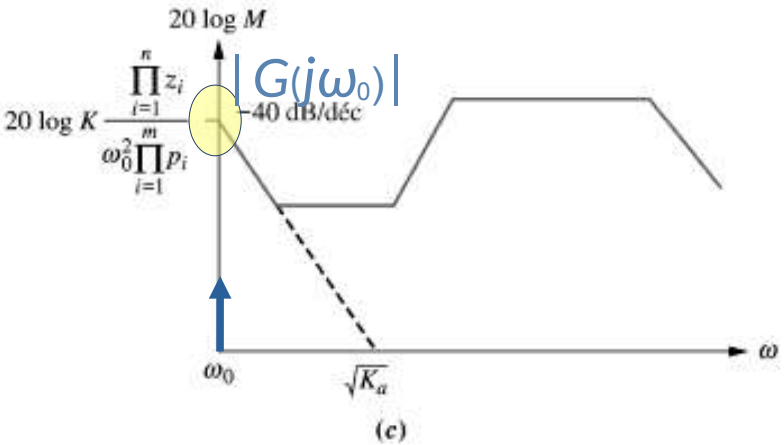


$$a.\,Tipo\,0: Kp = \lim_{s \to 0} G(j\omega_0)$$
$$b.\,Tipo\,1: Kv = \lim_{s \to 0} j\omega_{0G}(j\omega_0)$$
$$c.\,Tipo\,2: Ka = \lim_{s \to 0} j\omega_0^2 G(j\omega_0)$$
¿

$$K_v = \lim_{s \to 0} sG(s)$$

$$em\,|G(j\omega_0)|$$

$$\boxed{K_v = \omega_0 |G(j\omega_0)|}$$



$$K_a = \lim_{s \to 0} s^2 G(s)$$

$$\boxed{K_a = \omega_0^2 |G(j\omega_0)|}$$

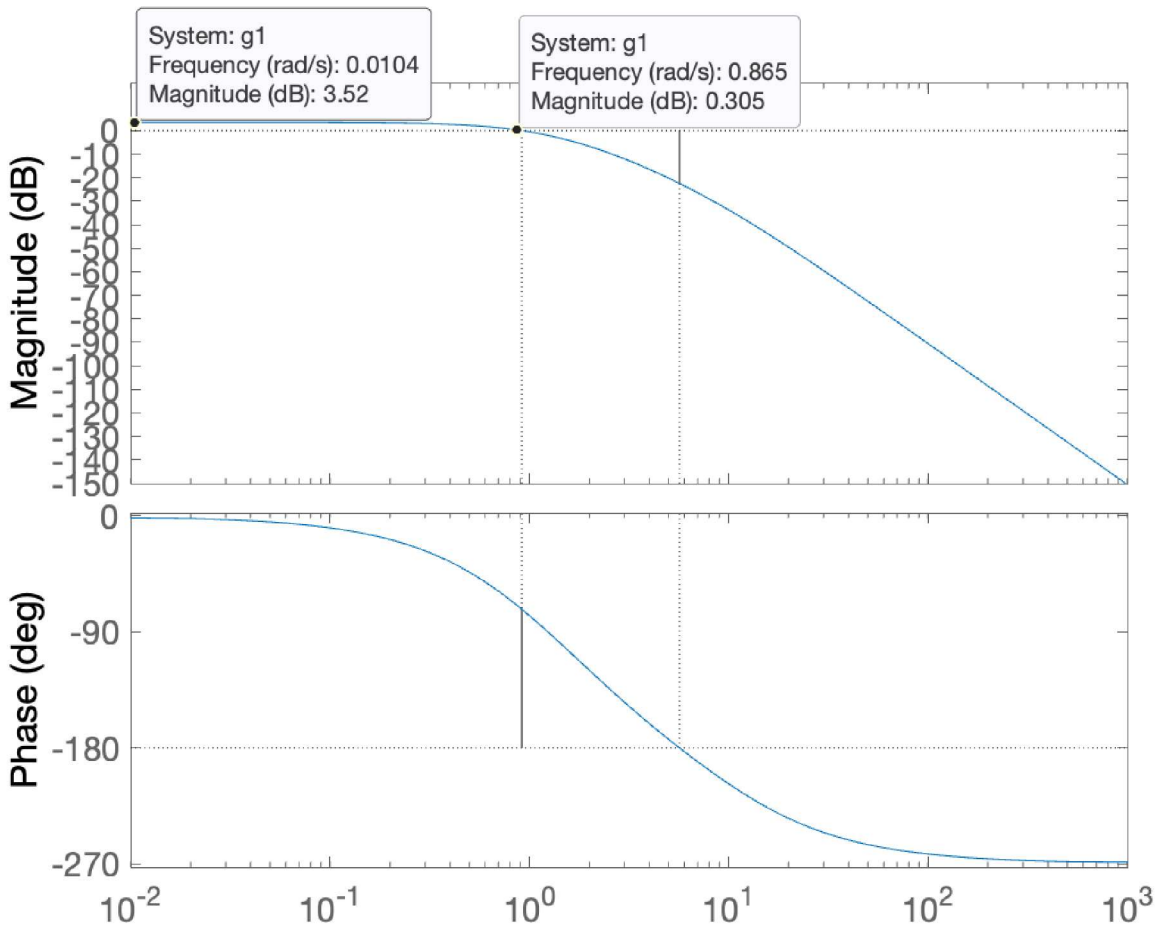
Especificações no domínio- ω

Erro em Regime Permanentes: K_p

$$Tipo\,0: Kp = \lim_{s \to 0} G(j\omega_0)$$

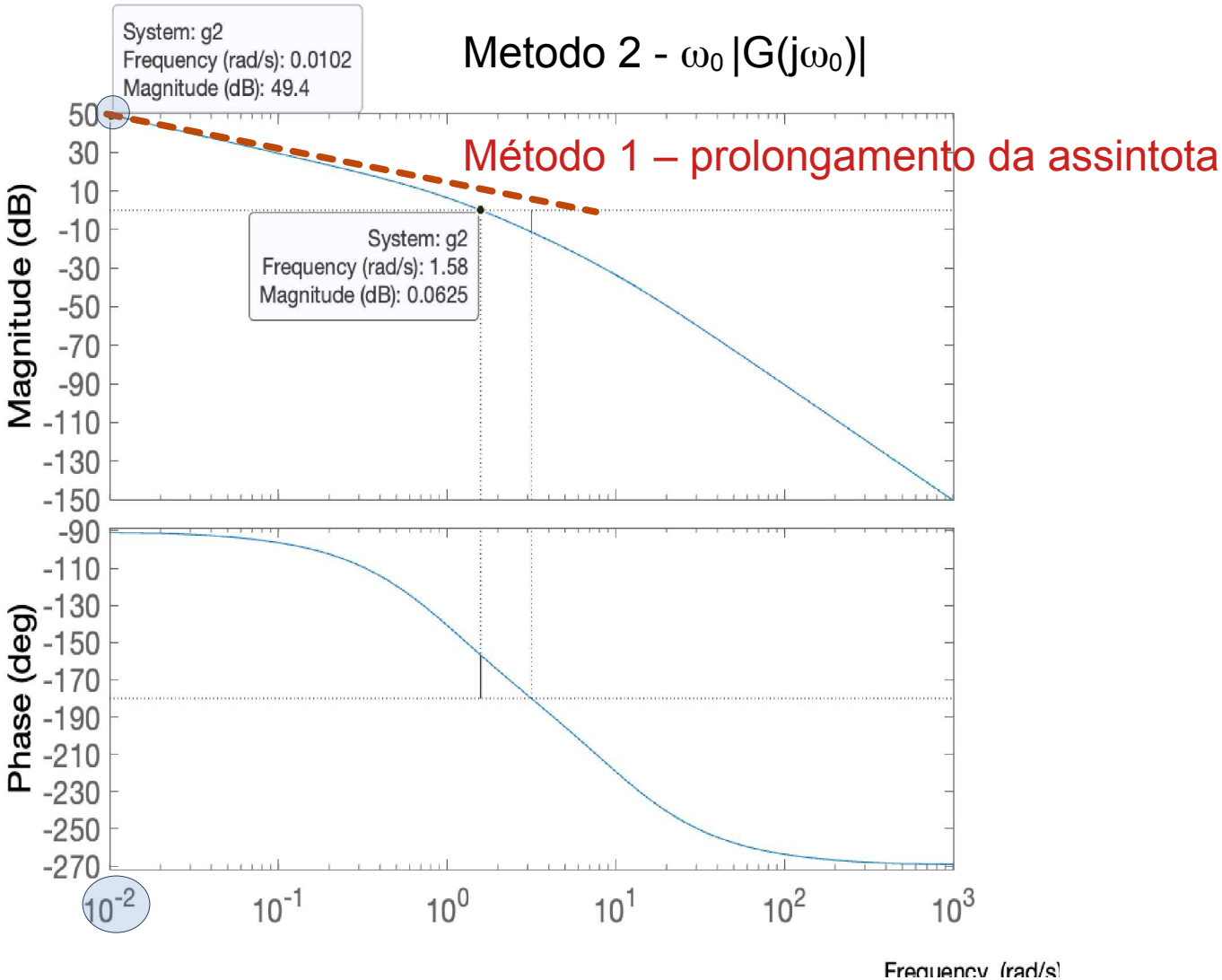
$$Kp = (3{,}52\,dB)$$

$$Kp = 10^{(3.52/20)} = 1{,}5$$



Especificações no domínio- ω

Erro em Regime Permanentes: K_v



$$Tipo\,1: Kv = \lim_{s \to 0} sG(s)$$
$$s = j\omega_0$$
$$Kv = \omega_0 G(j\omega_0)$$

$$Kv = 0{,}0102\, x\, 10^{(49.4/20)}$$
$$Kv = 3{,}01$$

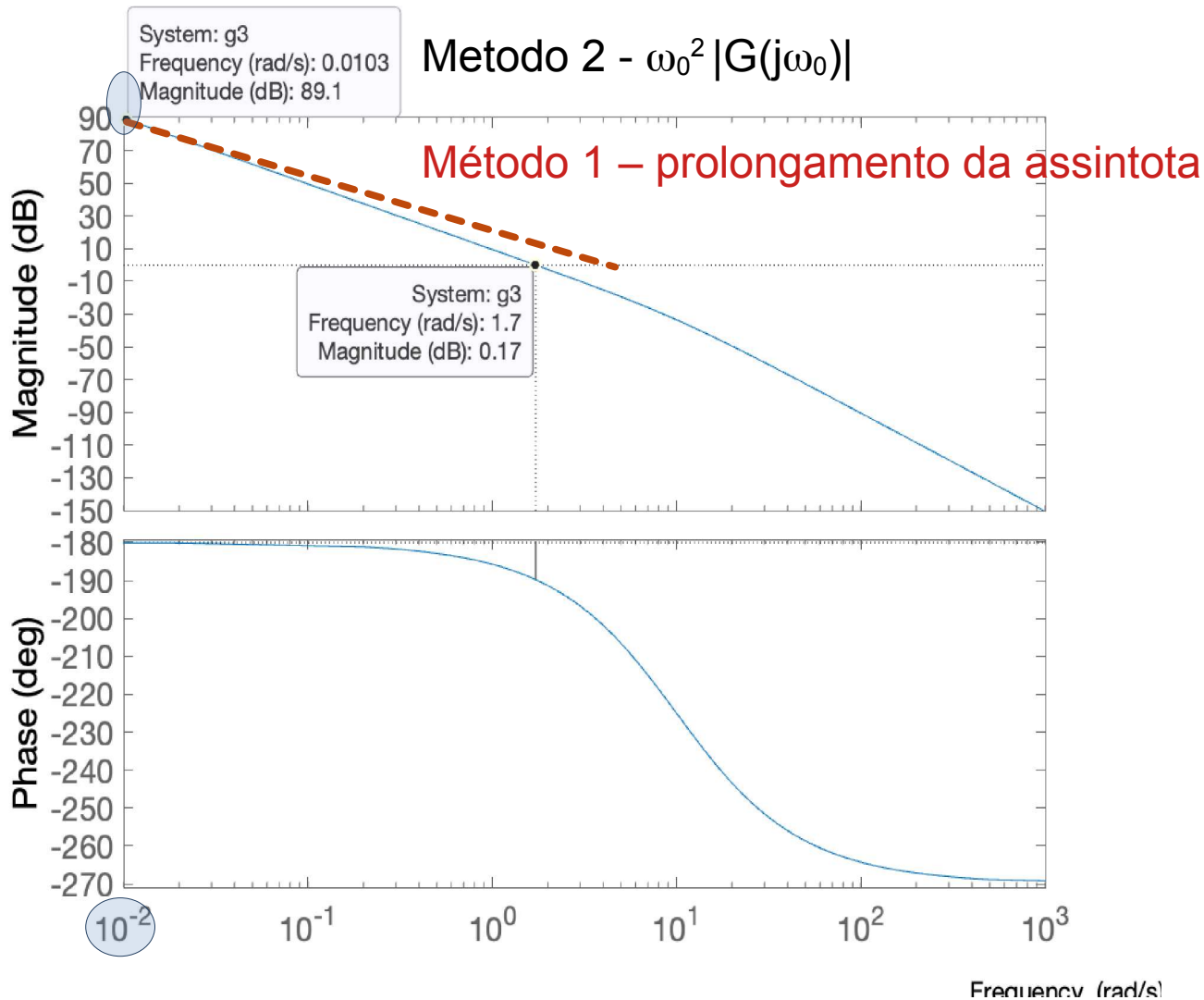
Especificações no domínio- ω

Erro em Regime Permanentes: K_a

$$Tipo\,2: Ka = \lim_{s \to 0} s^2 G(j\omega_0)$$

$$Ka = 0{,}0103^2 \, x \, 10^{(89.1/20)}$$

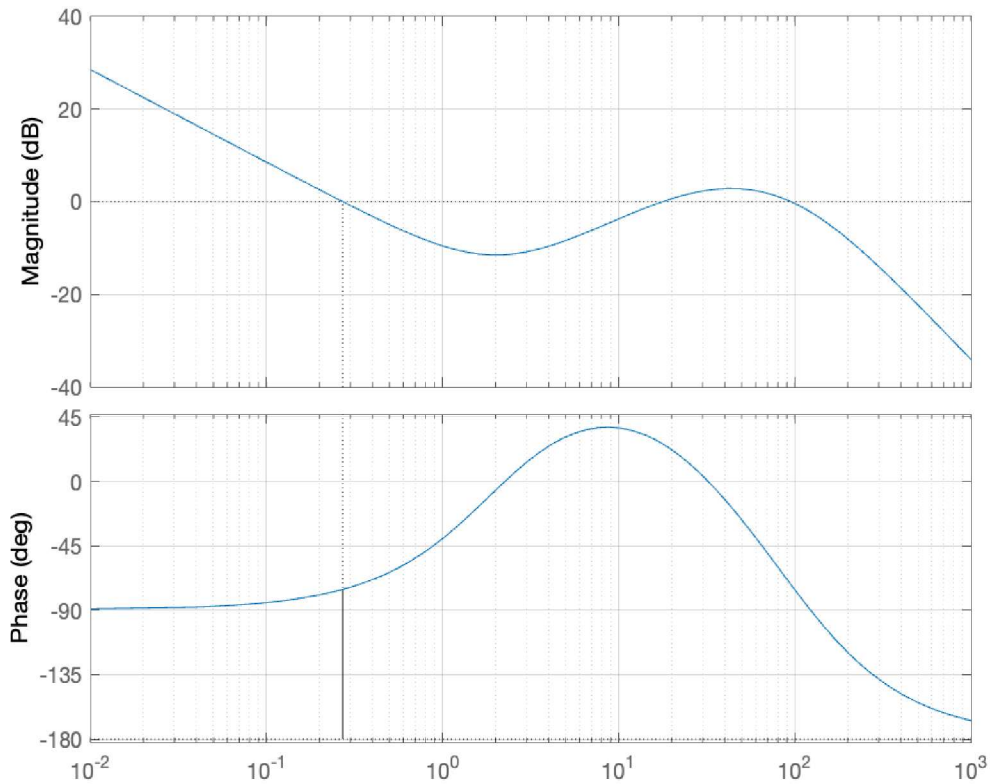
$$Ka = 3{,}024$$

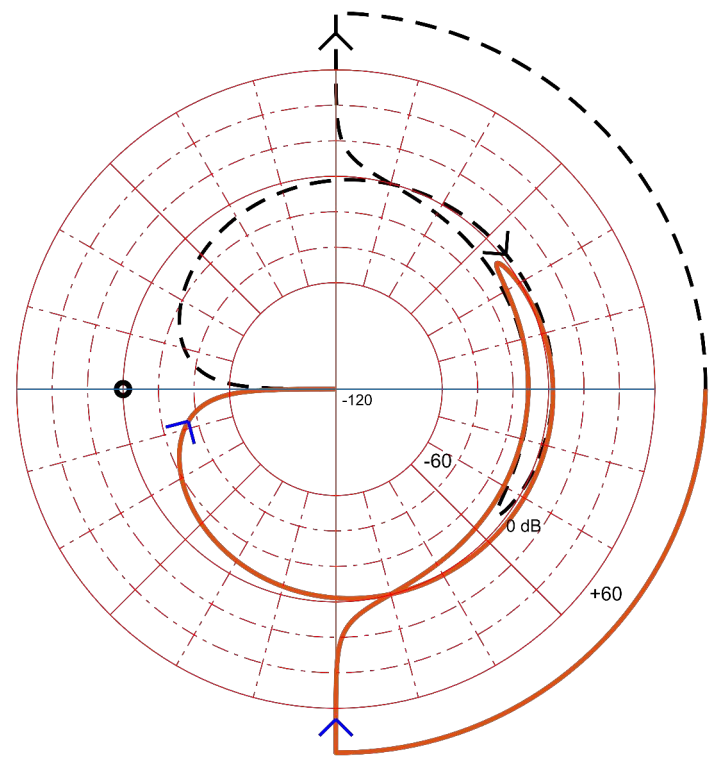


Especificações no domínio- ω

Erro em Regime Permanentes: K_v

$$Tipo\,1: Kv = \lim_{s \to 0} sG(j\omega_0)$$





Frequency (rad/s)

Exercício Extra 6

Considere a resposta em frequência G($j\omega$) de dois sistemas que tem um polo no SPD.

Para (I)--- e (II) —:

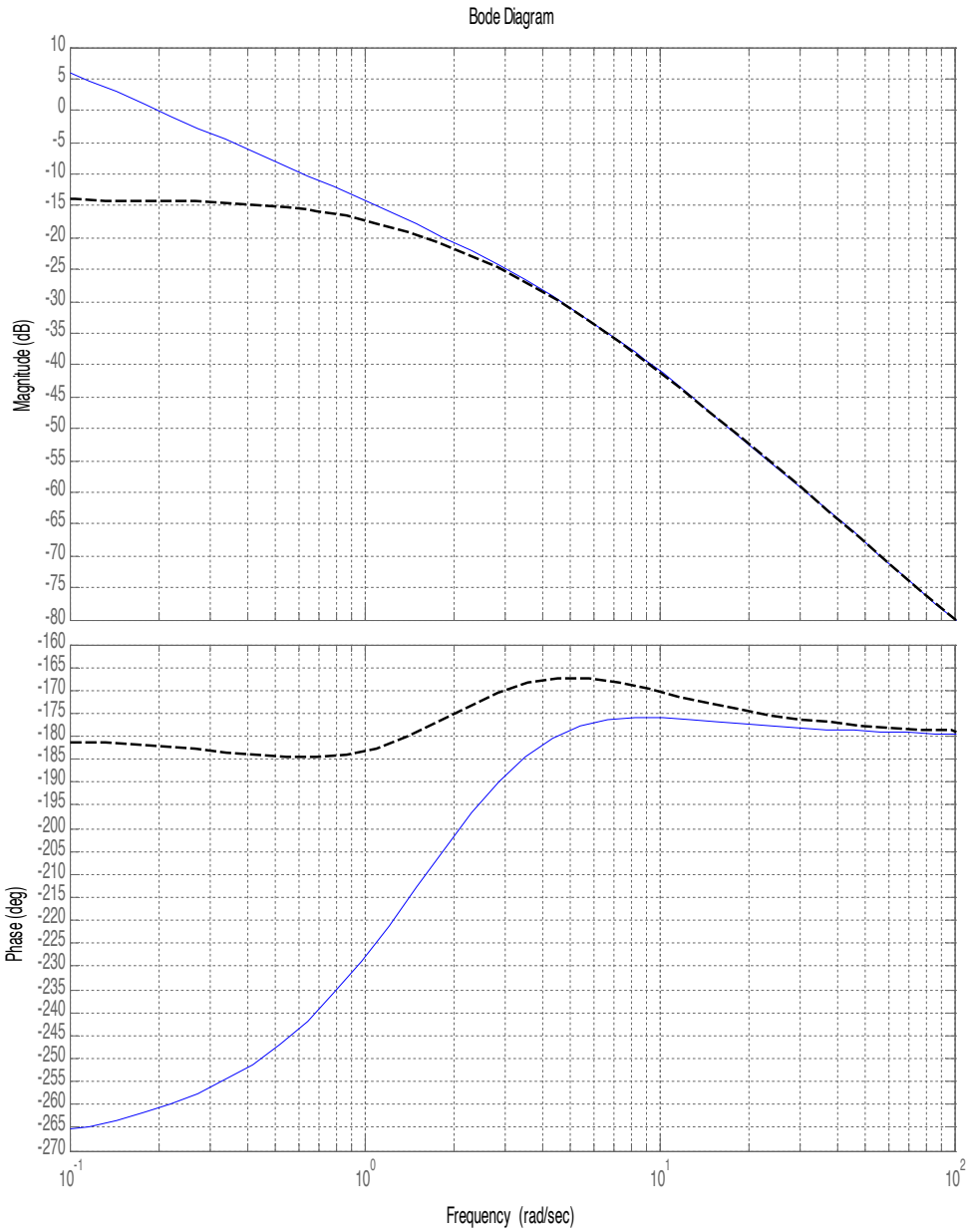
a) Esboce os diagramas de Nyquist correspondentes.

b) Em malha fechada, quais valores de K , $-\infty < K < \infty$, produzem respostas estáveis?

c) Para quais valores de K se tem a maior margem de fase possível?

d) Nas condições do item c), qual o erro em regime permanente para um degrau de referência.

e) Para quais valores de K o erro em regime permanente para um degrau de referência é $e_{ss} \leq 0{,}01$.



Compensação no domínio-ω

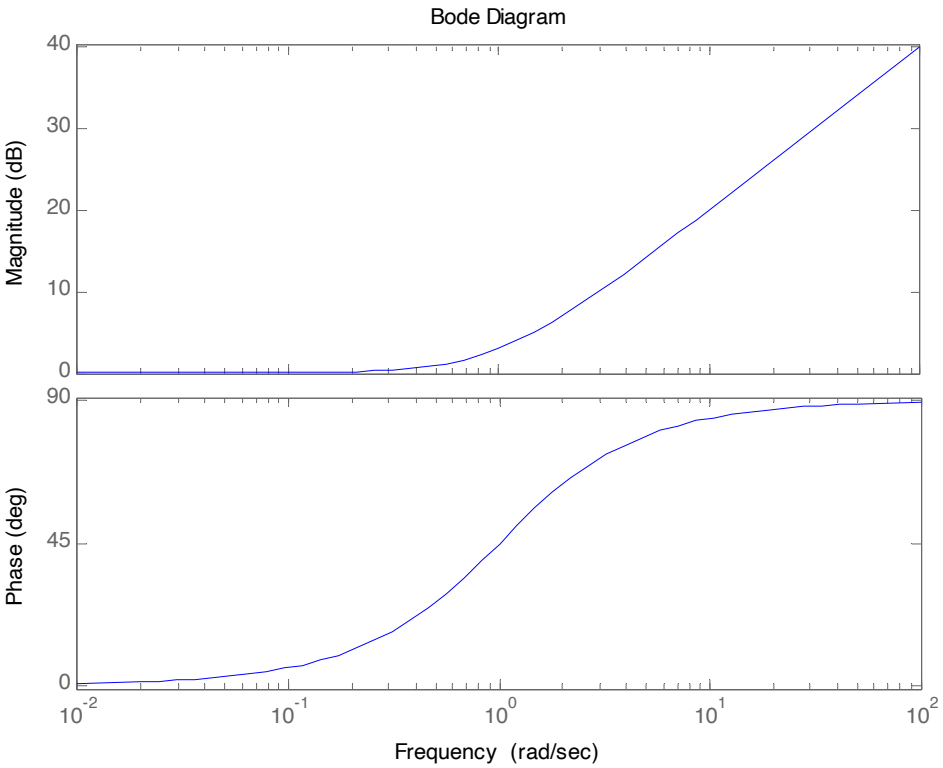
- Adição de elementos dinâmicos ao sistema de controle para melhorar a estabilidade e diminuir o erro em regime

- Compensador PD

  Pouco utilizado →

  amplifica ruído de medida

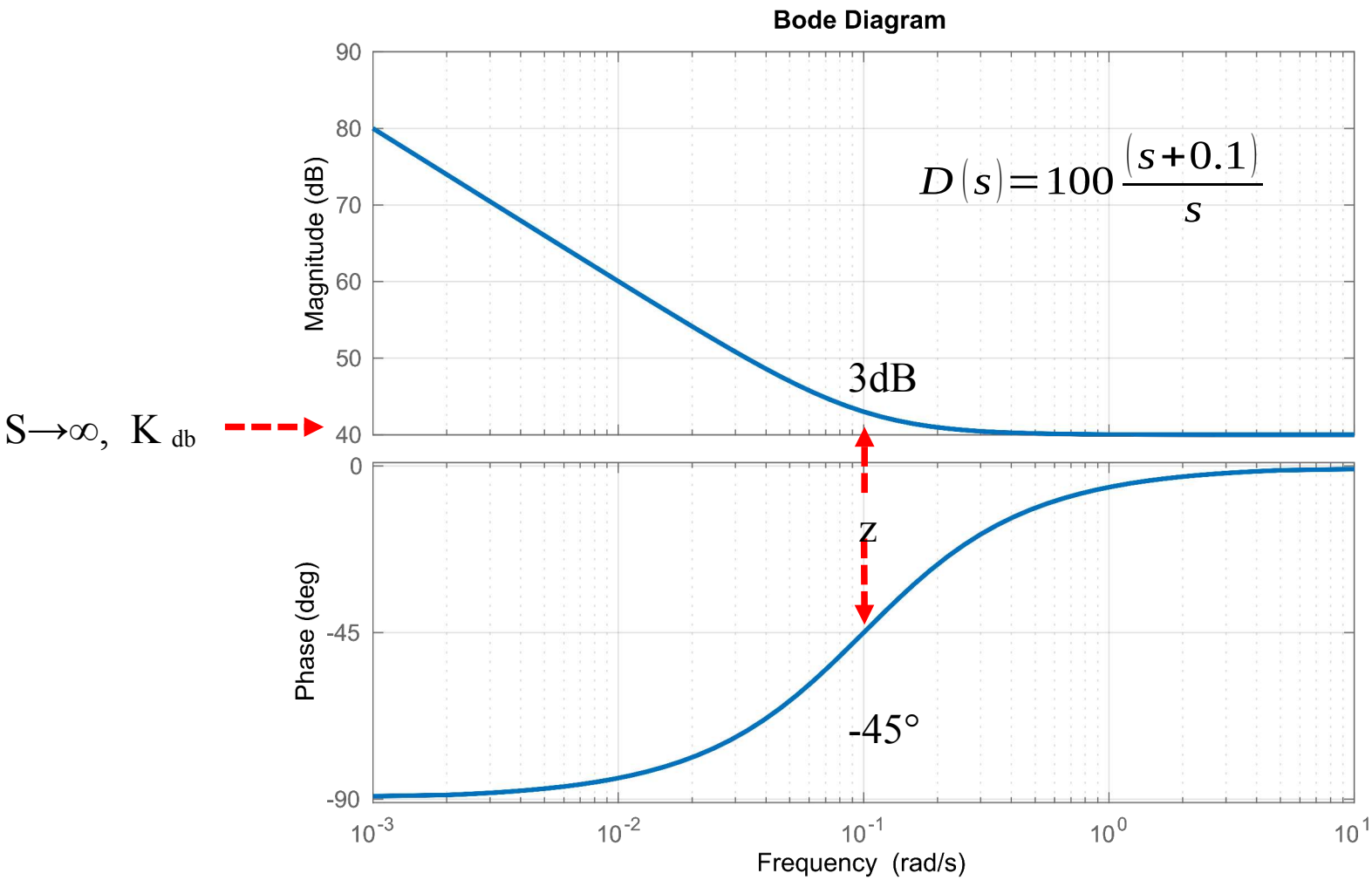
$$D(s) = K(T_d s + 1)$$



➢ Compensador PI

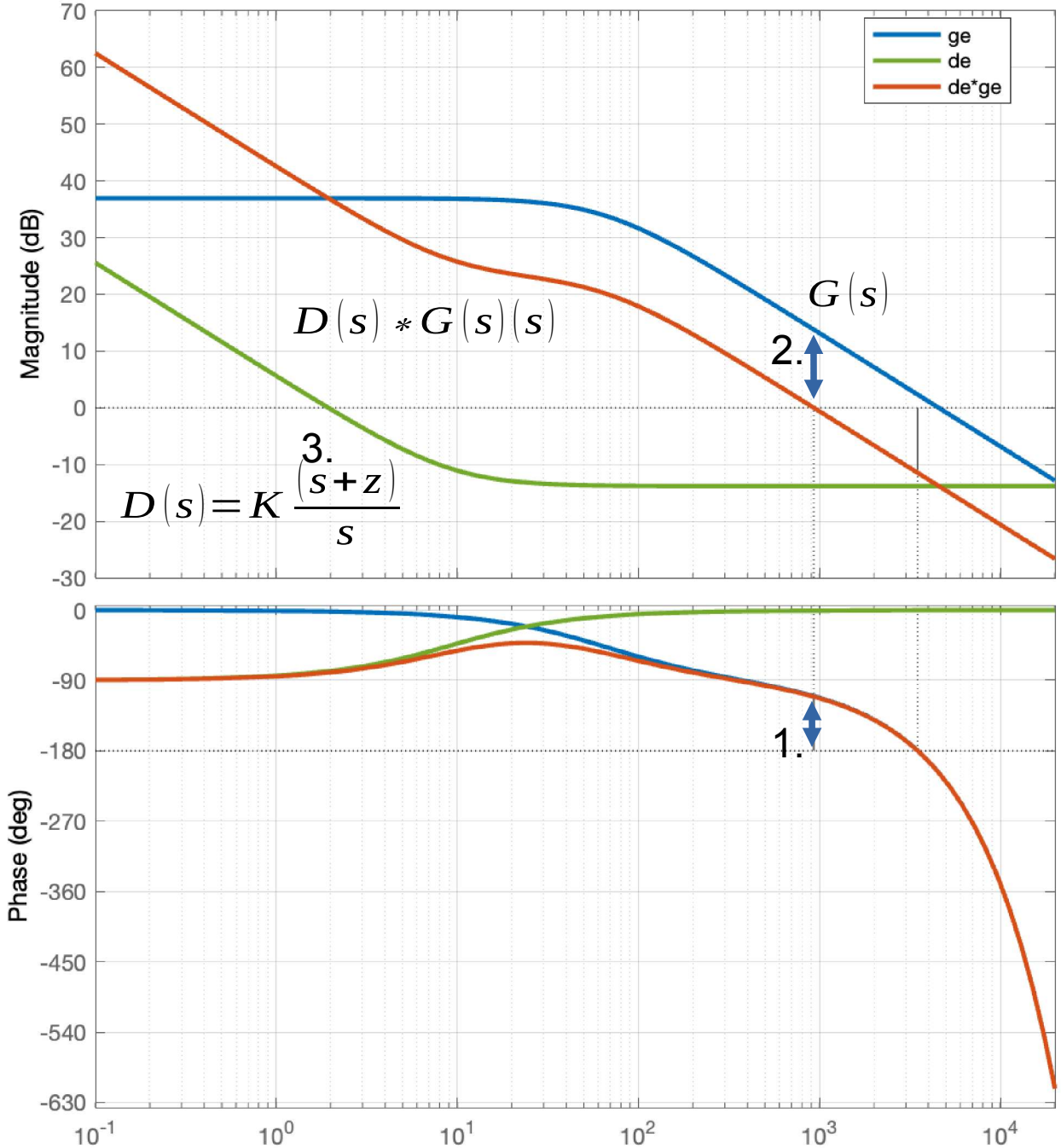
$$D(s) = K\frac{(s+z)}{s}$$

Muito utilizado



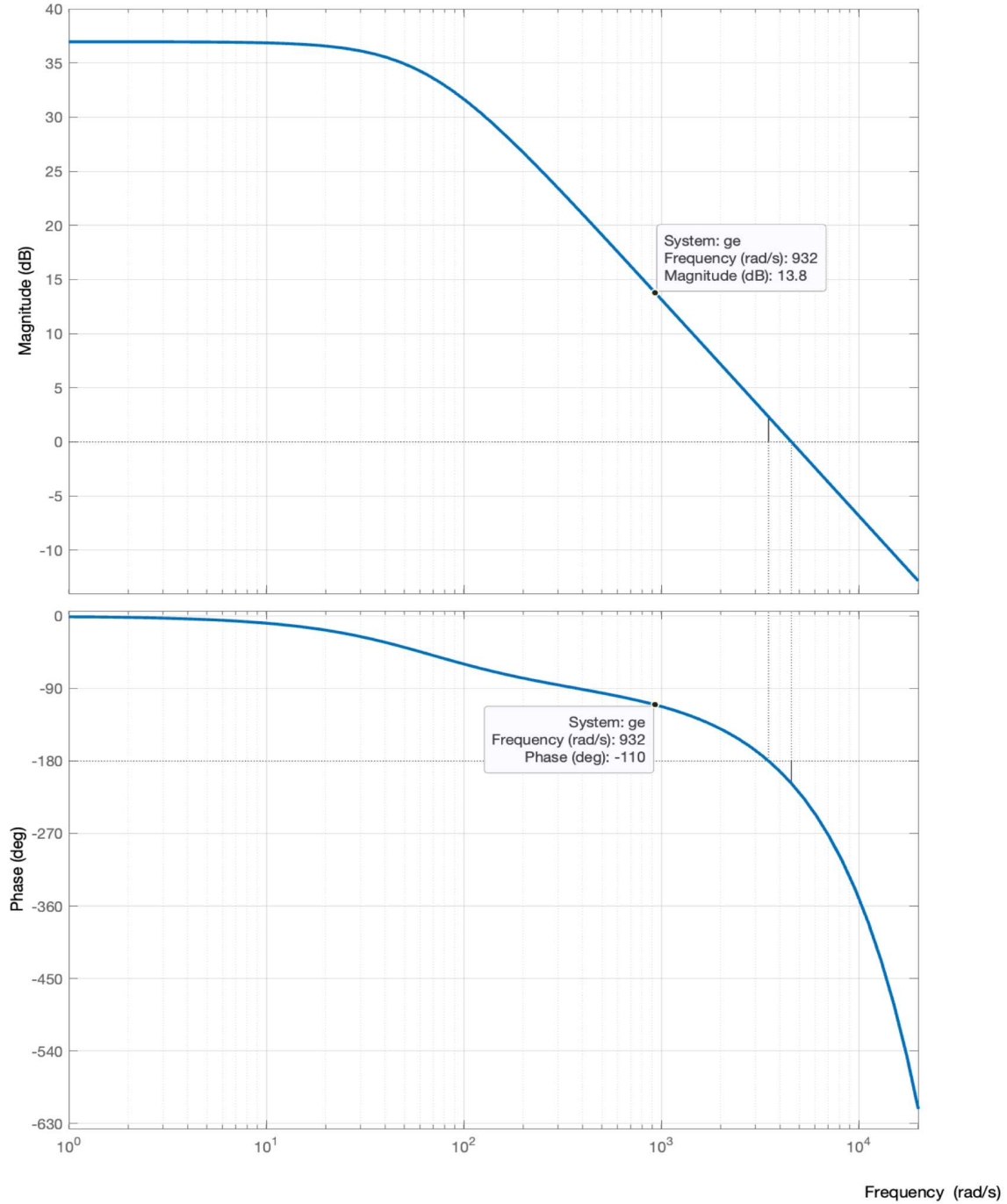
➢ Metodologia Projeto PI

- ➢ 1 . Buscar a fase especificada em ω_F
- ➢ 2. Buscar o ajuste de ganho K → 0dB em ω_F
- ➢ 3. Posicionar z = 0.01ω_F
- ➢ ($\underline{/D(j0.01\omega_F)}$ = 0,8°)



➢ Projeto PI

- ➢ Especificações:
  - MF $\geq$70°
- ➢ - max Kv



- ➢ A MF = 70° ocorre em ω = 932 rad/s
- ➢ → atenuar ganho 13,8 dB (0,2042)

- ➢ A posição do zero:

  Quanto maior z → maior Kv

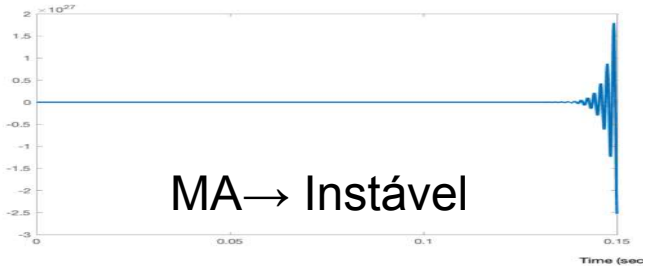
  Para z = 0.1 ω_C => MF -5.73° nOK

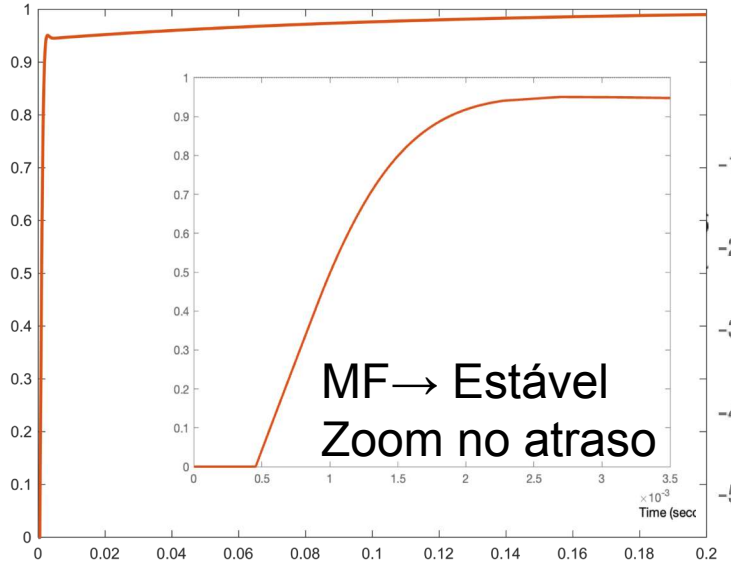
  Para z = 0.01 ω_C => MF -0,825° OK

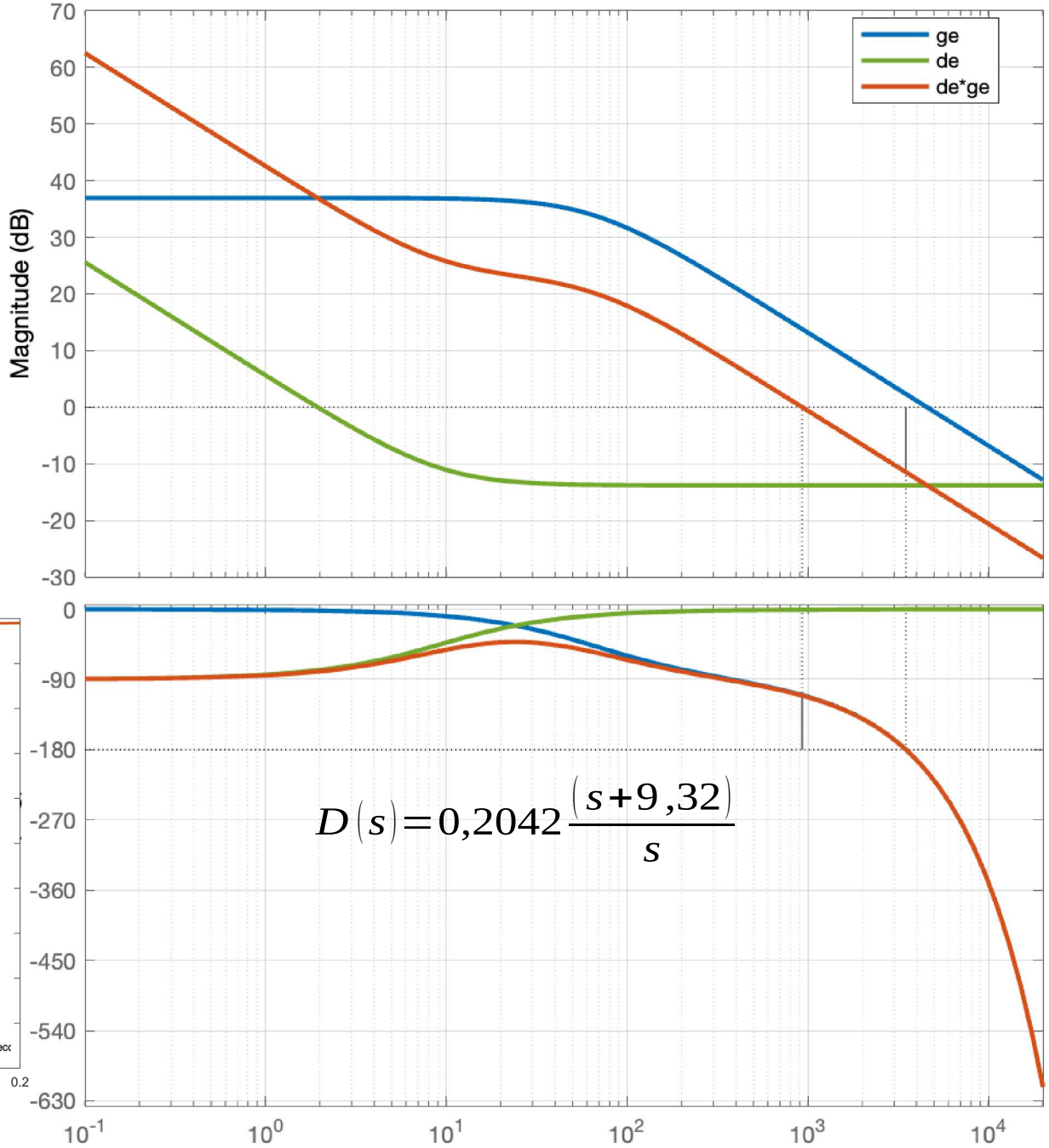
$$D(s) = 0{,}2042\frac{(s+9{,}32)}{s}$$

➢ Projeto PI

- ➢ Especificações:
  - MF ≥70°
- ➢ - max Kv







Compensador em avanço

- Muito Popular

$$D(s) = K\frac{Ts+1}{\alpha Ts+1}$$

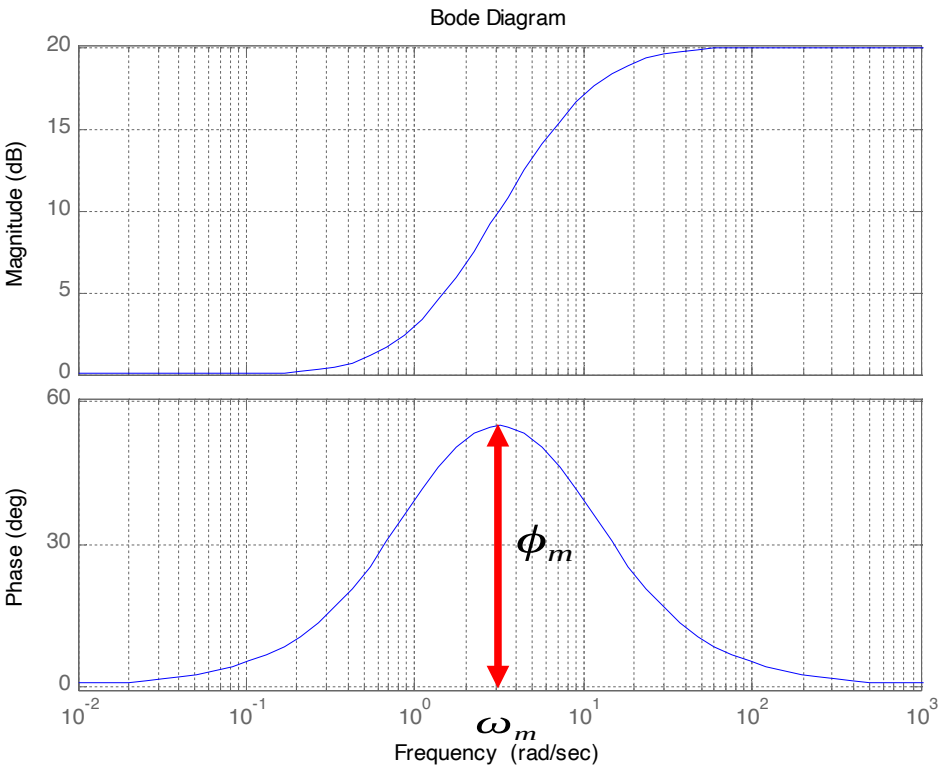
$$\alpha < 1$$

$$\phi = \tan^{-1}(\omega T) - \tan^{-1}(\alpha\omega T)$$

ω_m – *média geométrica das freqs de canto*

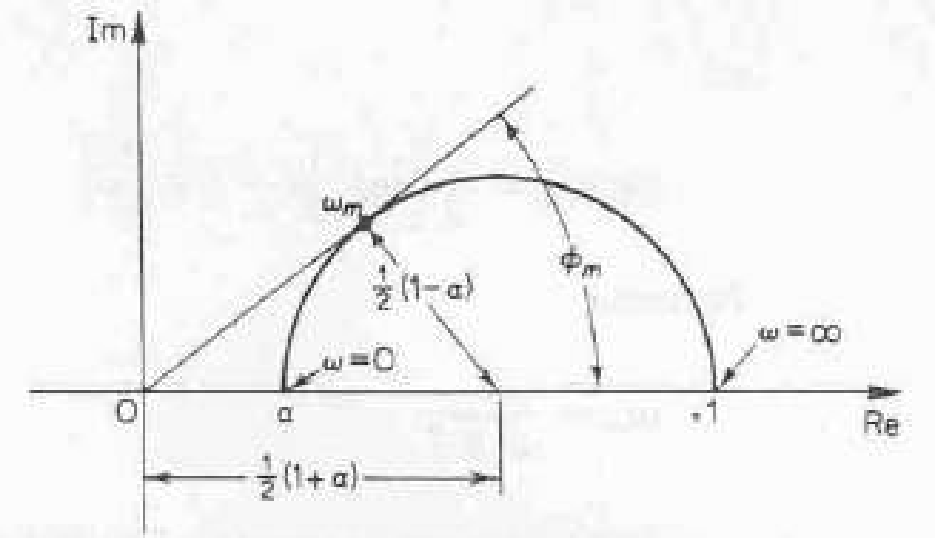
$$\log\omega_m = \frac{1}{2}(\log\frac{1}{T} + \log\frac{1}{\alpha T})$$

$$\omega_m = \frac{1}{\sqrt{\alpha}T}$$



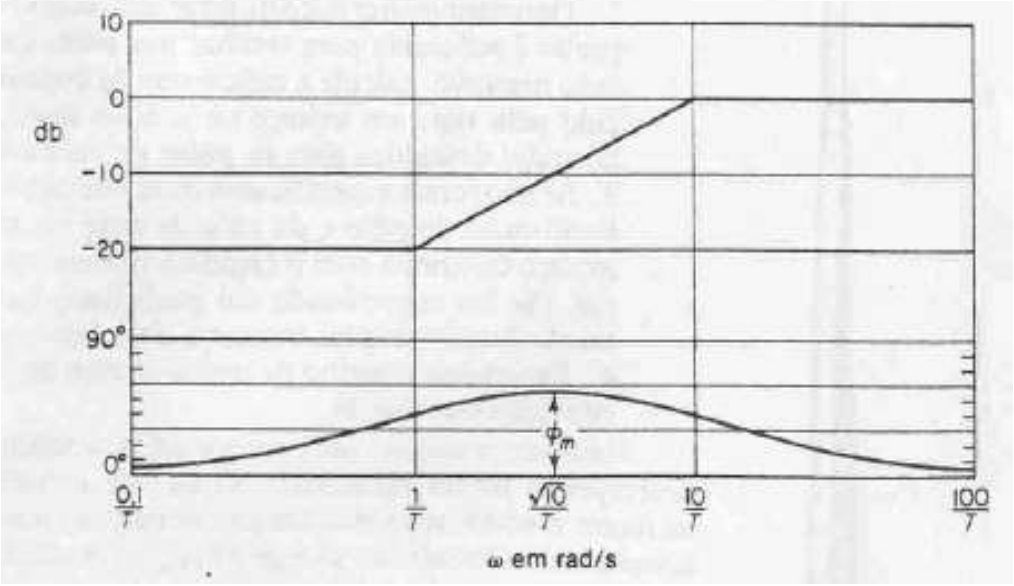
Compensador em avanço

- Avanço máximo depende apenas de α



$$Fator\ de\ avanço\ \frac{1}{\alpha} = \frac{1 + sen\phi_m}{1 - sen\phi_m}$$

$$sen\phi_m = \frac{1-\alpha}{1+\alpha}$$



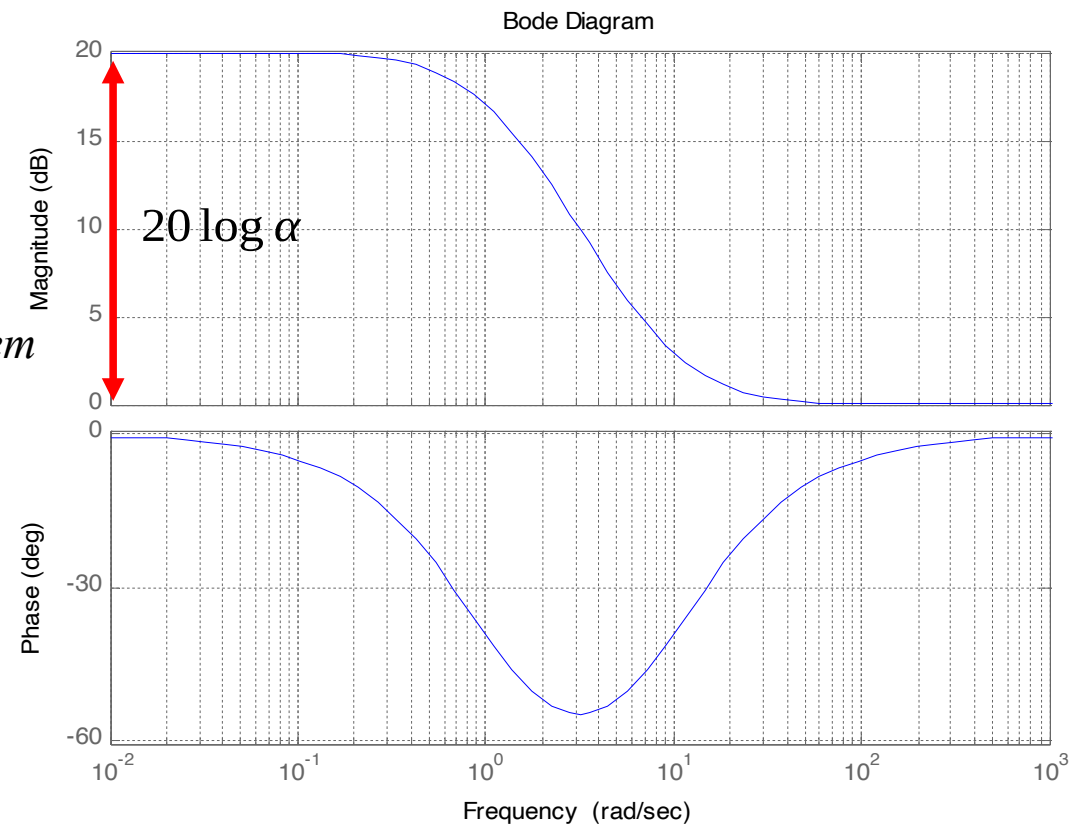
Compensador em atraso

$$D(s) = \alpha \frac{Ts + 1}{\alpha Ts + 1}$$

$\alpha > 1$

fornece ganho adicional de $+20\log\alpha$ *em baixas frequencias, mantendo margem de fase suficiente.*

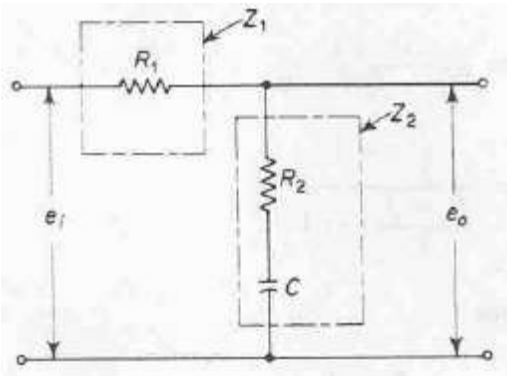
$\Rightarrow$ *pólo e zero bem próximos à origem para não afetar a resposta transitória.*



$$K_p' = \alpha K_p \left(e_{ss} = \frac{1}{1 + \alpha K_a} \right); K_v' = \alpha K_v \left(e_{ss} = \frac{1}{\alpha K_v} \right); K_a' = \alpha K_a \left(e_{ss} = \frac{1}{\alpha K_a} \right).$$

Compensador em atraso

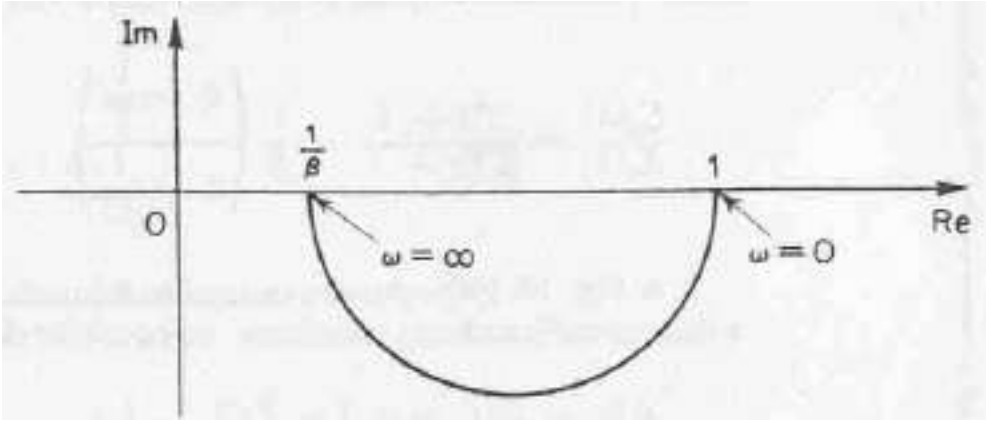
$$D(s) = \alpha \frac{Ts + 1}{\alpha Ts + 1}$$

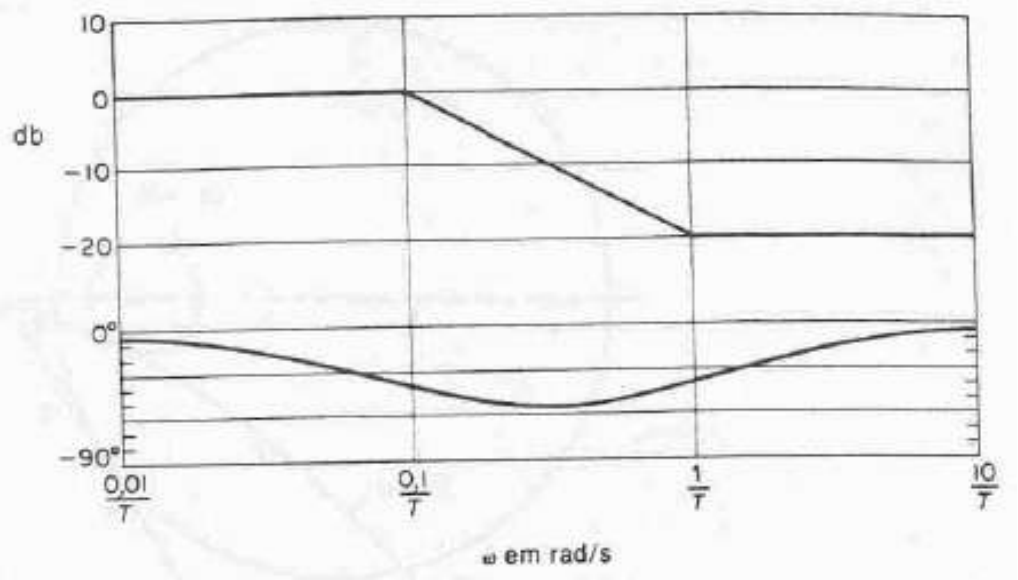


$$\frac{E_o(s)}{E_i(s)} = \frac{Z_2}{Z_1 + Z_2} = \frac{R_2Cs + 1}{(R_1 + R_2)Cs + 1}$$

$$R_2C = T, \qquad \frac{R_1 + R_2}{R_2} = \beta > 1$$

$$\frac{E_o(s)}{E_i(s)} = \frac{Ts + 1}{\beta Ts + 1} = \frac{1}{\beta}\left(\frac{s + \frac{1}{T}}{s + \frac{1}{\beta T}}\right)$$





Compensadores Dinâmicos em ω

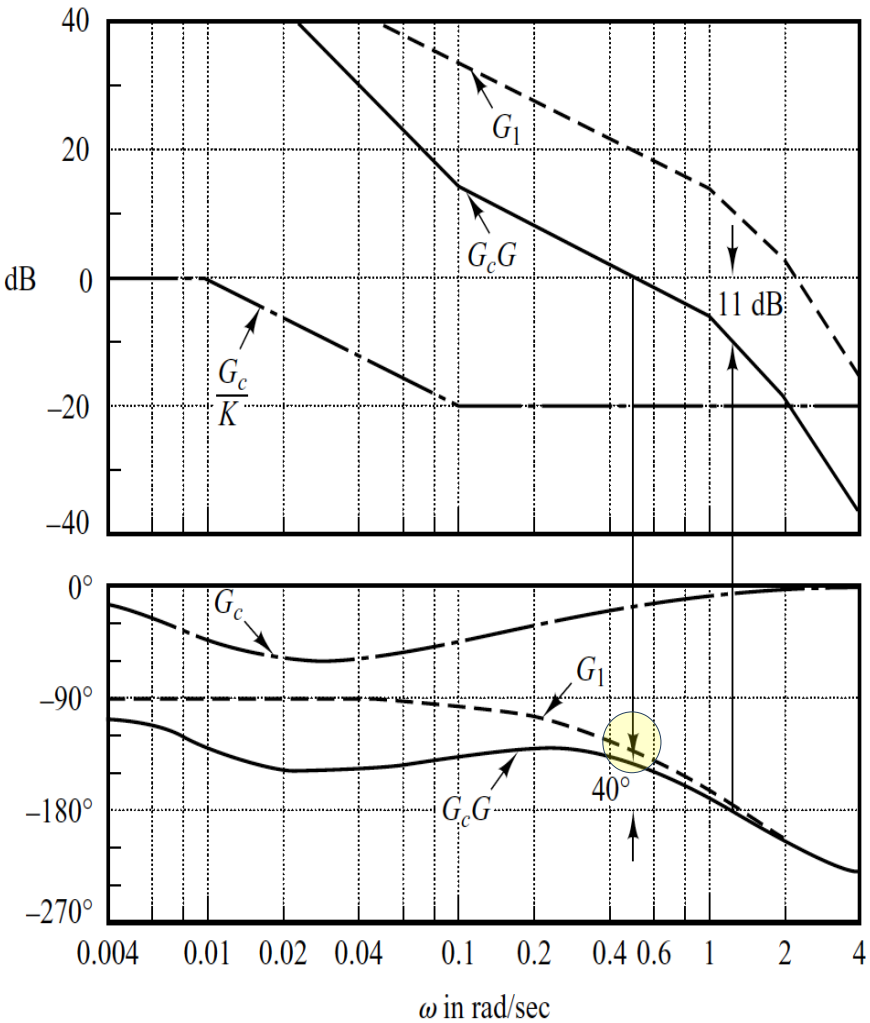
Estratégias de Projeto:

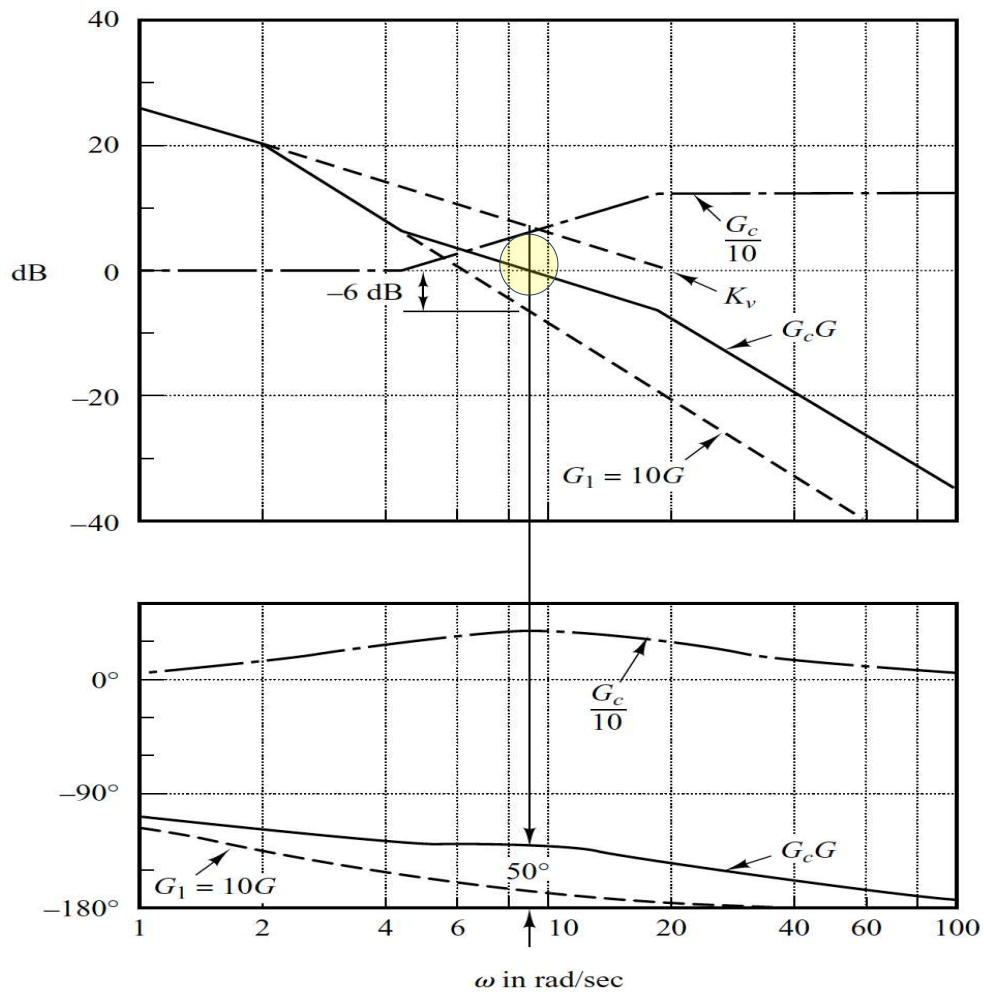
Atraso: 1 - Acertar e_{ss} (K_p)

2 - Usar MF da curva de G(jω)

Avanço: 1 - Acrescentar Fase à G(jω)

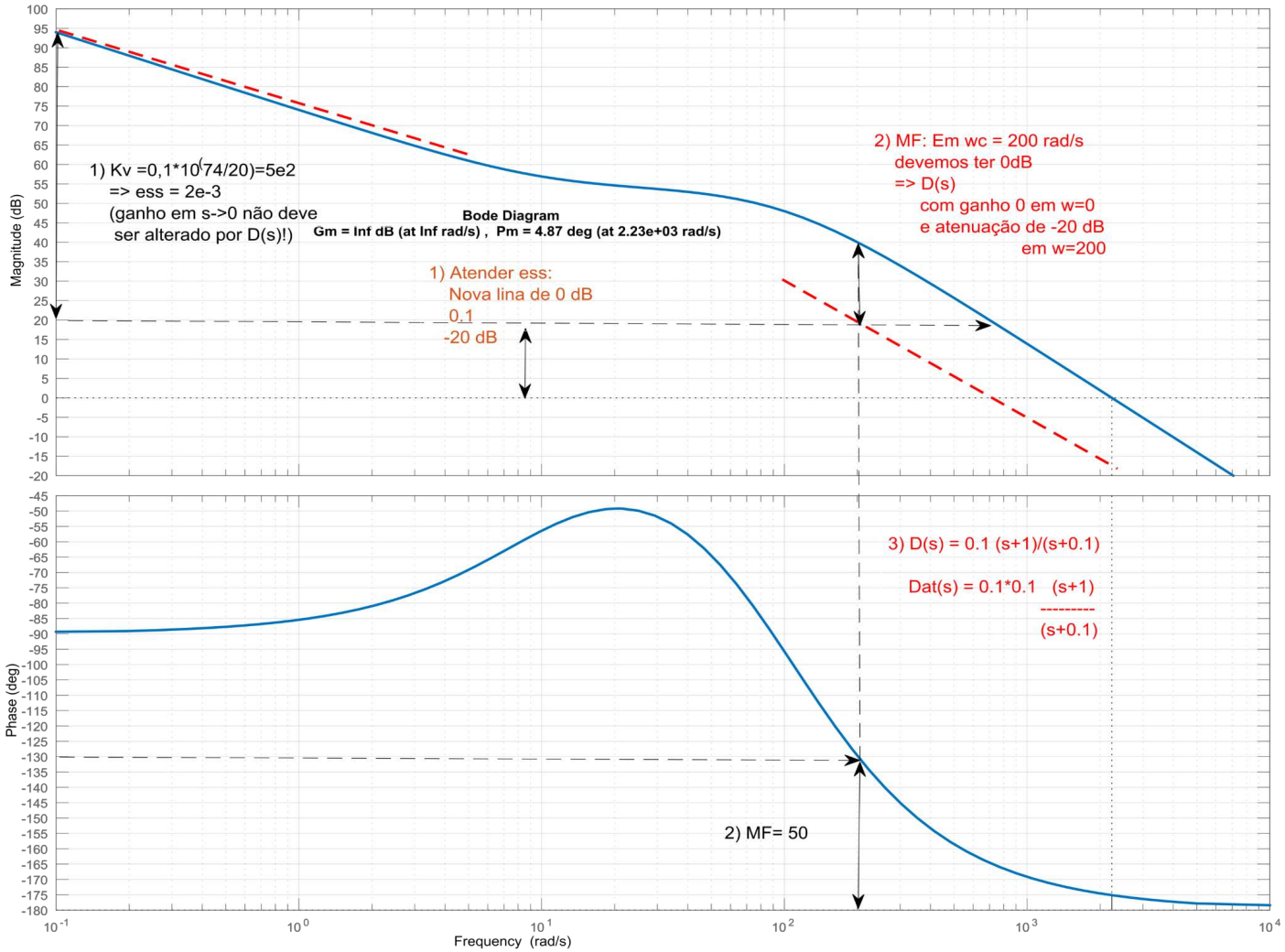
onde $|G(j\omega_a)D(j\omega_a)| = 0$ dB





Ex.: Projeto Ctrl. em Atraso

- MF = 50°
- e_{ss} = 0,002 (rampa unitária)



Exemplo - Compensador em atraso

- MF = 50°
- e_{ss} = 0,002 (rampa unitária)
- Menor redução da banda passante possível

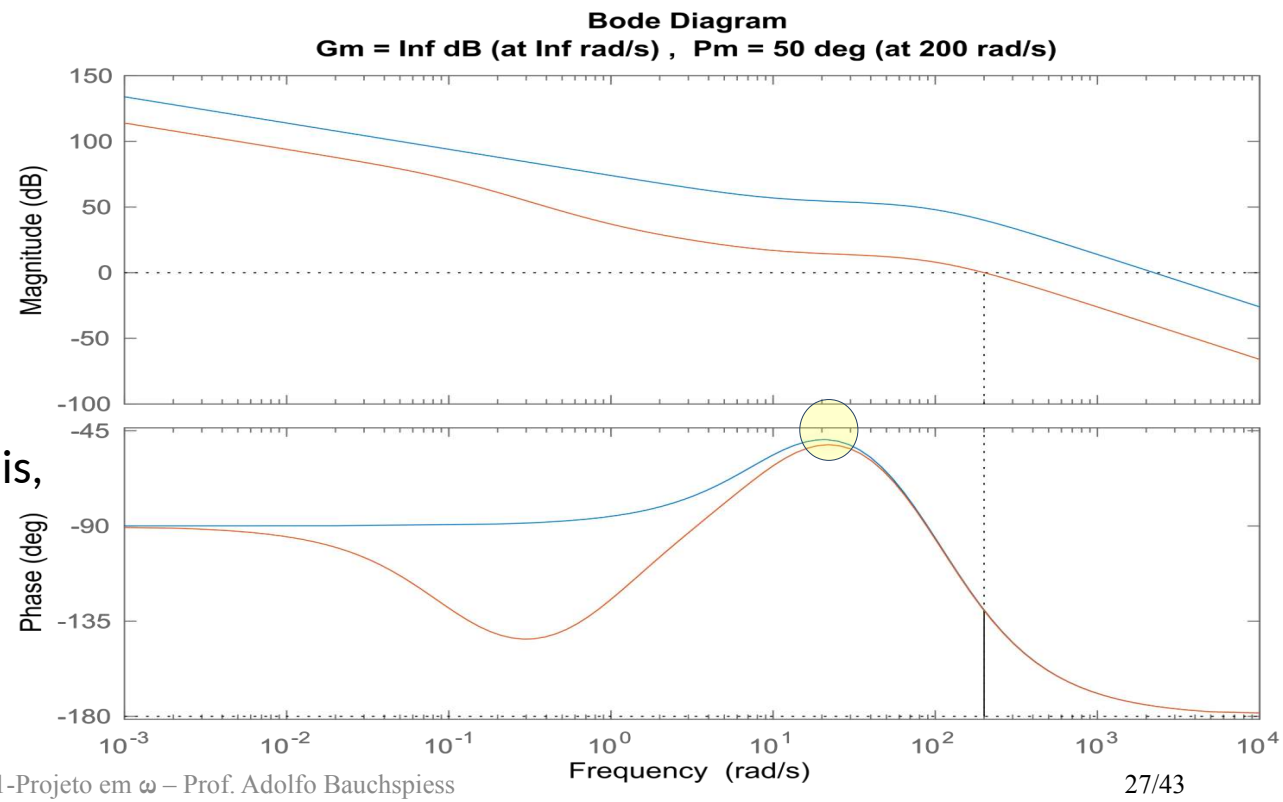
g1=zpk([-10],[0 -100 -100],5000000);

K1 = 0,01 (-40 dB) Kv' = 0,1 x 501,18 (54 dB) = 50,118
1/Kv'=0,02 => K2 =10

$$D(s) = 0{,}01 \frac{(s+1)}{s+0{,}1}$$

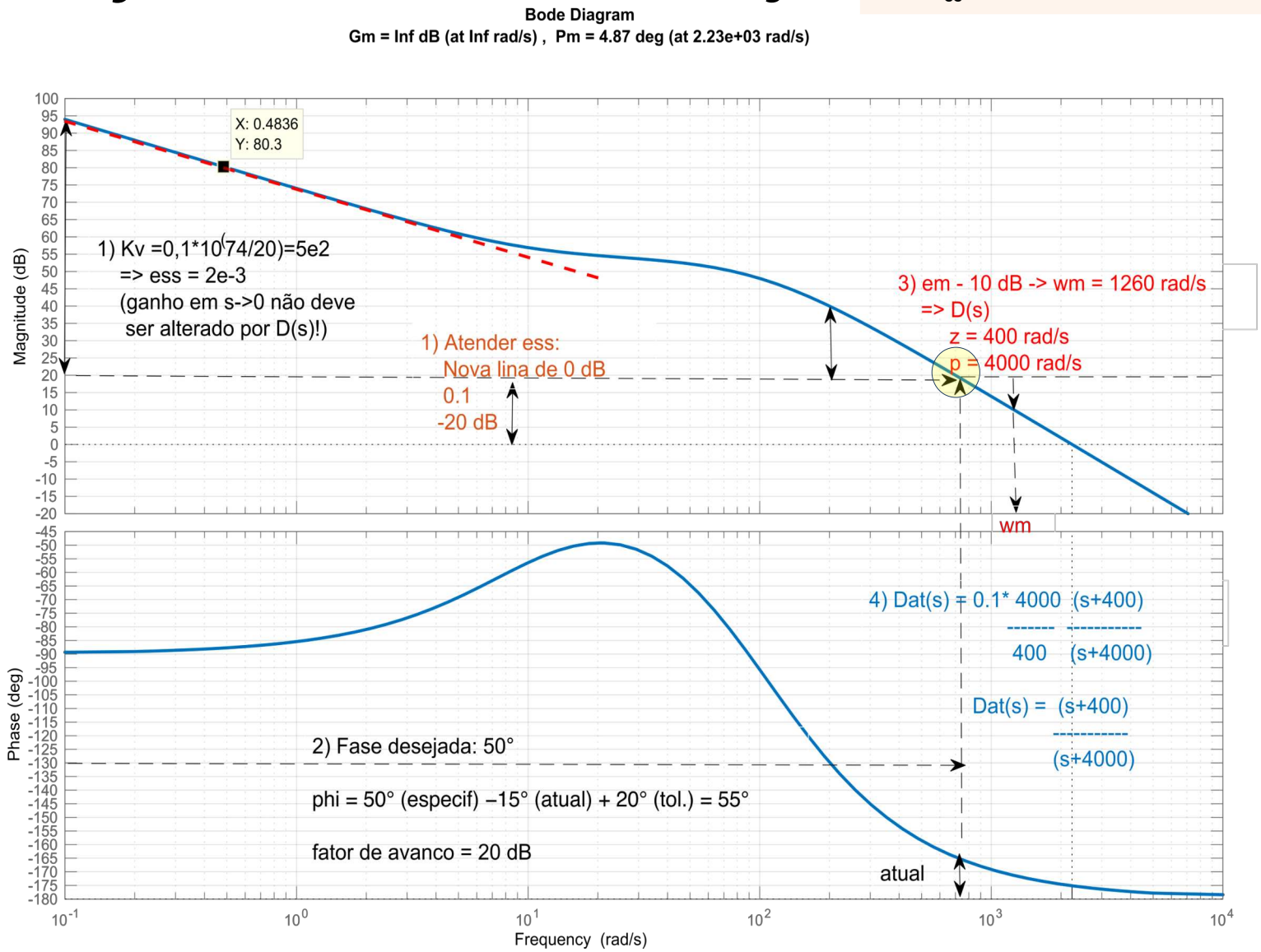
d=zpk(-1,-.1,0.01);
margin(d*g1)

Obs: várias soluções possíveis,
z/p = 10
e "longe" de 200 rad/s



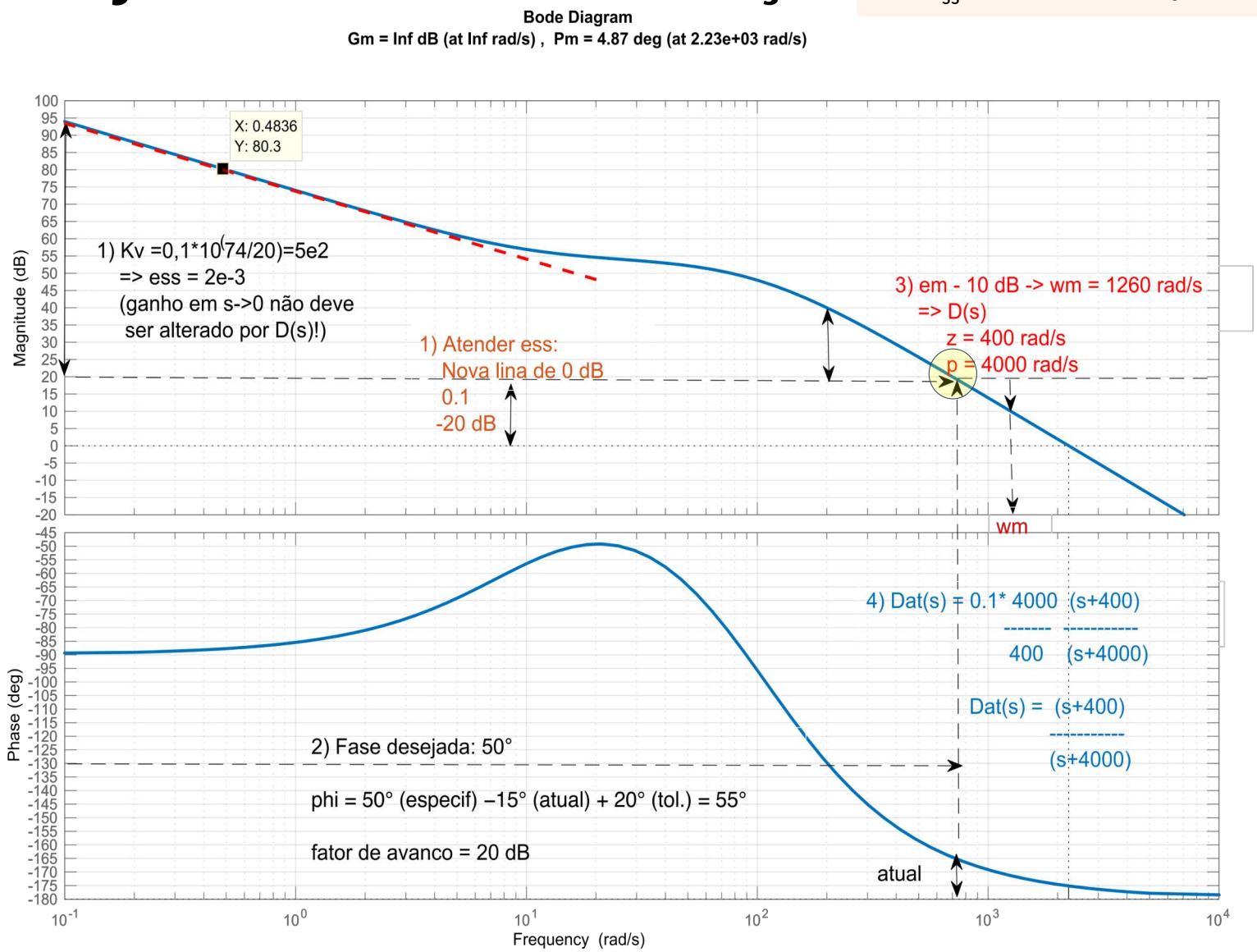
Ex.: Projeto Ctrl. em Avanço

- MF = 50°
- e_{ss} = 0,002 (rampa unitária)



Ex.: Projeto Ctrl. em Avanço

- MF = 50°
- e_{ss} = 0,002 (rampa unitária)



Exemplo - Compensador em avanço

- MF = 50°
- e_{ss} = 0,002 (rampa unitária)

$K_v = 10^{94/20} * 0{,}1 = 5e3 \rightarrow e_{ss} = 2e-4$
$\Rightarrow atenuar \;\; 20\,dB\; para\; obter \;\; e_{ss} = 2e-3$

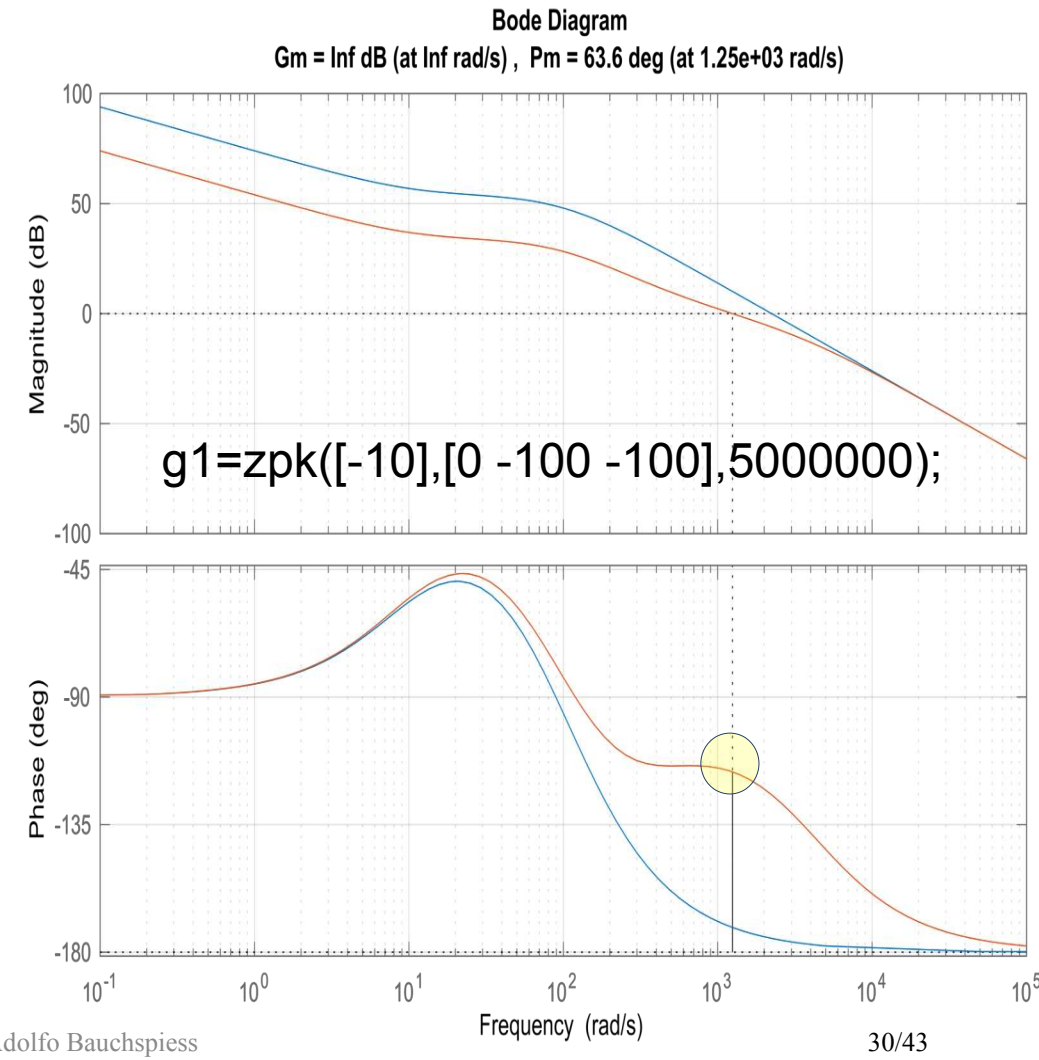
Fase: $50°\,(Espec.) - 15°\,(Atual) + 20°\,(Tol.) = 55° \Rightarrow fator\ de\ avanço = 10\ (20\ dB)$

Queda de 10 dB -> $frequência\ \omega_m = 1260\ rad/s$

$z = \sqrt{\alpha}\omega_m = 398{,}7 \approx 400$ rad/s
$p = \frac{\omega_m}{\sqrt{\alpha}} = 3981{,}6 \approx 4000$ rad/s

$$D(z) = 0{,}1\frac{4000}{400}\frac{s+400}{s+4000} = \frac{s+400}{s+4000}$$

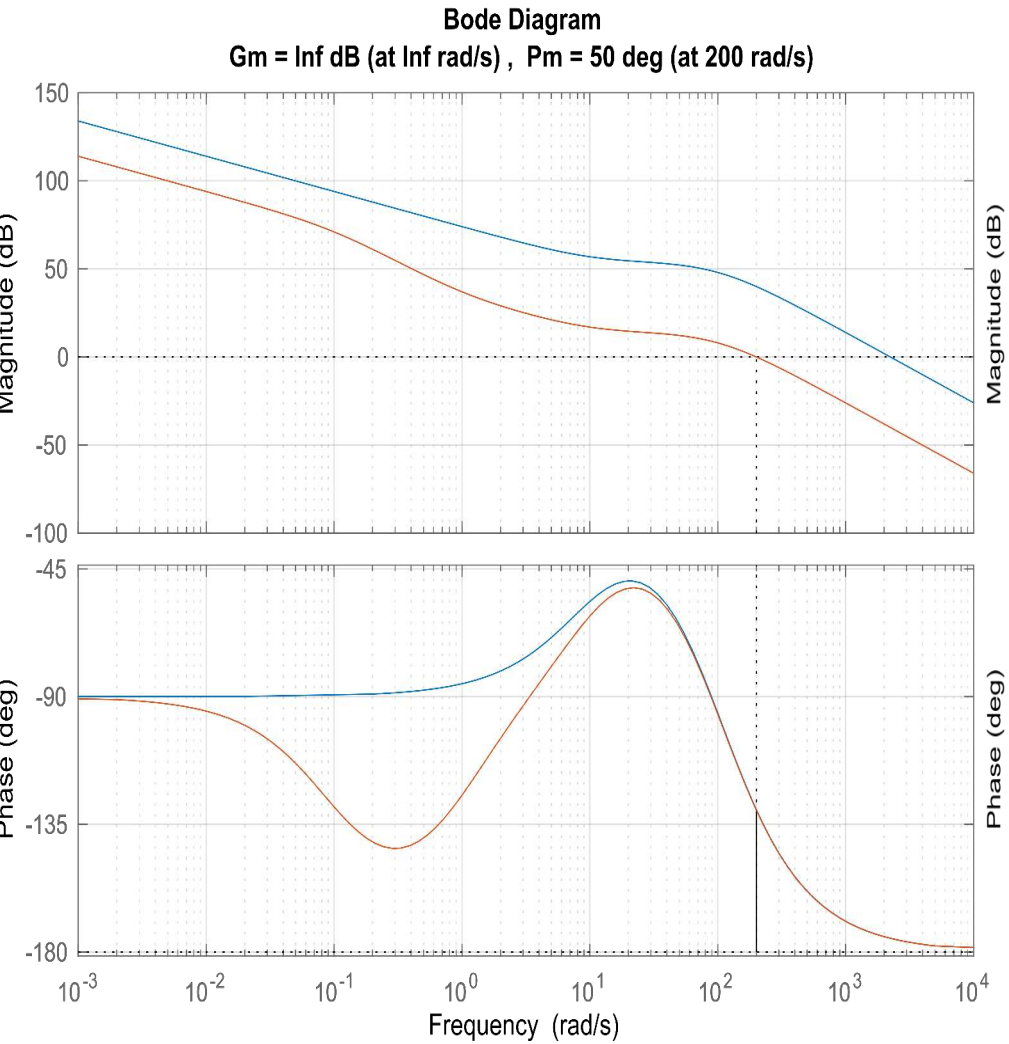
Obs: Margem de Fase obtida de fato: 64°

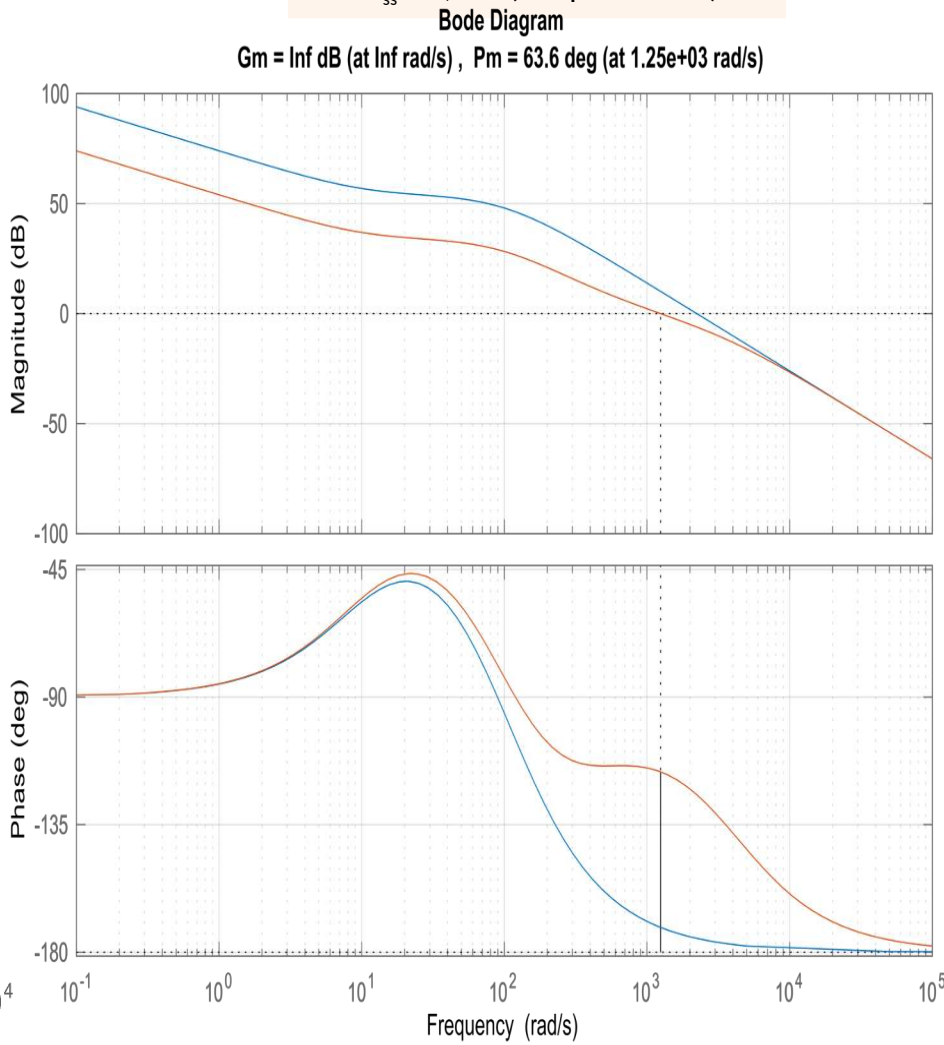


Comparação: Atraso x Avanço

g1=zpk([-10],[0 -100 -100],5000000);

- MF = 50°
- e_{ss} = 0,002 (rampa unitária)



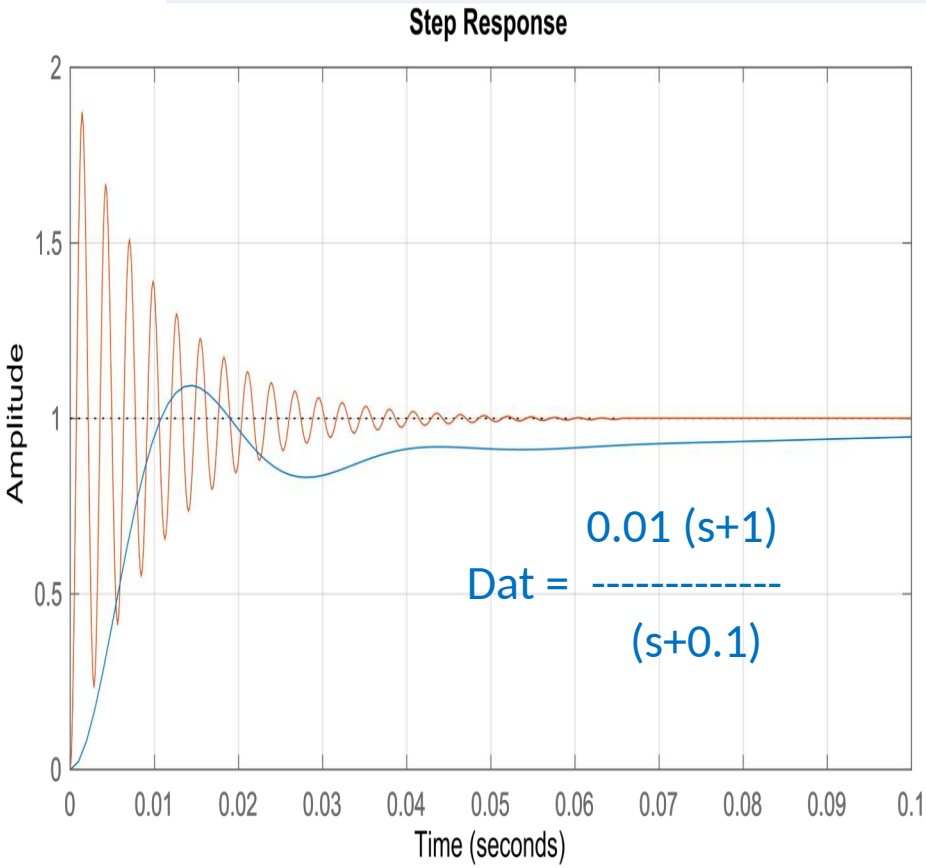


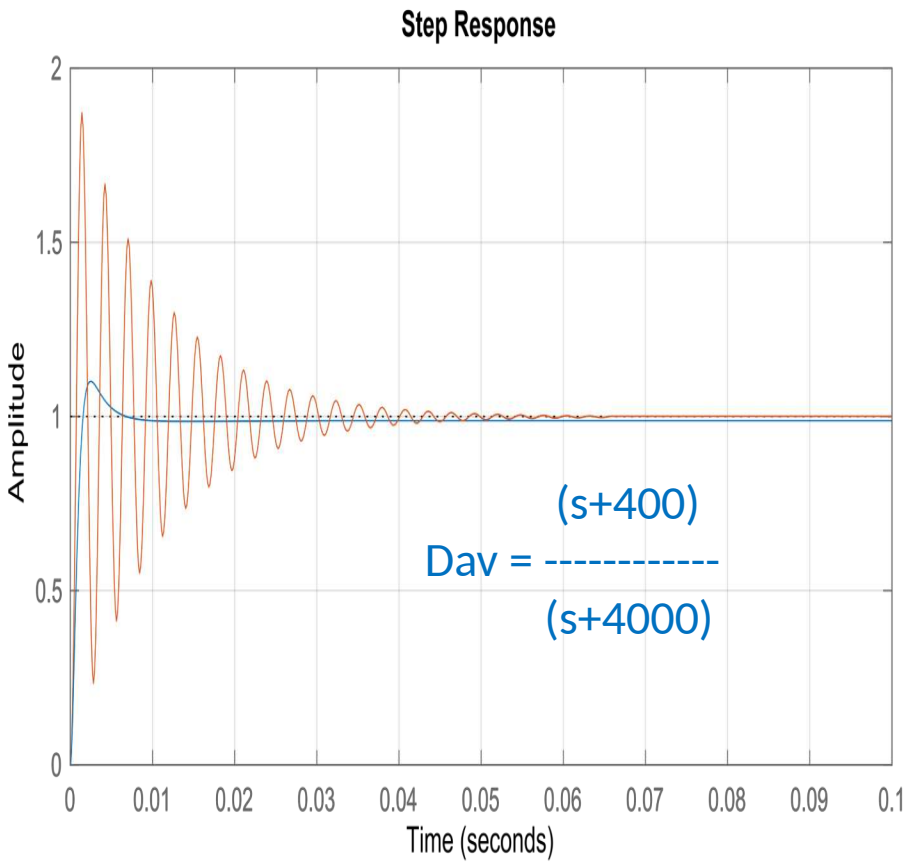
Comparação: Dat(s) x Dav(s) x D(s)=1

g1=zpk([-10],[0 -100 -100],5000000);

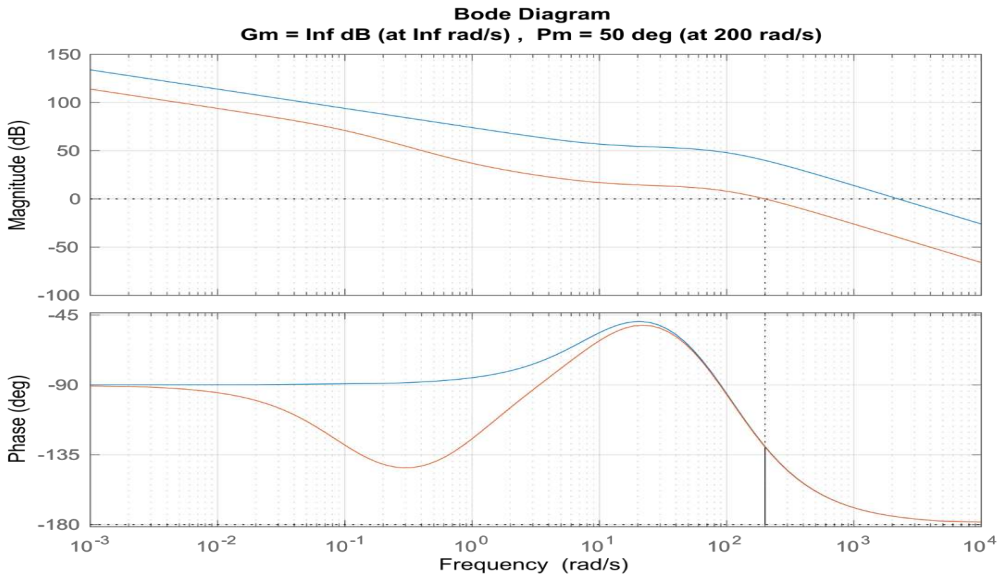
- MF = 50º
- e_{ss} = 0,002 (rampa unitária)

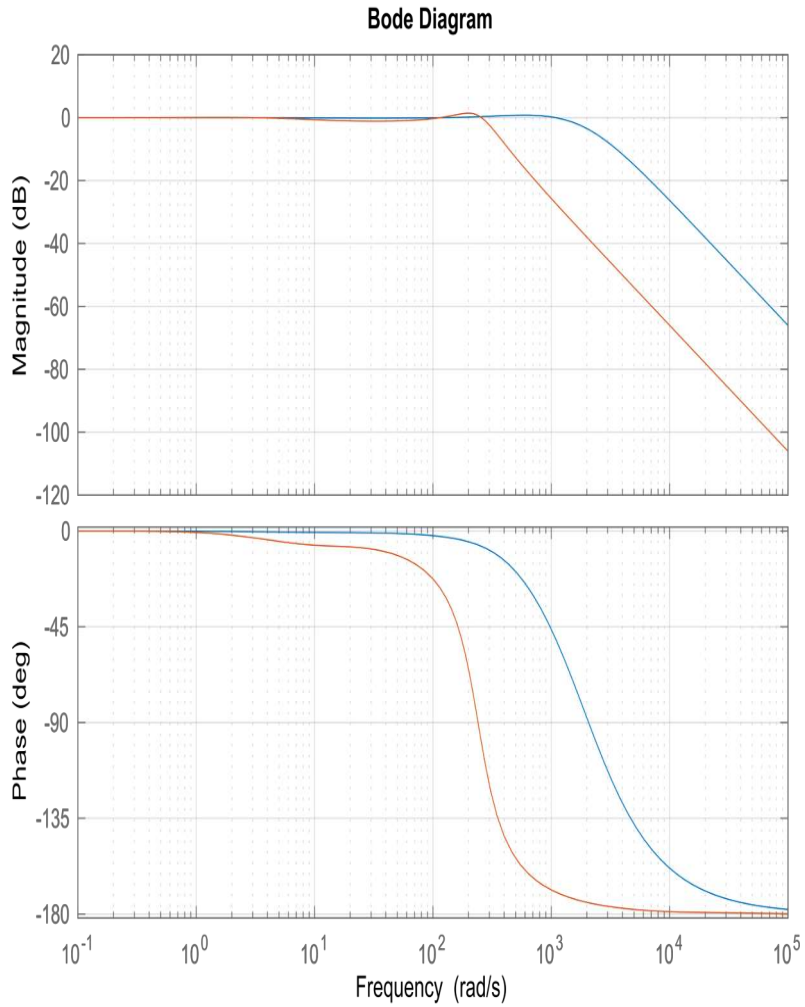
O compensador em avanço torna o sistema mais rápido!
O compensador em atraso deve ser utilizado quando há saturação do atuador!

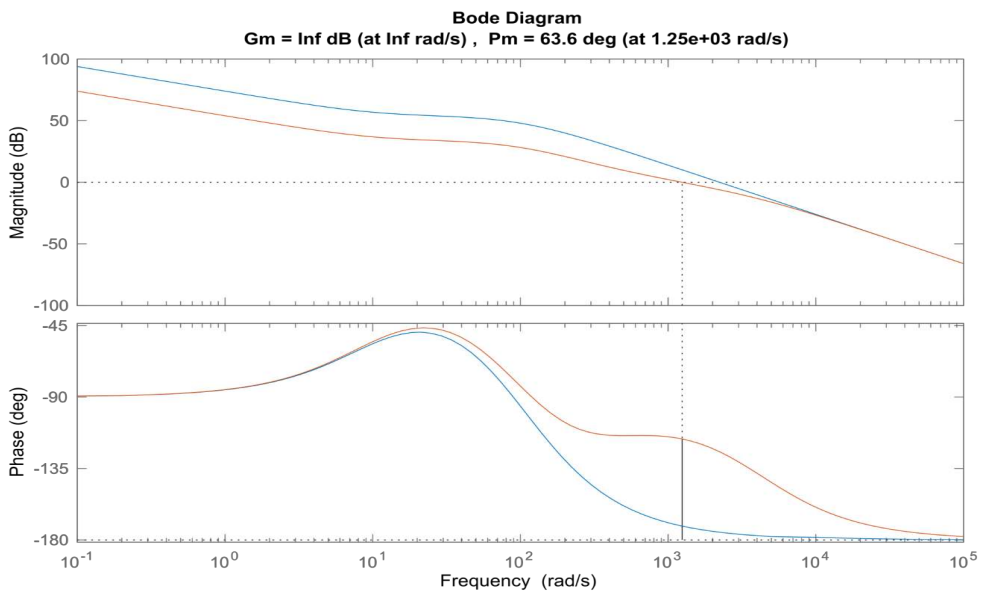




Comparação: Atraso x Avanço e Bode MF







A Banda Passante do compensador em avanço é mais larga!

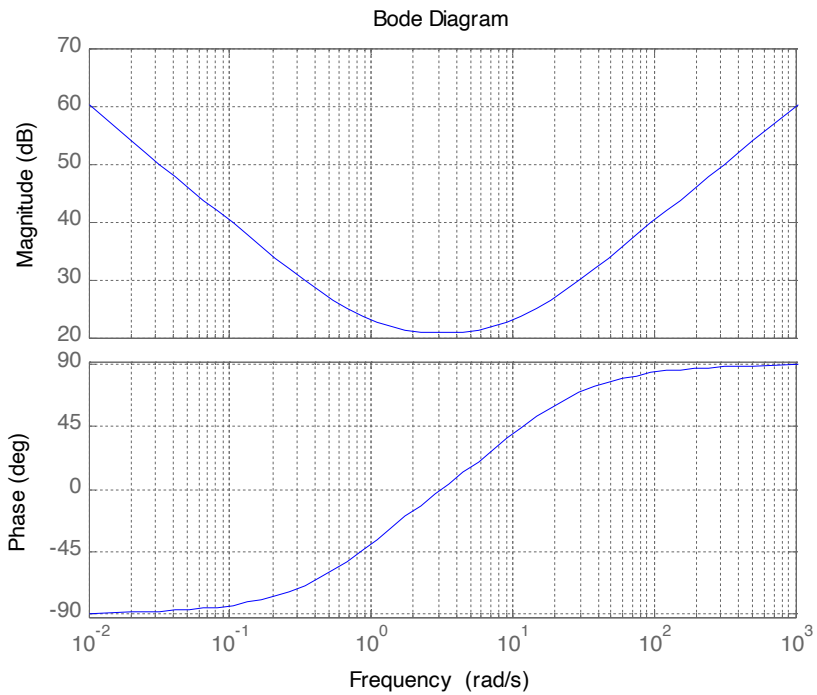
Verificar U(s)/R(s)!!

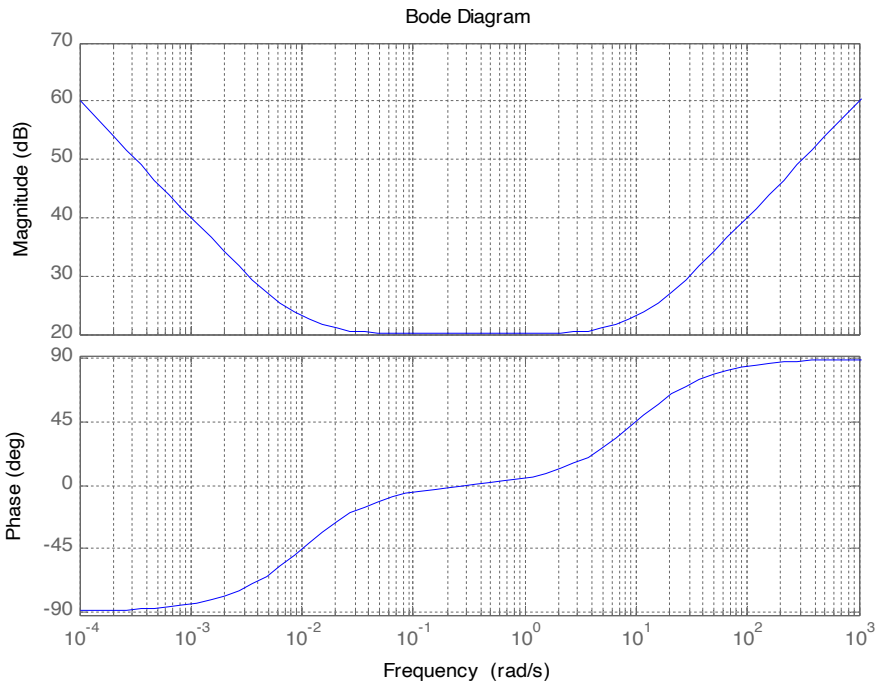
PID no domínio da frequência

$$D(s) = \frac{K(s+z_1)(s+z_2)}{s}$$

Ex.1: $$D(s) = \frac{(s+1)(s+10)}{s}$$

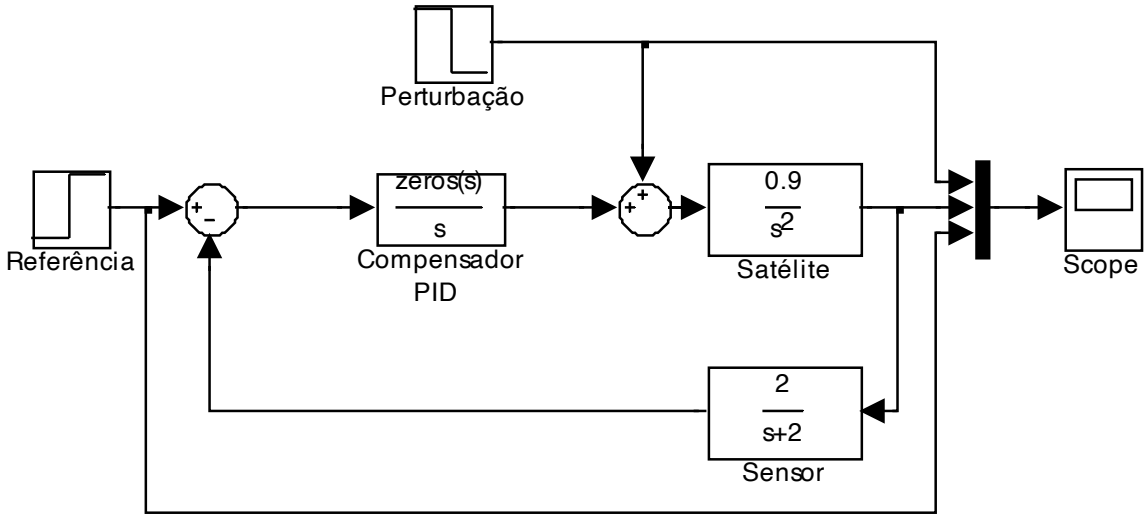
Ex. 2 $$D(s) = \frac{(s+0{,}01)(s+10)}{s}$$





Ex: PID satélite

$$G(s) = \frac{0{,}9}{s^2}\frac{2}{s+2}$$

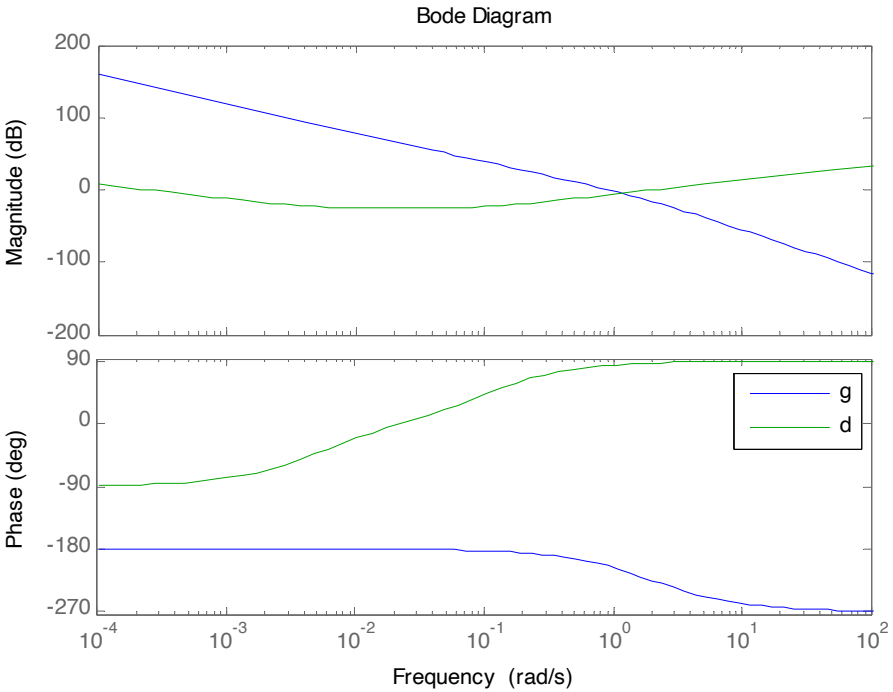


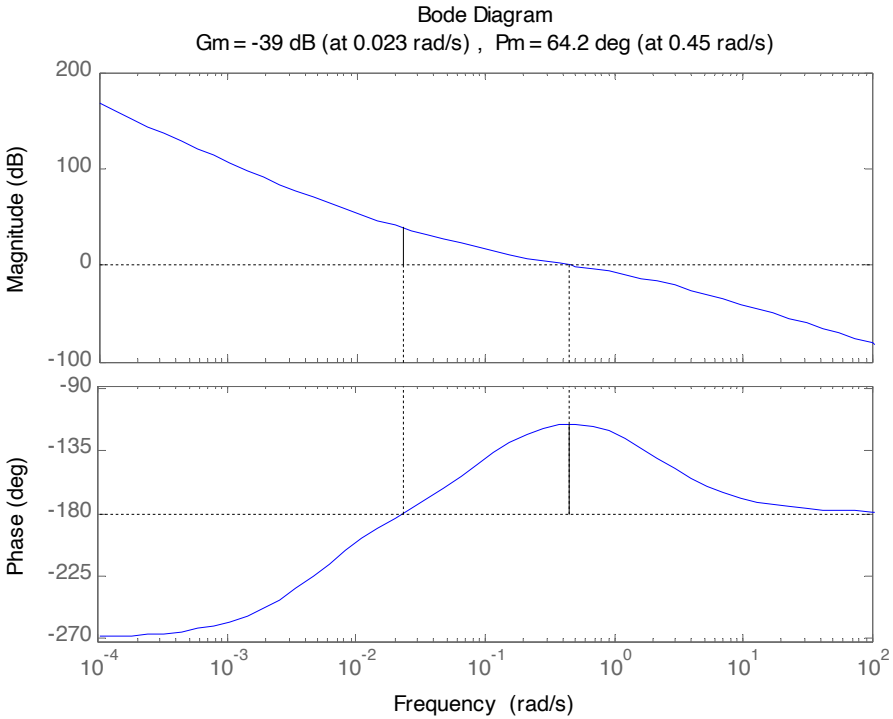
Especificações:

- e_{ss} = 0 para perturbações constantes
- MF = 65°
- Banda passante o mais larga possível

PID no domínio da frequência

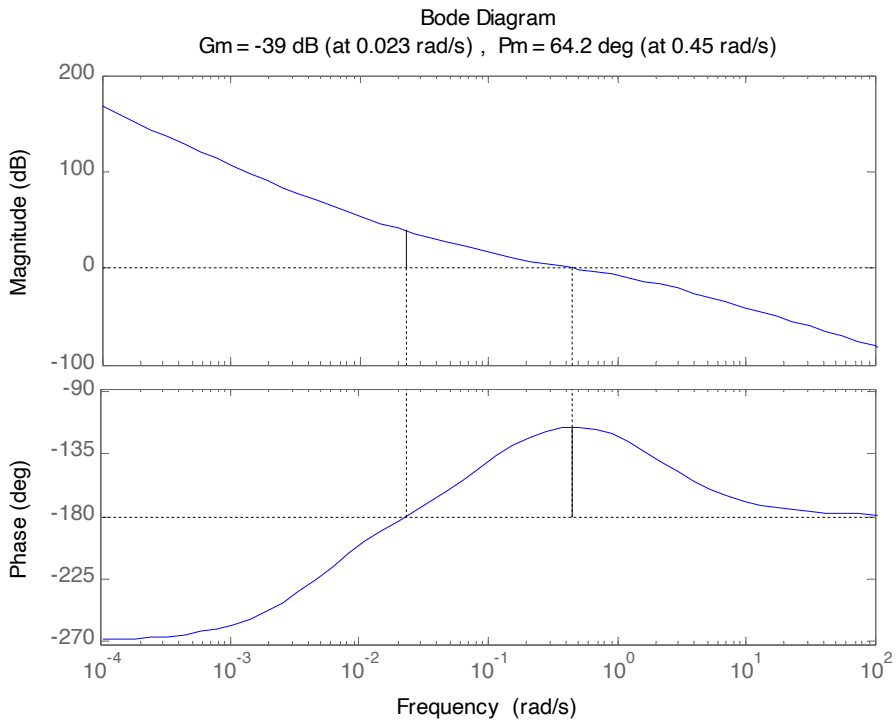
$$D(s) = \frac{K(s+z_1)(s+z_2)}{s}$$

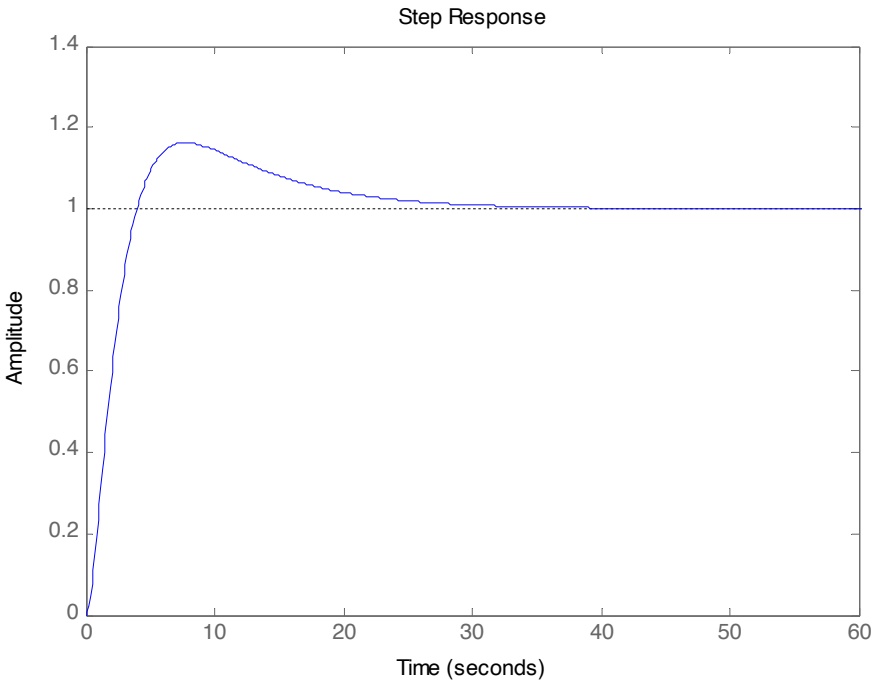




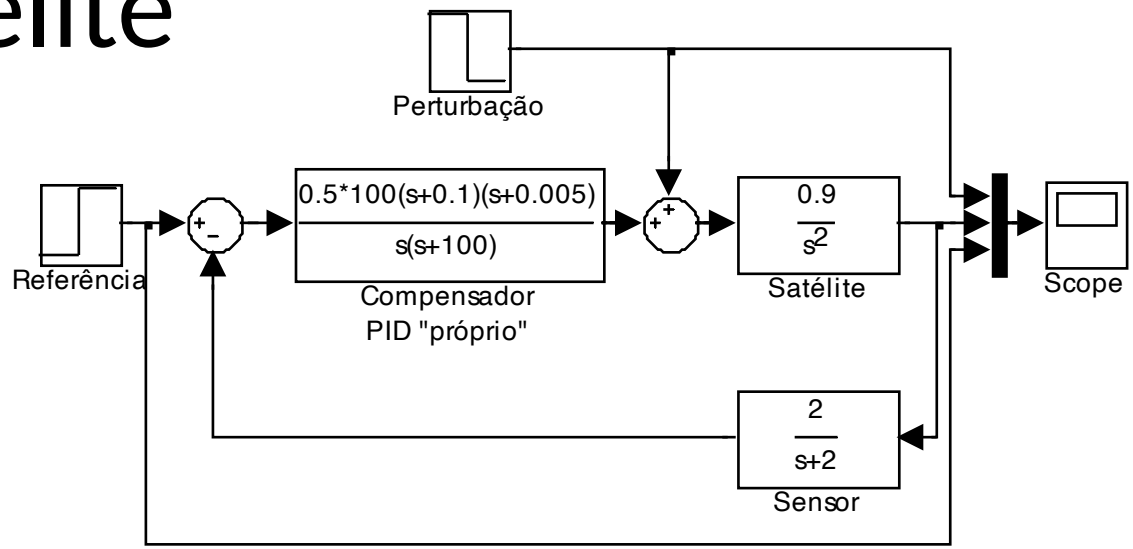
PID no domínio da frequência

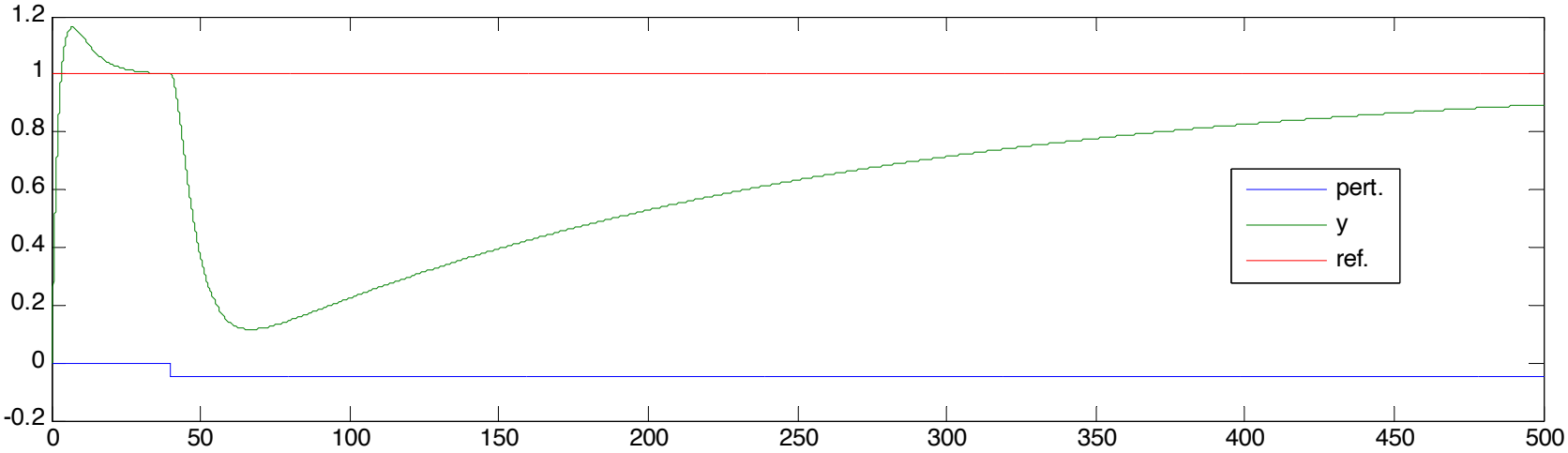
$$D(s) = \frac{0{,}5(s+0{,}1)(s+0{,}005)}{s}$$





PID satélite





Projeto PID em ω

$$D(s) = K_p + \frac{K_i}{s} + K_d s = K_c\left(1 + T_d s + \frac{1}{T_i s}\right) = K_c \frac{T_i T_d s^2 + T_i s + 1}{T_i s}$$

$$= \underbrace{\left(K\frac{1+sT_1}{s}\right)}_{\text{PI}} \underbrace{(1+sT_2)}_{\text{PD}}$$

Caso1: PI x PD

Especificações e_{ss} não definem Kc/Ti.
Espeficicação severas de e_{ss}.

$$= \underbrace{\frac{K}{s}}_{\text{I}} \underbrace{(1+sT_1)}_{\text{PD}} \underbrace{(1+sT_2)}_{\text{PD}}$$

Caso2: I x PD x PD

Especificações e_{ss} definem Kc/Ti.
Espeficicação severas de e_{ss}.

Veja apresentação detalhada em:

"Chapter 5 **Frequency-domain design,** Control Automático 3° Curso. Ing. Industrial Escuela Técnica Superior de Ingenieros Universidad de Sevilla", transparência 137 e seguintes:
http://control-class.com/ch_5/slides/frequency_domain_design.pdf

PID Método 2, I+PD+PD: $\frac{K}{s}(1+sT_1)(1+sT_2)$

Kc/Ti é definido pelas especificações de regime permanente.

- Escolha o *K* minimo para satisfazer e_{ss}.
- Plote o diagrama de Bode de G'(s)=K/s.G(s).
- Projete os 2 PDs para atender à especificação transitória (PO e/ou w_c') (Sugestão: Adote T_1=T_2)
- Calcule os parâmetros do PID:

$$K_c = K\frac{T_1+T_2}{T_1}; \quad T_i = T_1 + T_2; \quad T_d = \frac{T_1 T_2}{T_1+T_2}.$$

Ex. Projeto PID

$$G(s)=\frac{1}{s(s+1)(s+10)}$$

Especificações:

- Parábola $e_{ss} \leq 0{,}5$
- $Mp \leq 40\%$

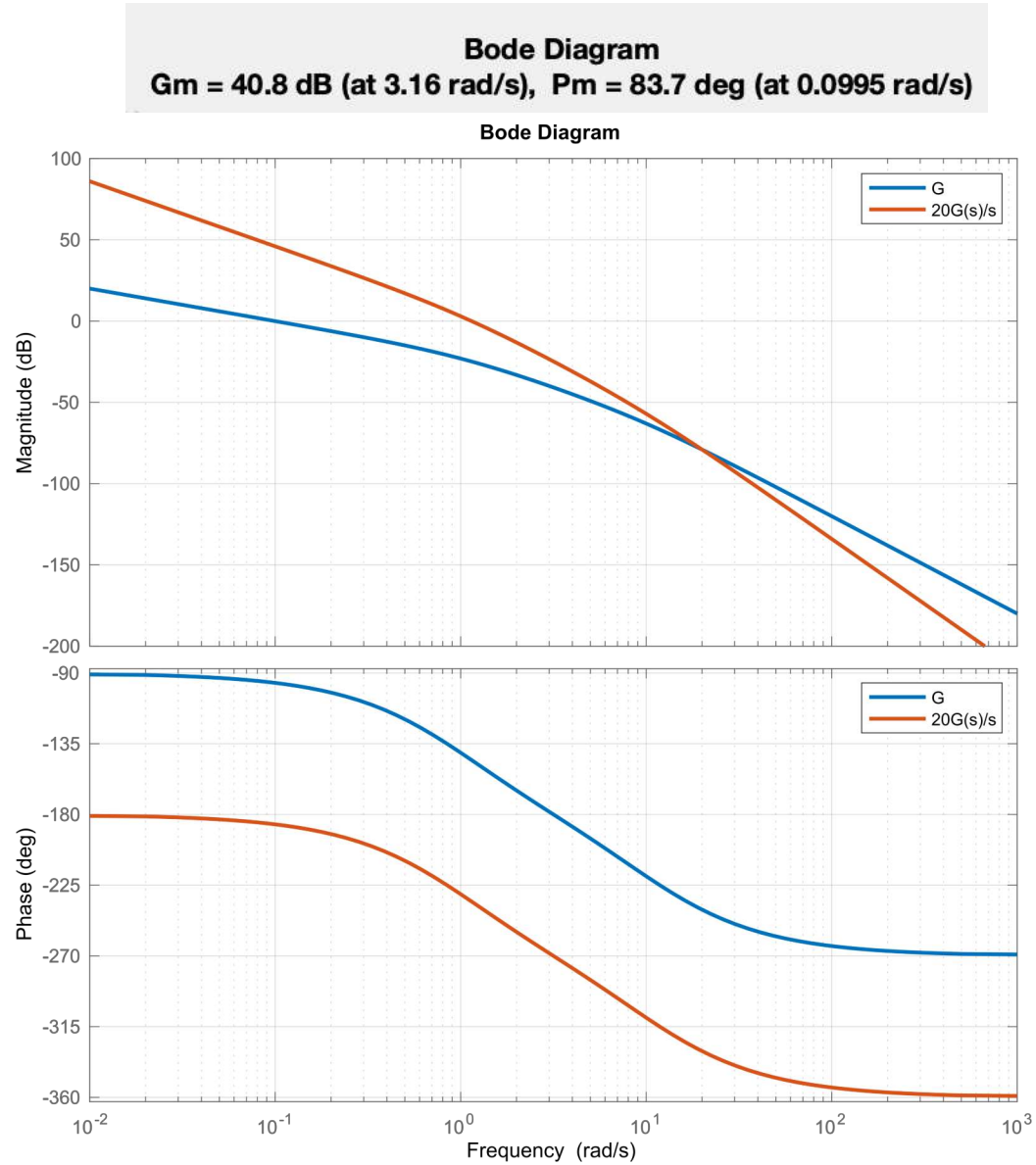
Projeto: 2o método

1. K G(s)/s
2. $(s+z_1)(s+z_2)$

e_{ss} só pode ser atendido por PI ou PID

Ka com canal I = lim s^2 G(s)/s = 0,1

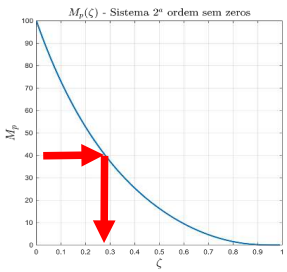
$e_{ss} = 1/K'a \rightarrow K'a \geq 2 \quad \rightarrow \quad K = 20$

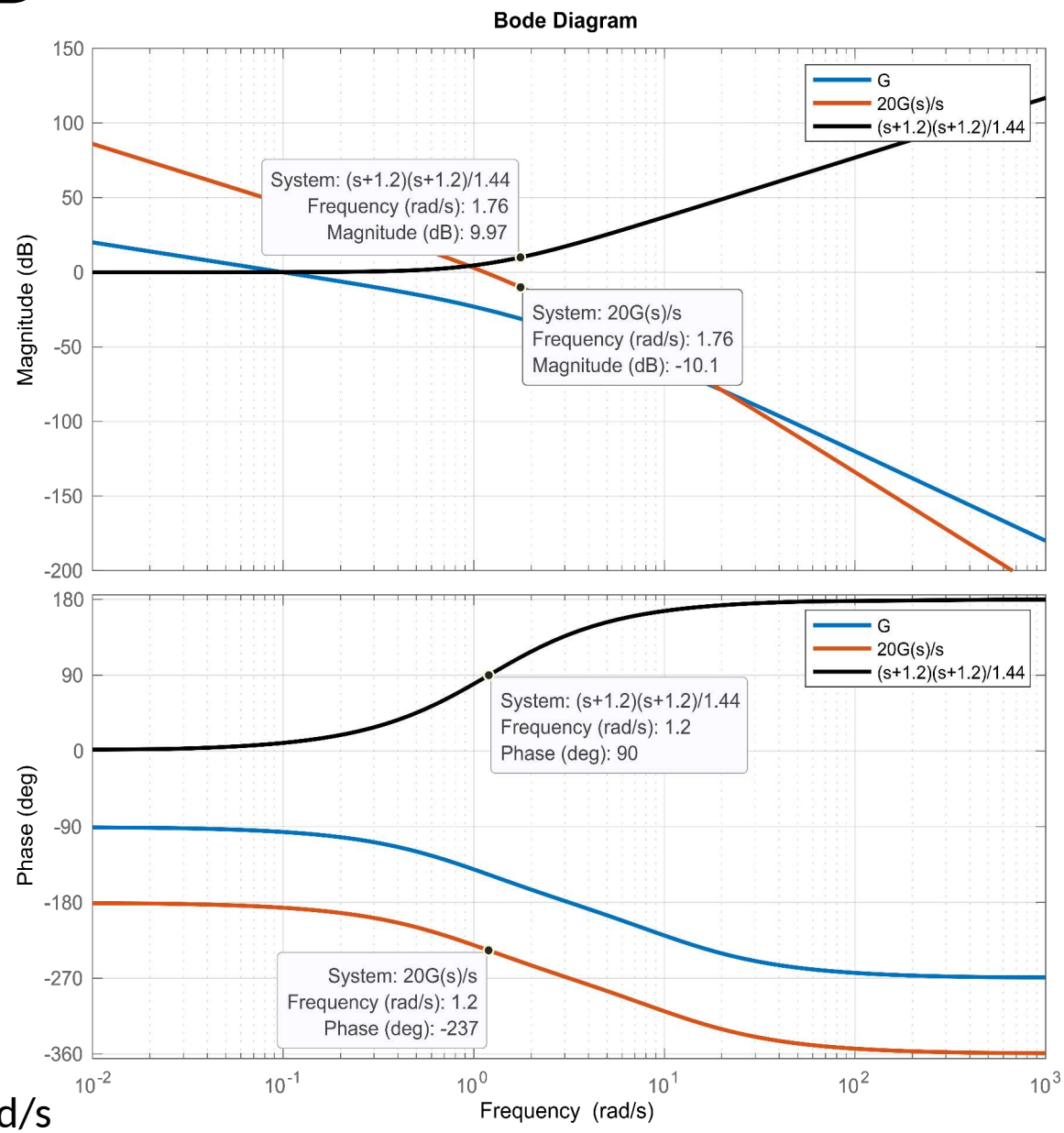


Ex. Projeto PID

–2º Método

1. K G(s)/s
2. $(s+z_1)(s+z_2)$





E_{ss} ->PI ou PID

$Mp \leq 40\% \rightarrow \zeta \geq 0{,}27 \rightarrow MF \geq 27°$

Ka com canal I = lim s^2 G(s)/s = 0,1

$e_{ss} = 1/K'a \rightarrow K'a \geq 2 \quad \rightarrow \quad K = 20$

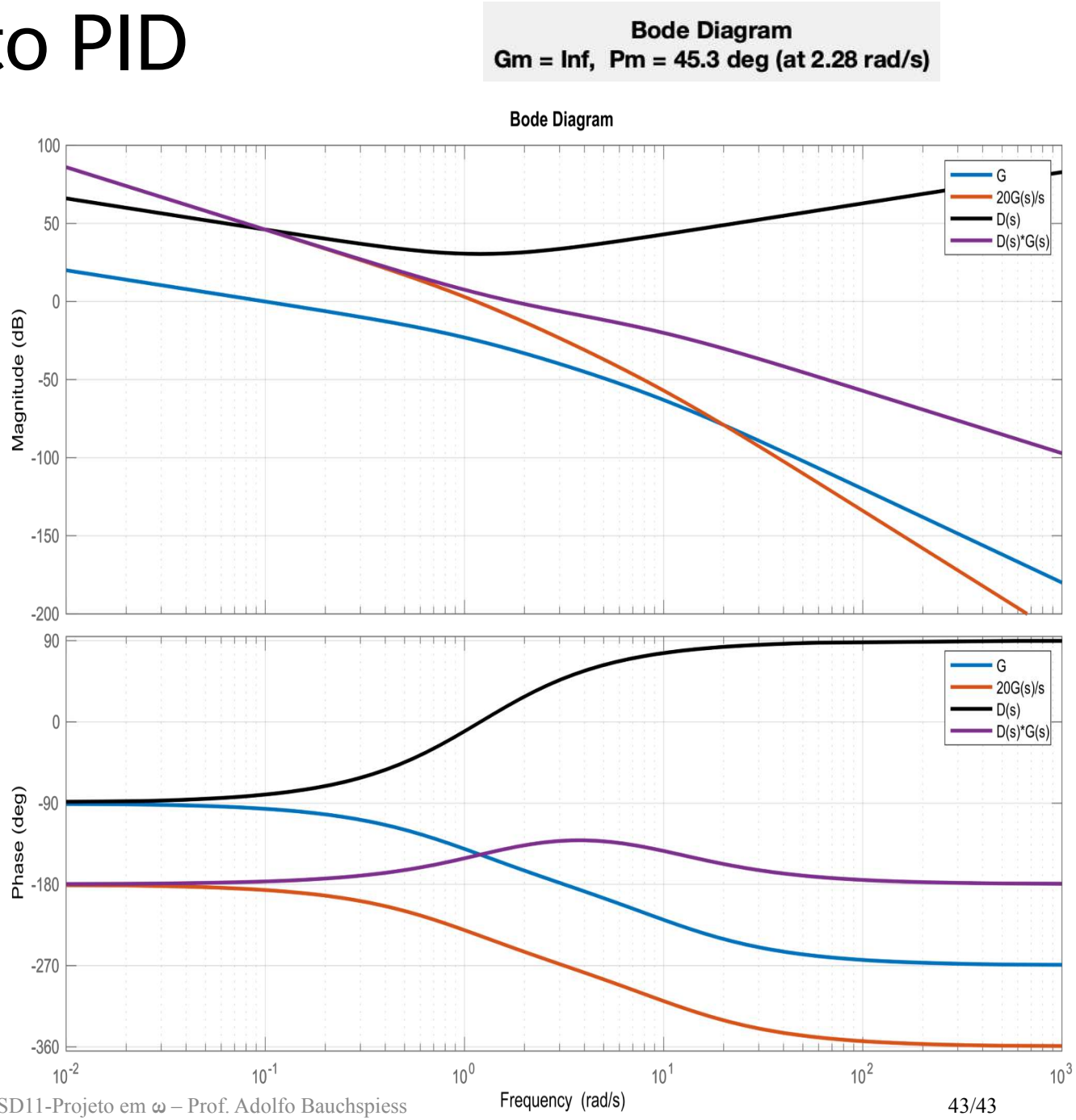
Com $z_1 = z_2 = -1.2$

MF seria 180-147=33° (1.2rad/s)

Porém ω_{MG} é deslocada para 1.76 rad/s

Ex. Projeto PID

–2º Método



$$G(s)=\frac{1}{s(s+1)(s+10)}$$

Com $z_1 = z_2 = -1.2$

$$PID(s)=\frac{20}{s}\frac{(s+1.2)^2}{1.2^2}$$

$$PID(s)=\frac{13.89(s+1.2)^2}{s}$$

→ MF = 40.9° (1.75 rad/s)