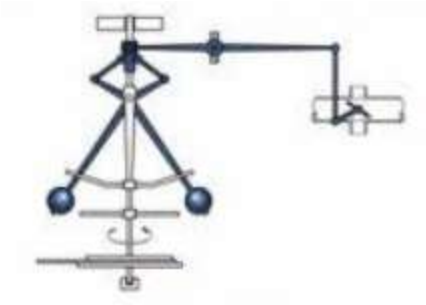
# Controle de Sistemas Dinâmicos

## CSD6-MAxMF_TipoEss



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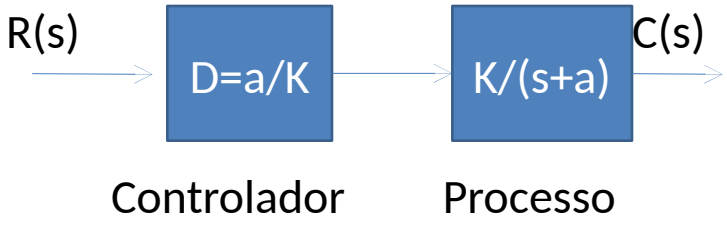
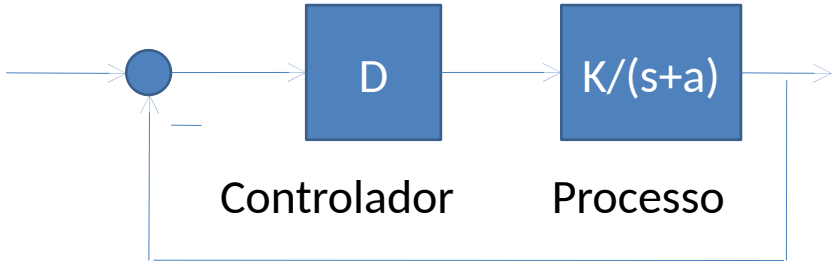
# Teoria da Realimentação

Qual esquema de controle é melhor, MA ou MF?

Critérios:

1. Sensibilidade à variação de parâmetros
2. Rejeição de Perturbações
3. Acompanhamento de Sinais



# 1- Sensibilidade

$$S_{F:P} = \lim_{\Delta P \to 0} \frac{Variação\ \%\ de\ F}{Variação\ \%\ de\ P}$$

*F*- Função
*P*- Parâmetro

$$S_{F:P} = \lim_{\Delta P \to 0} \frac{\Delta F / F}{\Delta P / P}$$

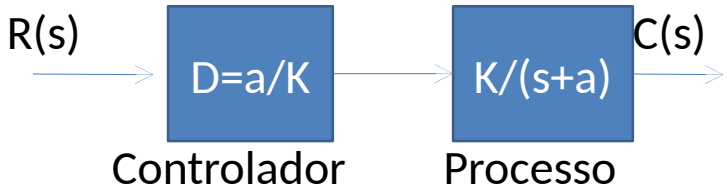
$$S_{F:P} = \lim_{\Delta P \to 0} \frac{P}{F} \frac{\Delta F}{\Delta P}$$

$$\boxed{S_{F:P} = \frac{P}{F} \frac{\partial F}{\partial P}}$$

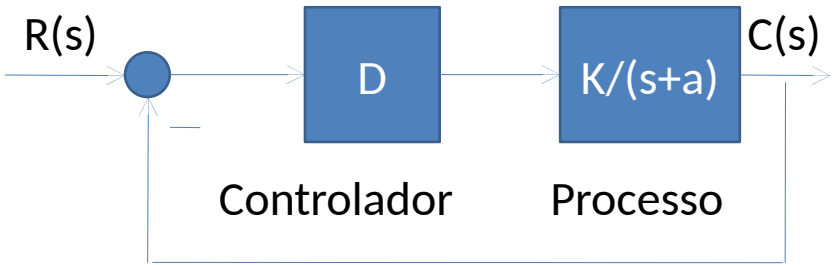
*Sensibilidade da função F em relação a variações do parâmetro P*

# Exemplo: MA x MF

$$S_{F:P} = \frac{P}{F}\frac{\partial F}{\partial P}$$




$$F_{MA} = \frac{DK}{s+a} \qquad S_{F_{MA}:K} = \frac{K}{\frac{DK}{s+a}}\frac{D}{s+a} = 1$$




$$F_{MF} = \frac{\frac{DK}{s+a}}{1+\frac{DK}{s+a}} = \frac{DK}{s+a+DK}$$

$$S_{F_{MF}:K} = \frac{K}{\frac{DK}{s+a+DK}}\frac{D(s+a+DK)-DDK}{(s+a+DK)^2} = \frac{s+a}{s+a+DK}$$

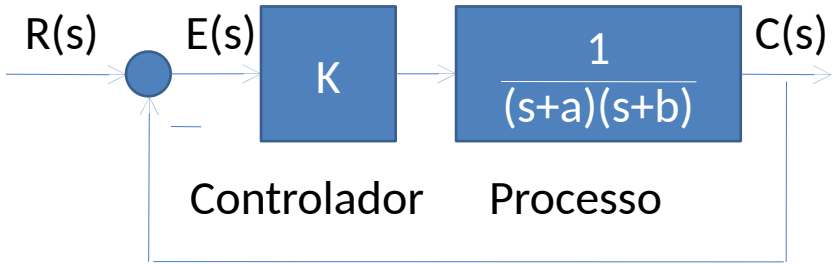
Se DK↑ → S↓ DK = Ganho de Malha

e.g. DK=99a → S=0,01

1% de variação em K
→ 0,01% de var. em F

# Sensibilidade do erro(∞) com entrada degrau

Kp=K/ab



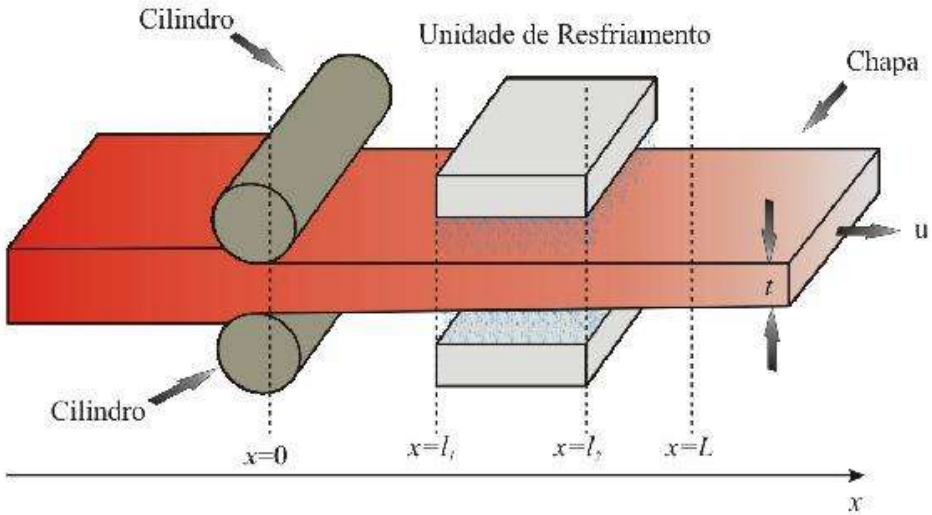

$$e(\infty) = \frac{1}{1+K_p} = \frac{1}{1+\frac{K}{ab}} = \frac{ab}{ab+K}$$

$$S_{e(\infty):a} = \frac{a}{e}\frac{\partial e}{\partial a} = \frac{a}{\frac{ab}{ab+K}}\frac{b(ab+K)-bab}{(ab+K)^2} = \frac{K}{ab+K}$$

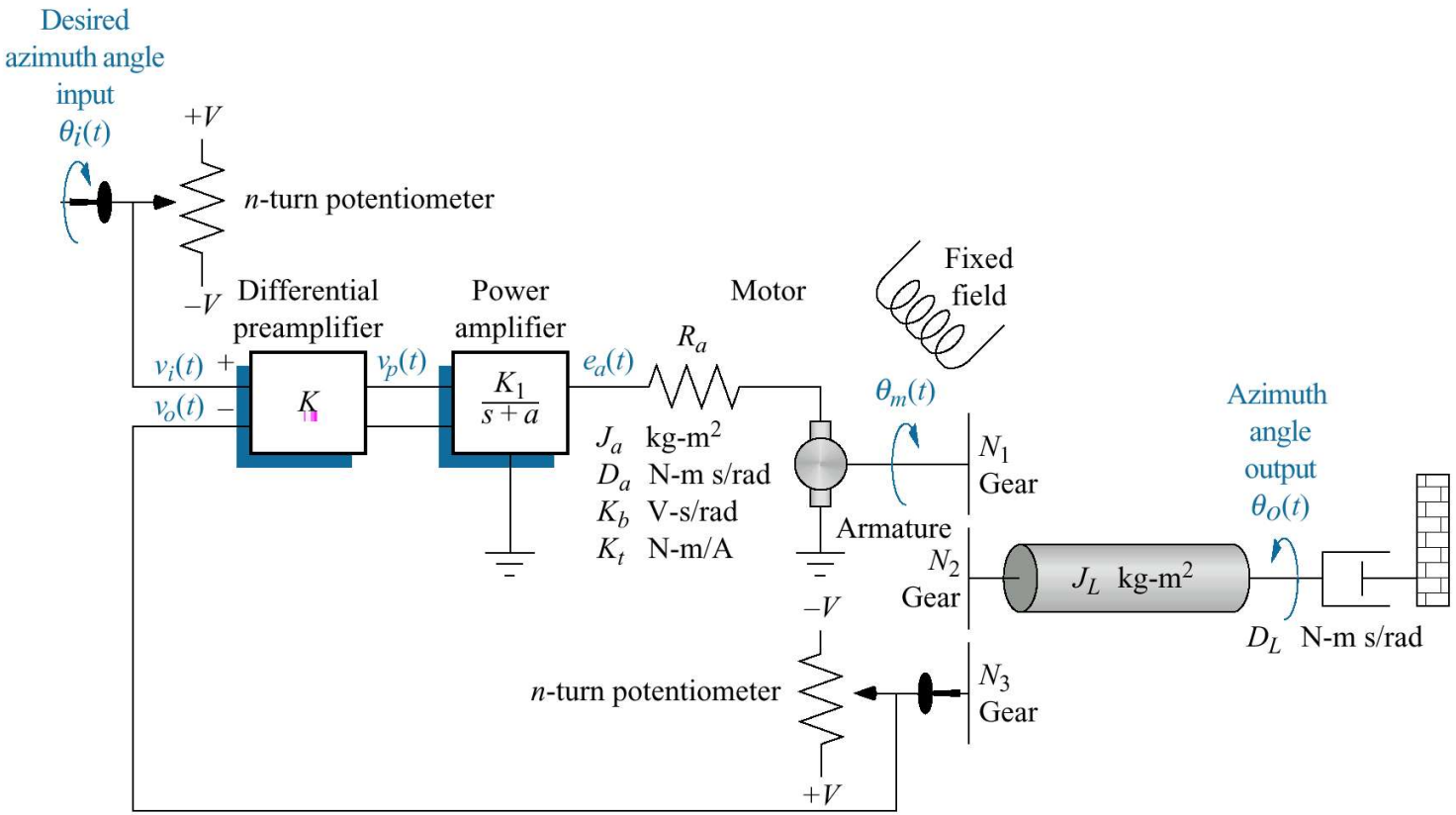
$$S_{e(\infty):K} = \frac{K}{e}\frac{\partial e}{\partial K} = \frac{K}{\frac{ab}{ab+K}}\frac{-ab}{(ab+K)^2} = \frac{-K}{ab+K}$$

# 2 – Rejeição de Perturbações (MA x MF)

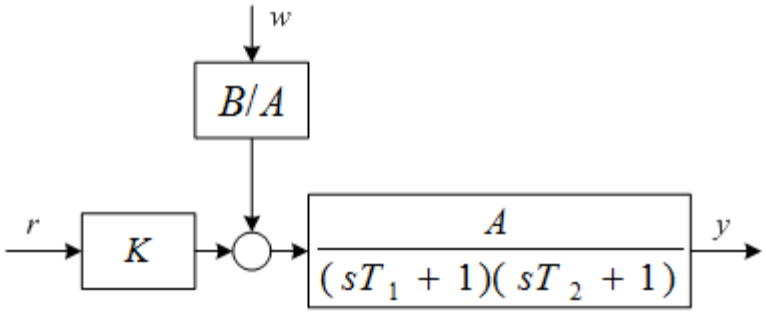
Laminação a quente

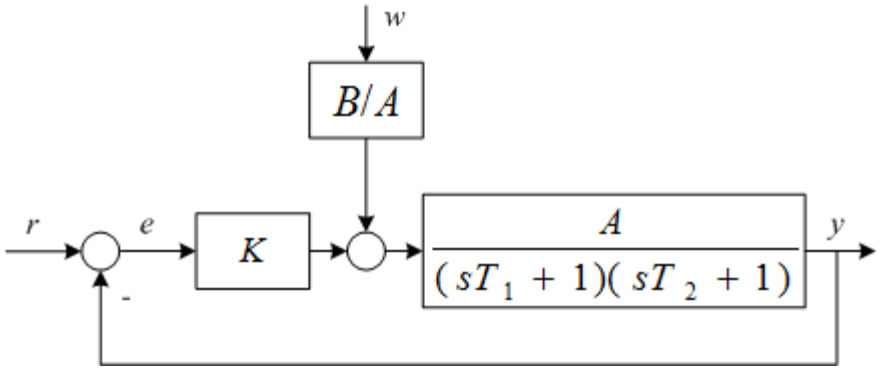


# Motor CC em MF



# Rejeição de Perturbações (MA x MF)




$$Y(s) = \frac{A}{(sT_1+1)(sT_2+1)}V_a(s) + \frac{B}{(sT_1+1)(sT_2+1)}W(s)$$

$$v_a = K(r - y)$$

Controlador Proporcional $K = 1/A$

$$y_{ss} = Av_a + Bw = AKr + Bw$$

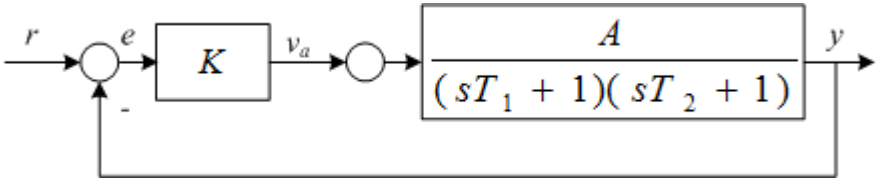
$$Y(s) = \frac{AKR(s) + BW(s)}{(sT_1+1)(sT_2+1) + AK}$$

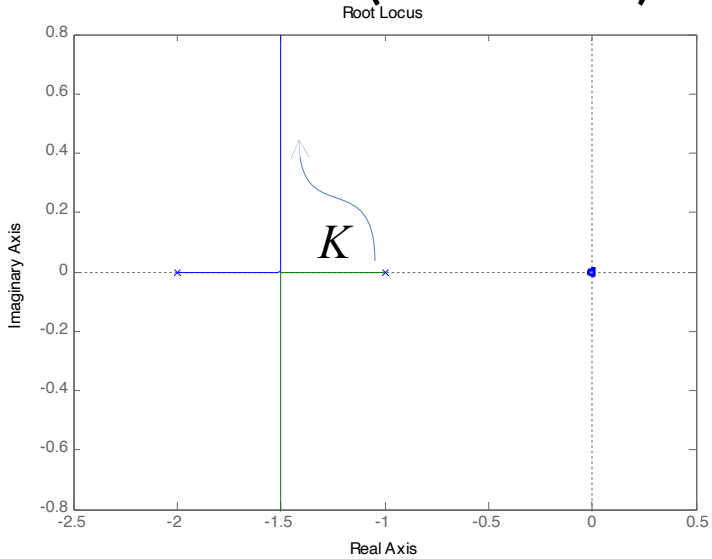
$$\boxed{y_{ss} = r + Bw}$$

$$\boxed{y_{ss} = \frac{AK}{1+AK}r + \frac{B}{1+AK}w}$$

# 3 - Acompanhamento de Sinais (MA x MF)

Se o sistema é lento
MA → novo projeto
MF → ajuste da resposta dinâmica

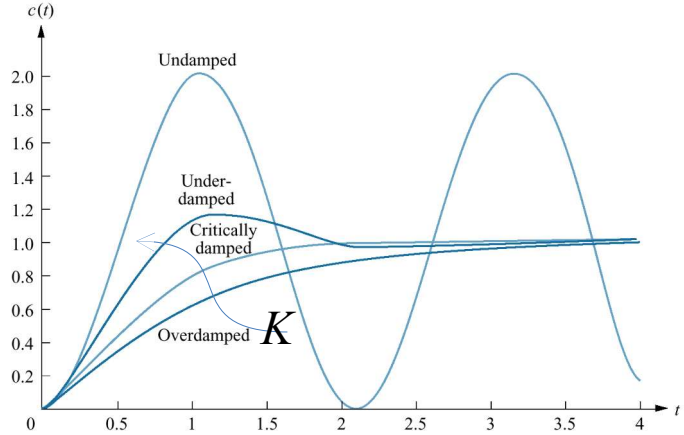



*eq. caracteristica*

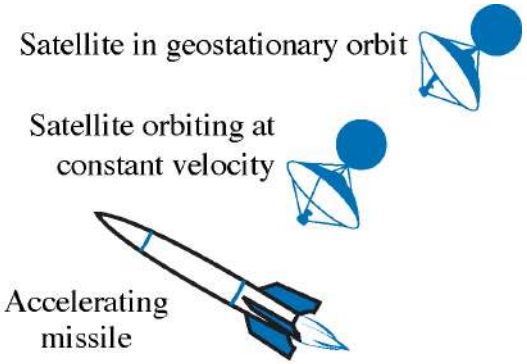
$$T_1T_2s^2 + (T_1+T_2)s + 1 + AK = 0$$

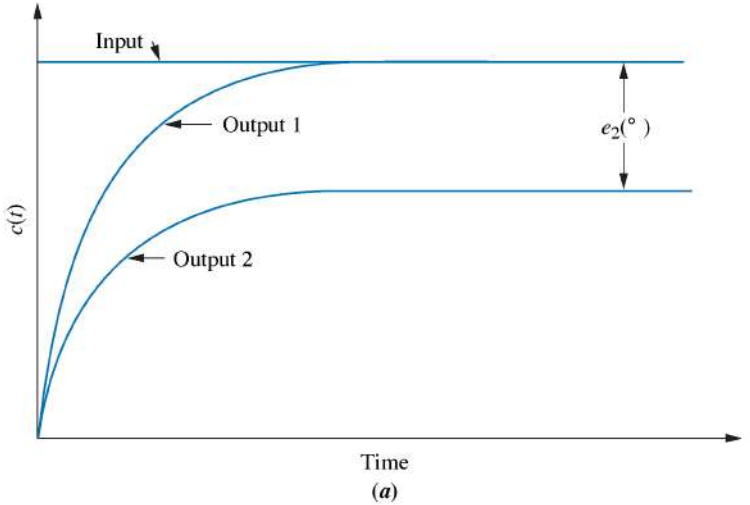
$$s = \frac{-(T_1+T_2) \pm \sqrt{(T_1+T_2)^2 - 4T_1T_2(1+AK)}}{2T_1T_2}$$

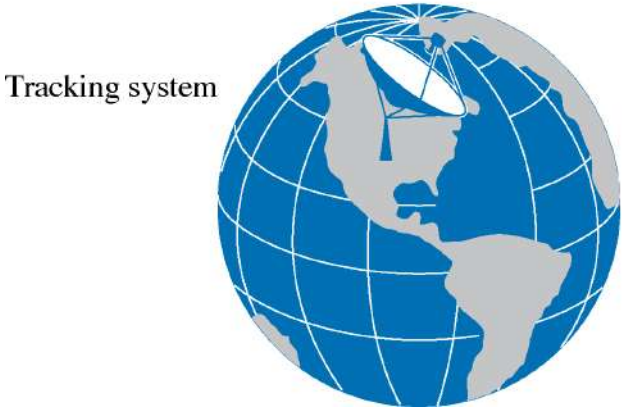


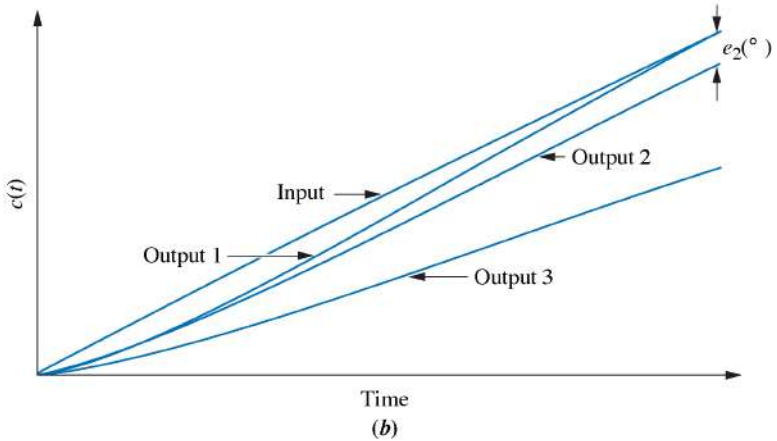
# Resposta em Regime Permanente

*Erro em Regime*: e(t→∞) Mede a capacidade de sistemas de acompanhar sinais em regime permanente



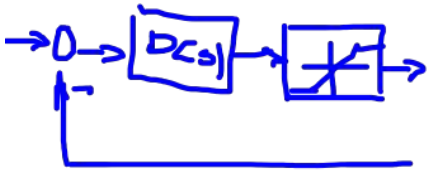



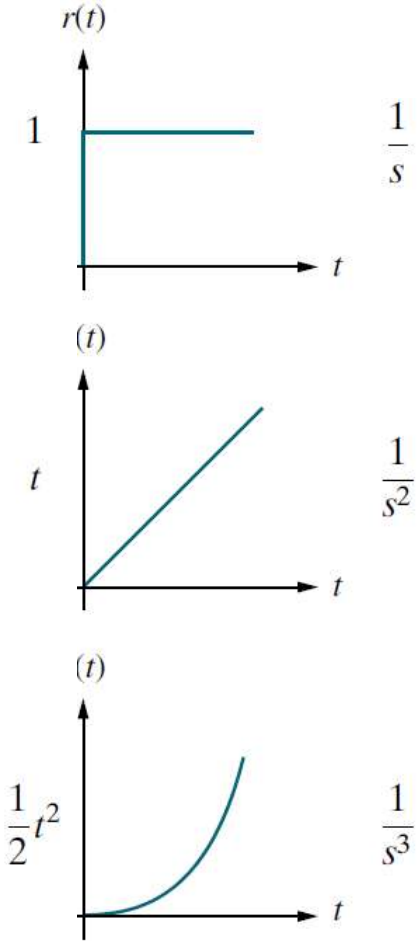



# Resposta em Regime Permanente

*Classificação de sistemas de controle*:
tipo 0,1,2,...
Quanto à habilidade de seguir entradas:
degrau, rampa, parábola,...





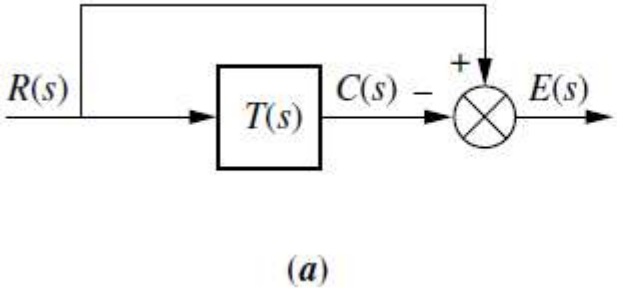

$$r(t) = \frac{t^k}{k!}1(t) \Leftrightarrow R(s) = \frac{1}{s^{k+1}}$$

Polinômios de grau k

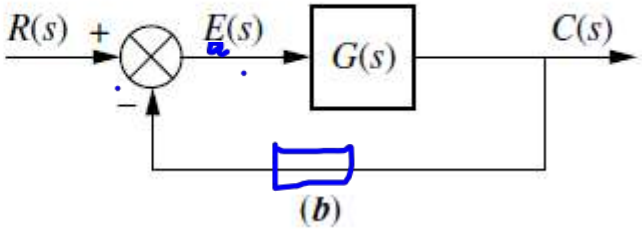
erro $\overset{\Delta}{=}$ saída desejada – saída medida

e = r - y

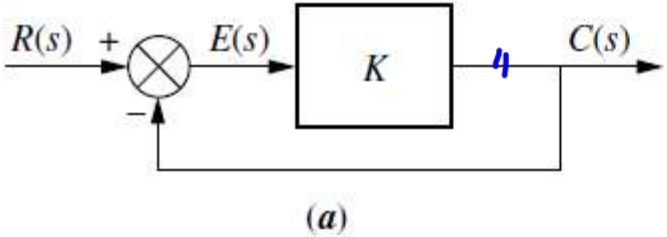
# Erro em malha fechada



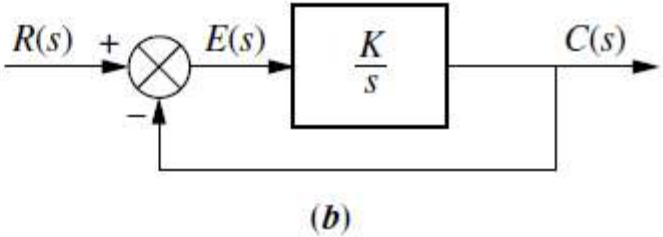

Erro genérico




Erro para realimentação unitária




$e_{ss}$ finito para degrau de referência




$e_{ss} = 0$ para degrau de referência

# Erro em função de G(s)

$$E(s) = R(s) - C(s)$$

$$C(s) = E(s)G(s)$$

$$E(s) = \frac{R(s)}{1 + G(s)}$$

$$\boxed{e(\infty) = \lim_{s \to 0} \frac{sR(s)}{1 + G(s)}}$$

$$com\ R(s) = \frac{1}{s}$$

$$\boxed{e(\infty) = e_{\text{step}}(\infty) = \lim_{s \to 0} \frac{s(1/s)}{1 + G(s)} = \frac{1}{1 + \lim_{s \to 0} G(s)}}$$

$K_p$

$$G(s) \equiv \frac{(s + z_1)(s + z_2) \cdots}{s^n (s + p_1)(s + p_2) \cdots}$$

$e_{ss} \to 0$ se $\lim_{s \to 0} G(s) = \infty$

# Erro em função de G(s)

$$G(s) \equiv \frac{(s+z_1)(s+z_2)\cdots}{s^n(s+p_1)(s+p_2)\cdots}$$

$$Rampa: R(s) = \frac{1}{s^2}$$

$$e(\infty) = e_{\text{ramp}}(\infty) = \lim_{s\to 0}\frac{s(1/s^2)}{1+G(s)} = \lim_{s\to 0}\frac{1}{s+sG(s)} = \frac{1}{\lim_{s\to 0} sG(s)}$$

$K_v$

$e_{ss} \to 0$ se $sG(s) \to \infty$

$$Parábola: R(s) = \frac{1}{s^3}$$

$$e(\infty) = e_{\text{parabola}}(\infty) = \lim_{s\to 0}\frac{s(1/s^3)}{1+G(s)} = \lim_{s\to 0}\frac{1}{s^2+s^2G(s)} = \frac{1}{\lim_{s\to 0} s^2G(s)}$$

$K_a$

$e_{ss} \to 0$ se $s^2G(s) \to \infty$

# Constantes de Erro

$$e(\infty) = e_{\text{step}}(\infty) = \frac{1}{1 + \lim_{s \to 0} G(s)}$$

Coeficiente de erro de posição:

$$K_p = \lim_{s \to 0} G(s)$$

$$e(\infty) = e_{\text{ramp}}(\infty) = \frac{1}{\lim_{s \to 0} sG(s)}$$

Coeficiente de erro de velocidade:

$$K_v = \lim_{s \to 0} sG(s)$$

$$e(\infty) = e_{\text{parabola}}(\infty) = \frac{1}{\lim_{s \to 0} s^2 G(s)}$$

Coeficiente de erro de aceleração:

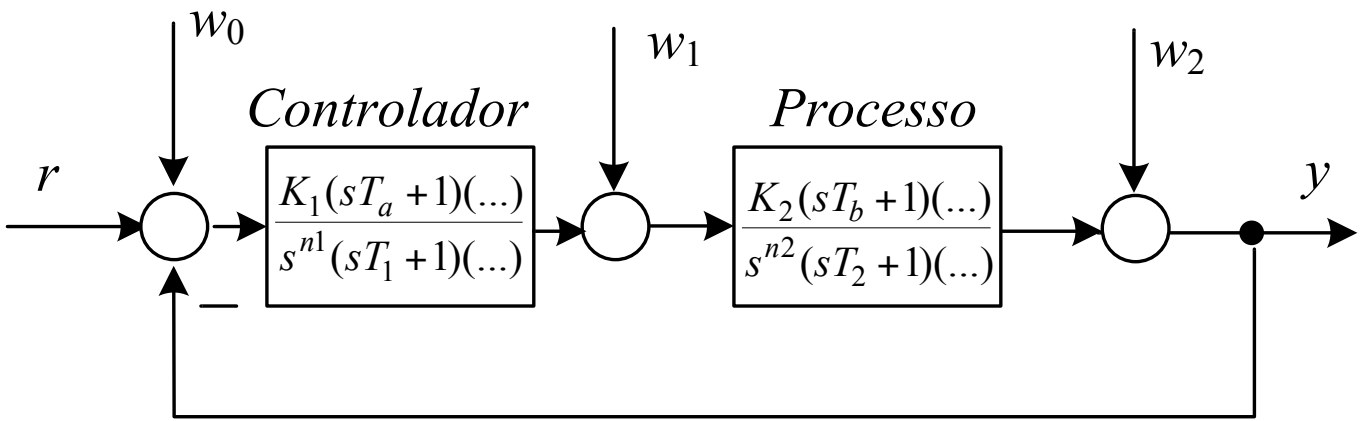
$$K_a = \lim_{s \to 0} s^2 G(s)$$

...Coeficiente de erro da derivada da aceleração

# Tabela: erro x tipo sistema x coef. erro x $e_{ss}$

| Input | Steady-state error formula | Type 0 Static error constant | Type 0 Error | Type 1 Static error constant | Type 1 Error | Type 2 Static error constant | Type 2 Error |
|---|---|---|---|---|---|---|---|
| Step, $u(t)$ | $\frac{1}{1+K_p}$ | $K_p = \text{Constant}$ | $\frac{1}{1+K_p}$ | $K_p = \infty$ | 0 | $K_p = \infty$ | 0 |
| Ramp, $tu(t)$ | $\frac{1}{K_v}$ | $K_v = 0$ | $\infty$ | $K_v = \text{Constant}$ | $\frac{1}{K_v}$ | $K_v = \infty$ | 0 |
| Parabola, $\frac{1}{2}t^2u(t)$ | $\frac{1}{K_a}$ | $K_a = 0$ | $\infty$ | $K_a = 0$ | $\infty$ | $K_a = \text{Constant}$ | $\frac{1}{K_a}$ |

# Múltiplas perturbações

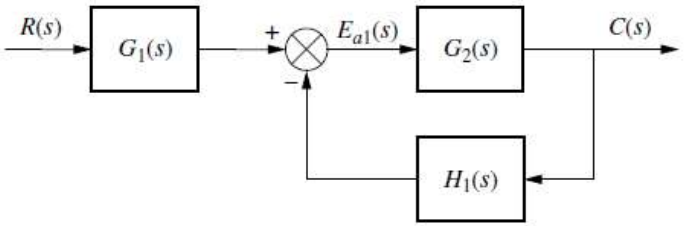



- “Rejeição: Devem haver integradores à esquerda da perturbação”

- O sistema é do tipo 0, $n1$, ($n1$+$n2$)
em relação às entradas de perturbação $w_0$,$w_1$ e $w_2$

- Perturbações na entrada ($w_0$) não são rejeitadas,
pois não é possível distinguí-las da entrada $r$

# Realimentação não-unitária



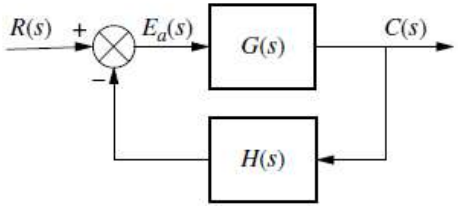

(a)

a) Calcular Direto

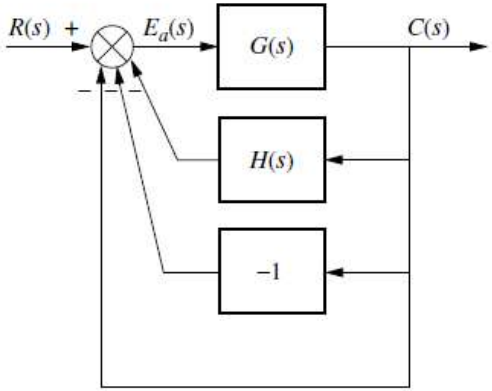
$$\frac{1}{s}$$

$$E = R - C$$

$$C = R\,\frac{G_1 G_2}{1 + G_1 G_2}$$

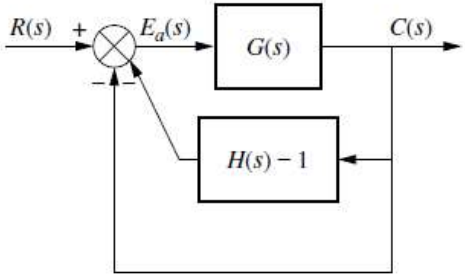



(b)

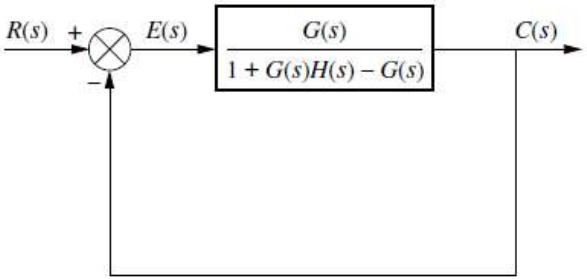



(c)

b) Rearranjo do diagrama de blocos

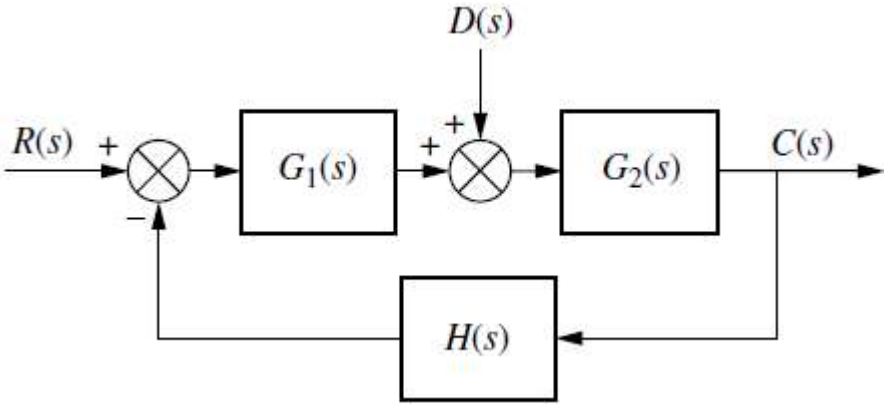



(d)



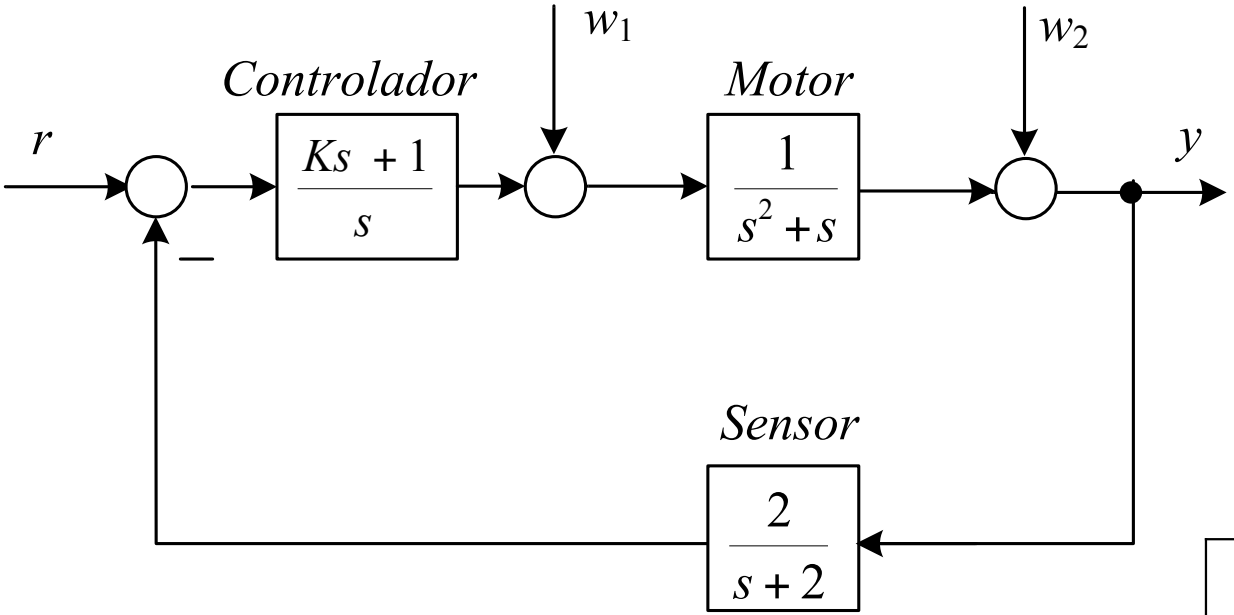

(e)

# Realimentação não unitária




$$e(\infty) = \lim_{s\to 0} sE(s) = \lim_{s\to 0} s\left\{\left[1 - \frac{G_1(s)G_2(s)}{1+G_1(s)G_2(s)H(s)}\right]R(s) - \left[\frac{G_2(s)}{1+G_1(s)G_2(s)H(s)}D(s)\right]\right\}$$

# Exercício Extra 1- $e_{ss}$




Preencha a tabela *com* $e_{ss}$(K=1)

| Entrada\ Sinal | $r$ | $w_1$ | $w_2$ |
|---|---|---|---|
| *Degrau* | 0 | 0 | 0 |
| *Rampa* | -1/2 | -1/K | 0 |
| *Parábola* | -∞ | -∞ | -1/K |

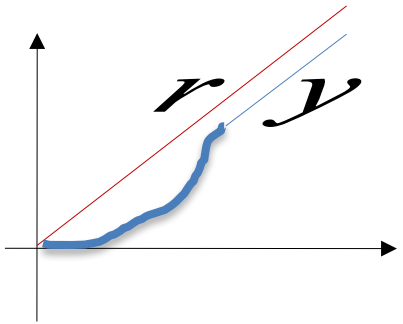
# Ex. Extra 1- $e_{ss}$

| | $w_1$ | $w_2$ |
|---|---|---|
| (r degrau) | | |
| $e_{ss}$ = 0 | =0 | =0 |
| | = 1/K | = 0 |
| (r rampa) | = ∞ | = 1/K |
| y →oo | | |
| $e_{ss}$ = -1/2 | | |

Obs: r,y → oo porém erro é finito!

(r parábola)

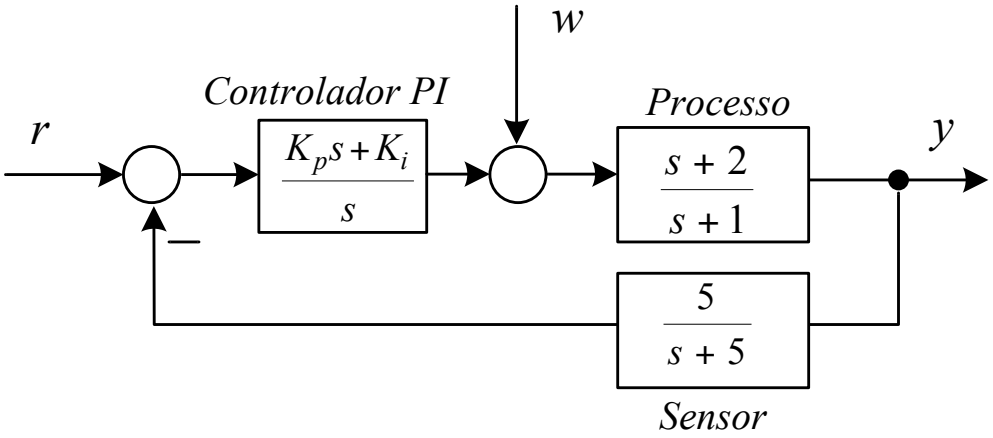
$e_{ss}$= -∞

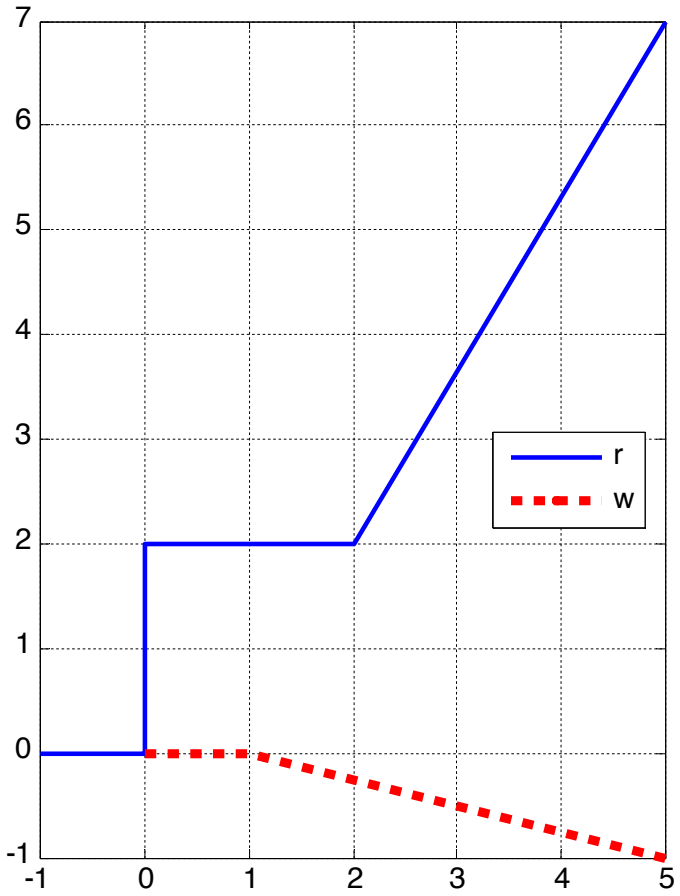


# Exercício Extra 2- $e_{ss}$

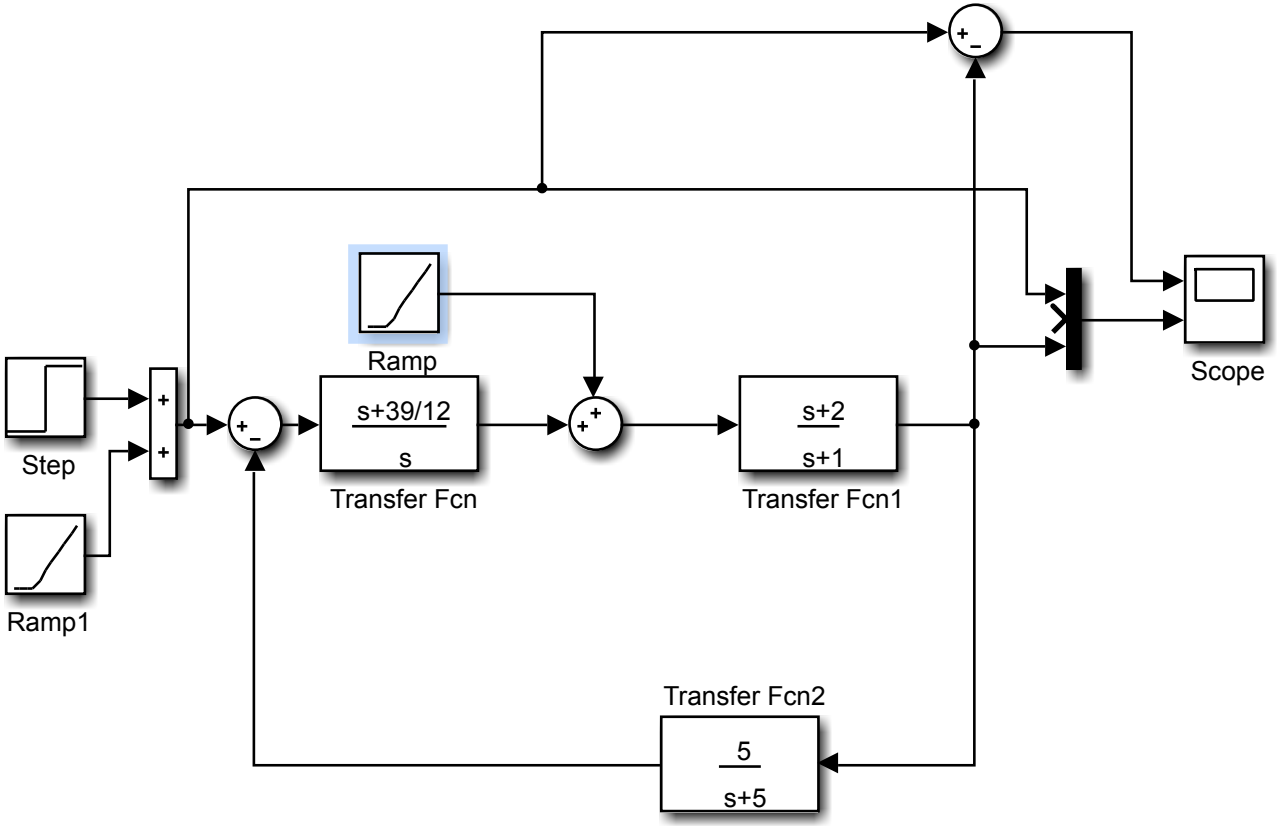
Considerando os sinais de referência e perturbação mostrados no gráfico, obtenha o erro em regime permanente para:

$K_i$ = 0,1*último dígito matrícula + 1;
$K_p$ = 10*penúltimo dígito matrícula + 1;

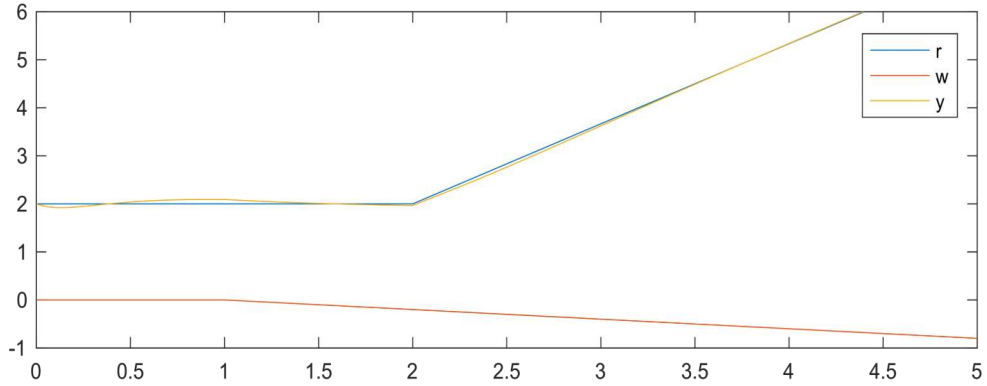


# Exemplo Extra 3 - $e_{ss}$

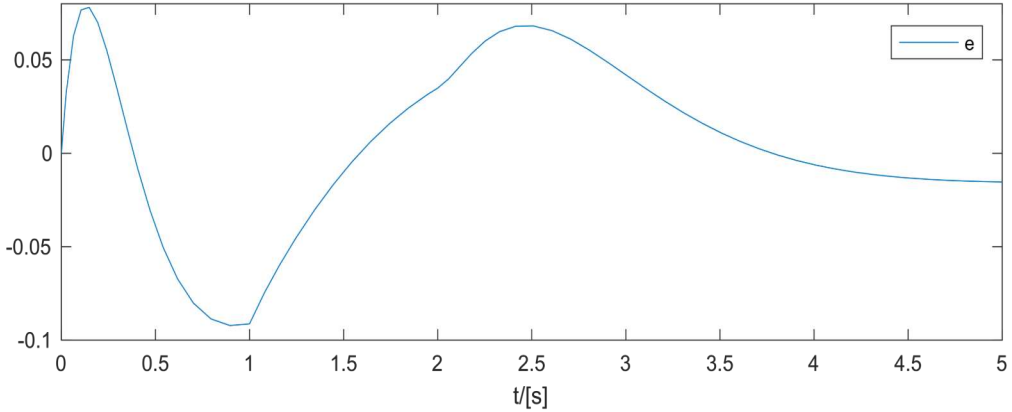


# Exemplo 3 - $e_{ss}$




Resposta completa:

$$y(t \to \infty) = y_r + y_w = 2 + \frac{5}{3}t.1(t-2) - \frac{5}{6K_i} + \frac{1}{3} - \frac{1}{4K_i}$$




$$y(t \to \infty) = \frac{5}{3}t.1(t-2) + \frac{7}{3} - \frac{13}{12K_i} \quad (1,0)$$

$$e(t \to \infty) = \frac{13}{12K_i} - \frac{1}{3}$$