1.º TESTE – 2/2014

Duração do teste: 20 minutos

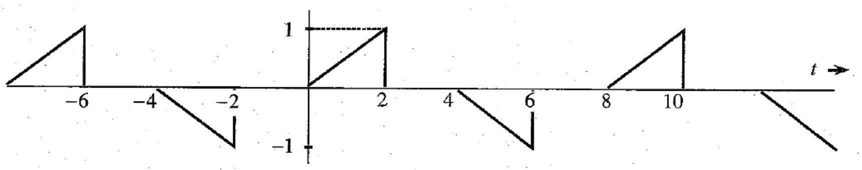
Nome: ____________________________________

Matrícula: ________________________________

Questão 1

Para o sinal periódico mostrado abaixo encontre a frequência fundamental f_0 e os coeficientes a_0, a_n e b_n de suas respectivas séries de Fourier trigonométrica.
Escreva os três primeiros termos da série trigonométrica de Fourier compacta.

(1)



$$f_0 = 1/8$$

$$a_0 = \frac{1}{T_s}\int_0^8 g(t)dt = 0$$

$$a_n = \frac{1}{4}\int_0^2 \frac{t}{2}\cos\left(\frac{\pi n}{4}t\right)dt - \frac{1}{4}\int_4^6 \frac{t-4}{2}\cos\left(\frac{\pi n}{4}t\right)dt$$

$$a_n = \frac{2}{\pi^2 n^2}\left[\cos\left(\frac{\pi n}{4}t\right) + \frac{\pi n}{4}t\sin\left(\frac{\pi n}{4}t\right)\right]_0^2 + \frac{2}{\pi^2 n^2}\left[\cos\left(\frac{\pi n}{4}t\right) + \frac{\pi n}{4}t\sin\left(\frac{\pi n}{4}t\right)\right]_4^6 + \left[\frac{2}{\pi n}\sin\left(\frac{\pi n}{4}t\right)\right]_4^6$$

$$a_n = \frac{2}{\pi^2 n^2}\left[\cos\left(\frac{\pi n}{2}\right) + \frac{\pi n}{2}\sin\left(\frac{\pi n}{2}\right) - 1 - \cos\left(\frac{3\pi n}{2}\right) - \frac{3\pi n}{2}\sin\left(\frac{3\pi n}{2}\right) + \cos(\pi n)\right] + \frac{2}{\pi n}\sin\left(\frac{3\pi n}{2}\right)$$

Para

$$n = 1,5,9,\ldots \Rightarrow a_n = \frac{2}{\pi^2 n^2}\left[0 + \frac{\pi n}{2} - 1 - 0 + \frac{3\pi n}{2} - 1\right] - \frac{2}{\pi n} = \frac{4}{n^2\pi^2}\left(\frac{n\pi}{2} - 1\right)$$

$$n = 0,2,4,\ldots \Rightarrow a_n = \frac{2}{\pi^2 n^2}[\pm 1 + 0 - 1 \mp 1 + 0 + 1] - 0 = 0$$

$$n = 3,7,11,\ldots \Rightarrow a_n = \frac{2}{\pi^2 n^2}\left[0 - \frac{\pi n}{2} - 1 - 0 - \frac{3\pi n}{2} - 1\right] + \frac{2}{\pi n} = -\frac{4}{n^2\pi^2}\left(\frac{n\pi}{2} + 1\right)$$

Agora, da mesma forma

$$b_n = \frac{1}{4}\int_0^2 \frac{t}{2}\sin\left(\frac{\pi n}{4}t\right)dt - \frac{1}{4}\int_4^6 \frac{t-4}{2}\sin\left(\frac{\pi n}{4}t\right)dt$$

$$a_n = \frac{2}{\pi^2 n^2}\left[\sin\left(\frac{\pi n}{4}t\right) - \frac{\pi n}{4}t\cos\left(\frac{\pi n}{4}t\right)\right]_0^2 + \frac{2}{\pi^2 n^2}\left[\sin\left(\frac{\pi n}{4}t\right) - \frac{\pi n}{4}t\cos\left(\frac{\pi n}{4}t\right)\right]_4^6 - \left[\cos\left(\frac{\pi n}{4}t\right)\right]_4^6$$

$$b_n = \frac{4}{n^2\pi^2}\sin\left(\frac{n\pi}{2}\right), \qquad n = 1,3,5,7,9,11,13\ldots$$

Fórmulas úteis

Série de Fourier Trigonométrica:

$$g(t) = a_0 + \sum_{n=1}^{\infty} a_n \cos(2\pi n f_0 t) + \sum_{n=1}^{\infty} b_n \sin(2\pi n f_0 t) = C_0 + \sum_{n=1}^{\infty} C_n \cos(2\pi n f_0 t + \theta_n)$$

$$a_0 = \frac{1}{T_0}\int_{T_0} g(t)dt; a_n = \frac{2}{T_0}\int_{T_0} g(t)\cos(2\pi n f_0 t); b_n = \frac{2}{T_0}\int_{T_0} g(t)\sin(2\pi n f_0 t)$$

$$C_0 = a_0; C_n = \sqrt{a_n^2 + b_n^2}, \quad \theta_n = \begin{cases} \tan^{-1}(-b_n/a_n) & , a_n \geq 0 \\ \tan^{-1}\left(-\frac{b_n}{a_n}\right) + \pi & , a_n < 0 \end{cases}$$

Identidades Trigonométricas:

$$\sin(2x) = 2\sin x\cos x$$

$$\cos(2x) = \cos^2 x - \sin^2 x$$

$$\sin^2 x + \cos^2 x = 1$$

$$\cos^2 x = \frac{1}{2}(1 + \cos 2x)$$

$$\sin^2 x = \frac{1}{2}(1 - \cos 2x)$$

$$\sin(x \pm y) = \sin x\cos y \pm \cos x\sin y$$

$$\cos(x \pm y) = \cos x\cos y \mp \sin x\sin y$$

$$\tan(x \pm y) = \frac{\tan x \pm \tan y}{1 \mp \tan x\tan y}$$

$$a\cos x + b\sin x = \sqrt{a^2 + b^2}\cos\left(x + \tan^{-1}(-b/a)\right)$$

Integrais:

$$\int x\sin ax\,dx = \frac{1}{a^2}(\sin ax - ax\cos ax)$$

$$\int x\cos ax\,dx = \frac{1}{a^2}(\cos ax + ax\sin ax)$$

$$\int x^2\sin ax\,dx = \frac{1}{a^3}(2ax\sin ax + 2\cos ax - a^2x^2\cos ax)$$

$$\int x^2\cos ax\,dx = \frac{1}{a^3}(2ax\cos ax - 2\sin ax + a^2x^2\sin ax)$$

$$\int xe^{ax}dx = \frac{e^{ax}}{a^2}(ax - 1)$$

$$\int x^2e^{ax}dx = \frac{e^{ax}}{a^3}(a^2x^2 - 2ax + 2)$$

$$\int e^{ax}\cos bx\,dx = \frac{e^{ax}(a\cos bx + b\sin bx)}{a^2 + b^2}$$

$$\int e^{ax}\sin bx\,dx = \frac{e^{ax}(a\sin bx - b\cos bx)}{a^2 + b^2}$$

$$\int \frac{1}{x^2 + a^2}dx = \frac{1}{a}\tan^{-1}\frac{x}{a}$$

$$\int \frac{x}{x^2 + a^2}dx = \frac{1}{2}\ln(x^2 + a^2)$$

$$\int \frac{x}{a^4 + x^4}dx = \frac{1}{2a^2}\tan^{-1}\frac{x^2}{a^2}$$