State Space Control and Extended Kalman Filter Estimation Applied to a 3rd Order Liquid Level Process

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Abstract **- This paper presents results of the use of simultaneous state feedback control state/parameter estimation of a third order liquid level process. In this process, a state space controller with integral channel driven by the estimated signals is investigated. Effects such as valve opening and operating point changing are correctly identified by an Extended Kalman Filter (EKF). On the other hand, non-modeled stick-slip of the level sensor, leads to an erroneous interpretation of process behavior by the EKF.**

Index Terms **— State Space Control, Extended Kalman Filter, Non-linear effects, stick-slip, liquid level process.**

I. INTRODUCTION

In order to investigate the application of control techniques and estimation, a third order liquid level process was implemented. This system, shown in Figure Fig. 1, is highly configurable. Depending on the positioning of the valves, one can have a first, second or third order process, as well as the number of inputs/outputs can be modified. For each connection, the liquid flow depends on the square root of the liquid level, which imposes a strong non-linear characteristic of this system. Such a system is commonly found in the industry, and measurements are perturbed my sensor noise and imperfections.

Fig. 1 - Picture of the liquid level process

In this work, results on state feedback control and parameter estimation applied to a third order liquid level process are discussed. State space control needs a model and the measurement (or observation) of the state variables in order to feedback them. Theoretically, an arbitrary pole placement could be obtained. As a linear approach, an operating point of the non-linear process is assumed. In practice, however, an accurate model of the system is difficult to be obtained, in part due to approximations on modeling as well as on changes on the parameters of valves models. In order to automatically identify the new model parameters, a stochastic approach as been implemented. Indeed, an Extended Kalman Filter (EKF) [7][9] is used to simultaneously estimate the process model parameters (valves load loss) and signals (levels) that are used by a state space controller with integral channel. Such a strategy has shown satisfactory results in other areas, such as in mobile robotics [2] or in catalytic reactors [10].

The block diagram of Figure 2 shows the main components of this research. The plant (liquid level process), is controlled by a state space controller, which uses smooth estimates of the reservoirs levels as state feedback. These estimates are provided by an EKF, which also computes estimates of the plant parameters. However, for the current investigation, there has been no interest on feeding back the estimated parameters, resulting on adaptive control. The main concern is on the validity of the estimated parameters.

This paper is organized as follows. Section II presents the non-linear continuous time model of the liquid level system. Control structure and real-time parameter estimation models are presented in Section III. The context of this study is on remote laboratory experiments, for which the general structure is presented in Section IV. Experimental results are discussed in Section V.

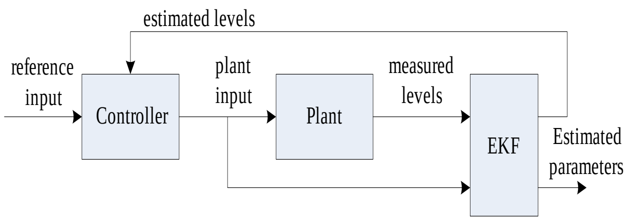

Fig. 2 – Block diagram of the control system.

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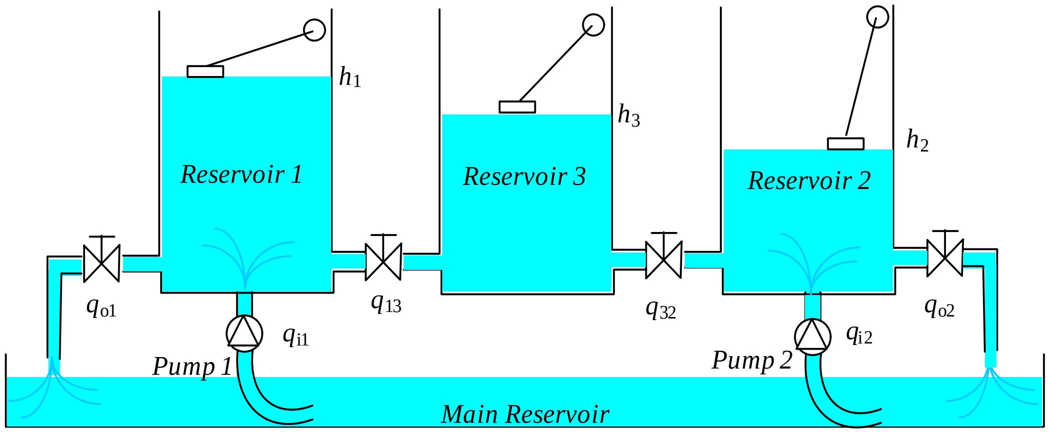

Fig. 3 - Multi-variable liquid level schematics.

II. PROCESS MODEL

Considering the process in Figure 3, the following variables can be defined:

- q_i , input flow [cm^3/s] in reservoir 1;
- q_{13}, q_{32} , flows between reservoir [cm^3/s];
- q_o , output flow [cm^3/s] from reservoir 2;
- h_1, h_2, h_3 , liquid levels [cm] of reservoirs 1, 2 and 3;
- A - reservoir transversal section area [cm^2], supposed the same for the three reservoirs.

By applying Bernoulli's law in the mass balance, one has the following non-linear dynamic model:

$$
\begin{aligned}
A\frac{dh_1}{dt} &= q_i + k_{13}\sqrt{|h_1 - h_3|} \\
A\frac{dh_2}{dt} &= k_{32}\sqrt{|h_3 - h_2|} - k_2\sqrt{h_2} \\
A\frac{dh_3}{dt} &= k_{13}\sqrt{|h_1 - h_3|} - k_{32}\sqrt{|h_3 - h_2|}
\end{aligned}
\tag{1}
$$

A.Linearized Model

The levels h_1, h_2 *and* h_3 are the state variables, named henceforth x_1,x_2 and x_3, and represented in vector form as $\boldsymbol{x} = [x_1\ x_2\ x_3]^T$. In this work, the flow q_i is the input, x_2 is the output of the system. Linearizatiuon of the non-linear model around an operating point leads to the following model:

$$
\begin{bmatrix} \dot{\delta x}_1 \\ \dot{\delta x}_2 \\ \dot{\delta x}_3 \end{bmatrix} =
\begin{bmatrix}
0 & 0 & \frac{1}{CR_{13}} \\
0 & -\frac{1}{C}\frac{R_2+R_{32}}{R_2R_{32}} & \frac{1}{CR_{32}} \\
-\frac{1}{CR_{13}} & -\frac{1}{CR_{32}} & -\frac{1}{C}\frac{R_{13}+R_{32}}{R_{13}R_{32}}
\end{bmatrix}
\begin{bmatrix} \delta x_1 \\ \delta x_2 \\ \delta x_3 \end{bmatrix} +
\begin{bmatrix} \frac{1}{C} \\ 0 \\ 0 \end{bmatrix} [\delta u]
$$

and $y = x_2$ being the system output. In such a model, δx_1, δx_2, δx_3 and δu are small variations around the operating point [8]. R_i and C are obtained from the non-linear model parameters. They are equivalent to resistences and capacitances in electric circuits.

III. SIMULTANEOUS STATE SPACE CONTROL AND PARAMETER ESTIMATION

State space control can take benefit of all the values estimated by the EKF. This approach is, however, very dependent on correct model structure and is quite sensitive to sensor errors. Thus, correct estimation of model parameters can improve control performance. *A priori* estimates of the dynamic model parameters can be obtained from steady state of the system. However, such parameters can change according to operating points due to non-modeled effects. This makes necessary the use of real-time parameter estimation simultaneously with system control.

A. State space control

Parameter estimation is performed with the system in closed-loop with a controller. With the controller, in order to achieve zero steady error, the actuator signal includes an integral channel, considering the steady state operating point, the pole placement and reference signal:

$$u(t) = U_{SS} - K(\mathbf{x}(t) - X_{SS}) - K_I \int (y(t) - r(t))dt$$

with *r(t)* being the desired h_2 level. The operating point is represented by U_{SS} and X_{SS} for the input u and the state vector $\boldsymbol{x}$. The gains K and K_I are determined based on the desired location of linearized system poles.

B.Real-time parameter estimation

Consider $\theta = [k_2 \quad k_{13} \quad k_{32}]^T$ the vector of model parameters. The recursive and simultaneous estimation of these parameters and the state vector $\boldsymbol{x}$ is performed using a linearized version of the Kalman filter [9], the suboptimal Extended Kalman Filter (EKF). In order to achieve this, the following augmented model for the *discrete time* describes the evolution of parameters and states [7]:

$$\mathbf{z}_k = \begin{bmatrix} \mathbf{x}_k \\ \boldsymbol{\theta}_k \end{bmatrix} = \mathbf{F}(\mathbf{x}_{k-1}, \boldsymbol{\theta}_{k-1}) + \mathbf{w}_k \tag{2}$$

with

$$\mathbf{F}(\mathbf{x}_{k-1}, \boldsymbol{\theta}_{k-1}) = \begin{bmatrix} \mathbf{f}_\mathbf{x}(\mathbf{x}_{k-1}, \boldsymbol{\theta}_{k-1}) \\ \boldsymbol{\theta}_{k-1} \end{bmatrix} \tag{3}$$

and $\mathbf{w}_k \sim N(\mathbf{0},\mathbf{Q}_k)$ being a white Gaussian noise, representing uncertainty on the model and parameter evolution. The following model governs the measurements of the process:

$$\mathbf{y}_k = \begin{bmatrix} \mathbf{I}_{3x3} & \mathbf{0}_{3x3} \\ \mathbf{0}_{3x3} & \mathbf{0}_{3x3} \end{bmatrix} \cdot \mathbf{z}_k + \mathbf{v}_k \tag{4}$$

with $\mathbf{v}_k \sim N(\mathbf{0},\mathbf{R}_k)$ being a white Gaussian noise, representing measurement uncertainty. According to the EKF formalism, measurements integration is carried at each discrete time using EKF prediction and updating equations, resulting in estimates $\hat{\mathbf{z}}_k$ and $\hat{\mathbf{P}}_k$ of the augmented state and its associated covariance matrix, respectively. A previous tuning of the filter gave values for $\mathbf{Q}_k$ and $\mathbf{R}_k$. *A priori* information about extended model state and covariance matrix is embodied in the initial estimates $\mathbf{z}_0$ and the associated covariance matrix $\mathbf{P}_0$.

IV. REMOTE LABORATORY

The system discussed in this paper is available as a remote laboratory [3][6]. This means, allows remote use of a real experiment. In this scenario, TCP/IP and *http* protocols can be used for keeping real time communication between user and process. In the distance education context, remote laboratories are known to be very efficient, due to the high level of interaction. The main idea of the proposed remote lab is to use the World Wide Web as supporting communication platform and a Web Browser as its interface [11]. A similar system proposed by other authors is presented in [12].

The client software required to run the experiment is the everywhere available Web Browser and Java Applet plug-in. This second is available for the most commom web browsers. The Web server implements the interface between the remote client and the physical experiment in the laboratory. In real-time the user can adjust controller parameters, reference signals and see them on a continuously updated plot of chosen signals as well as on 2-D visualization of the water levels.

Remote laboratories can support one or more online experiments [1][3][13]. Fig. 4 shows a possible solution for a remote lab with an arbitrary number of experiments, based on a unique Web server.

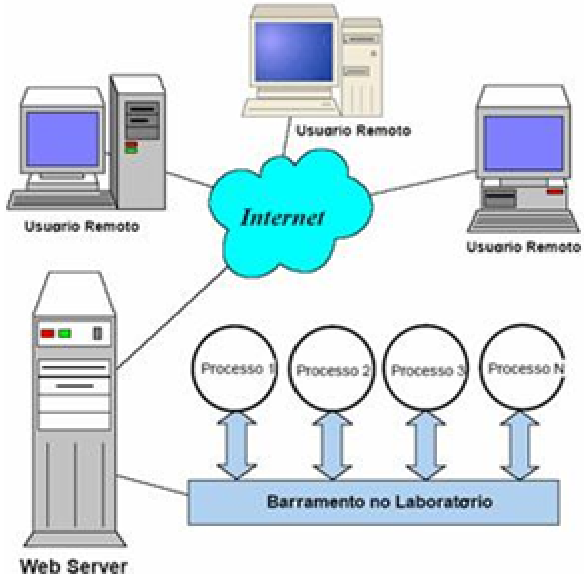

Fig. 4 – Remote laboratory architecture with an arbitrary number of experiments

A.Web Server Characteristics

The Web Server used in this remote laboratory is Apache TomCat, which supports Java Server Pages and Servlets and allow the use of Java 2 Enterprise Edition Technologies – J2EE. To manage the experiments data the server uses FireBird. FireBird is a full-featured SQL open source database engine. All results and parameters are stored on database [5] for future evaluation.

Another important aspect on the server side is a Java Socket Server application that receives the client connection and send back the experiment results in real time at each 0,5 seconds [5]. The client is a Java applet application that plots a 2D graphics of the liquid levels and actuator signals. In this work the server side is composed of only open source applications. The complete remote laboratory communication architecture is presented on Fig 5.

In this experiment, the controller was designed using the linearized model and without integral term. In the second run, the integral term was considered, with the controller achieving satisfactory results, *i.e.*, small steady state error even after change on reference input (see Fig. 7). Even with satisfactory results, the integral term may lead to undesired effects such as overshoot if model mismatches. This made adjusting the integral gain a difficult task

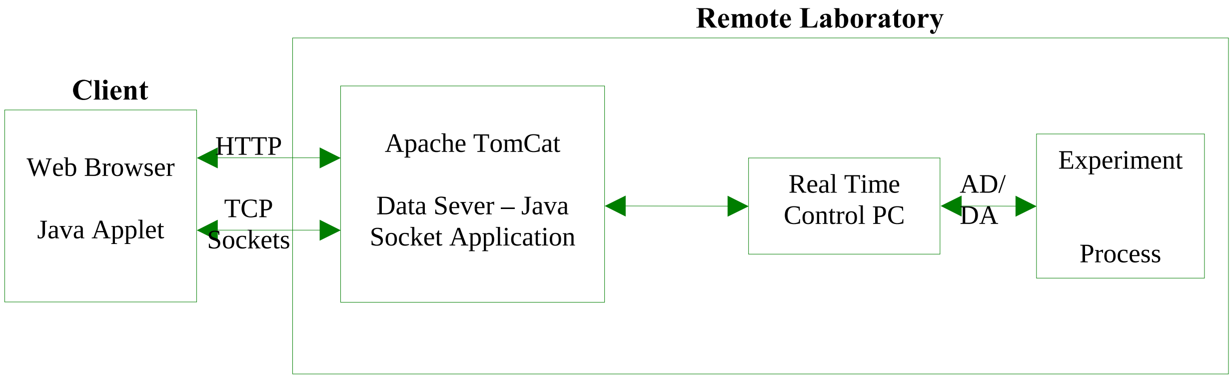

Fig 5 - Remote Laboratory Communication Architecture

B.Real Time Control Program

An IBM-PC implements interrupt-based real time control at 10Hz. This approach makes possible process communication in every control cycle. The communication frequency is 2Hz, since it is not necessary to display every data results. This leads to a very low data rate on the client side.

V. EXPERIMENTAL EVALUATION

This section presents some experimental results, following the system setup of Fig. 3. Fig. 6 shows the estimated levels (h_i-hat) obtained in a run with reference changing from 4cm to 7cm at time t=400s. The controller used no integral action (K_i=0), thus steady state error is expected if model mismatches. With t<400s, the controller presented satisfactory results. The model parameters were k_2 = 22, k_{13} = 31 and k_{32} =59. These are approximated parameters, previously determined using steady level measurements taken with constant inflow u. Thus, EKF was not used for determining these parameters. After change of operating point, the system performance has been degraded.

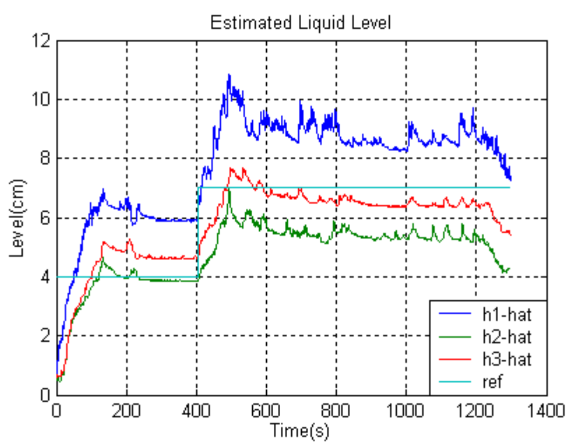

Fig. 6 - Estimated levels and reference for controller without integral action using approximated parameters.

The estimated parameters during this experiment are shown in Fig. 8. The initial values are the same used in the previous run, but after some time the estimated values stabilize around k_2 = 17.61, k_{13} = 32.06 and k_{32} =36.68. These are now considered as estimated parameters.

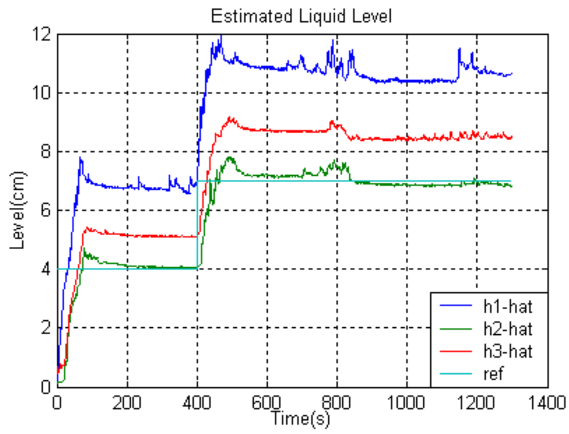

Fig. 7 - Estimated levels and reference for controller with integral action using approximated pareameters

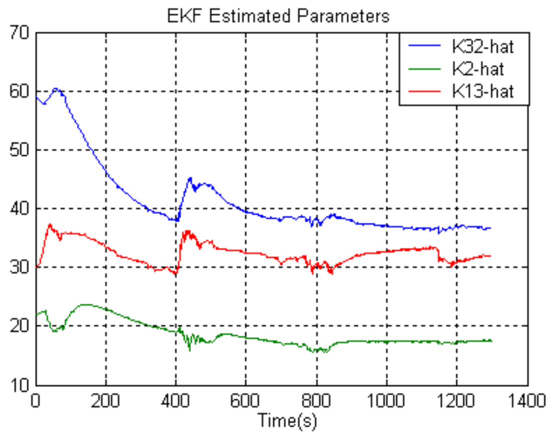

Fig. 8 - Estimated valve parameters.

In the third run, the first experiment without integral action was repeated, but considering the estimated parameters in computing the control law terms. Since there is no integral action, steady state errors mean model mismatch, at least for small frequencies. The obtained results shown in Fig. 9 presented satisfactory performance for the entire experiment.

It should be pointed out that the stochastic model used for estimation incorporates only Gaussian noise. Thus, non-zero mean perturbations can lead to divergence on the estimated parameters. In the case of the used sensor, composed of a floating buoy connected to a resistive potentiometer, stick-slip due to static friction acting on sensor axis. This can be verified in the curves of Fig 9 where stick-slip effect affects directly the estimated parameters of Fig. 11.

Stick-Slip effect difficult control and leads to a wrong interpretation on the EKF. It has been verified in Figure 10 at about 500s as well as at 1000s (on level $\bar{h}_1$ only). These effects were noticed in practice and the wrong interpretation on EKF is shown in Fig. 11.

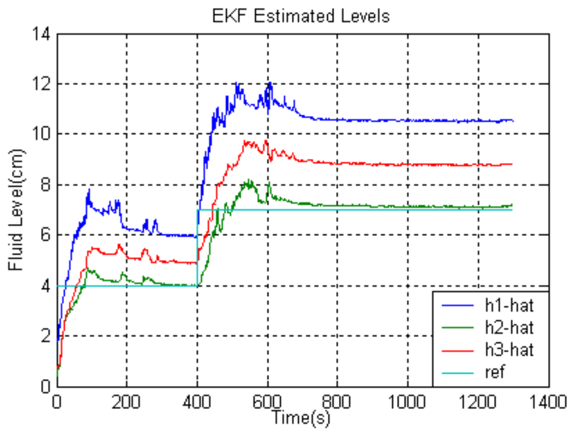

Fig. 9 - Estimated levels and reference for controller without integral action using estimated parameters.

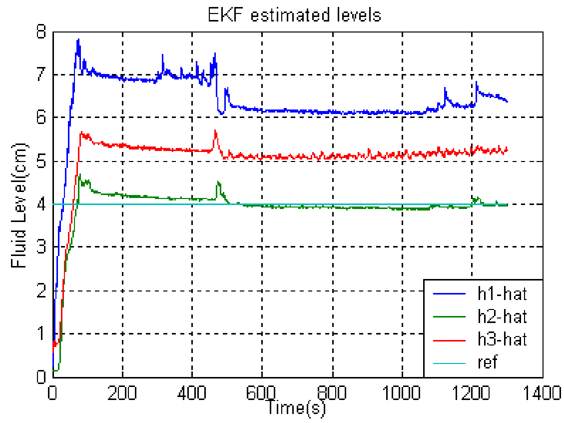

Fig 10 - Stick-Slip occuring during an experiment.

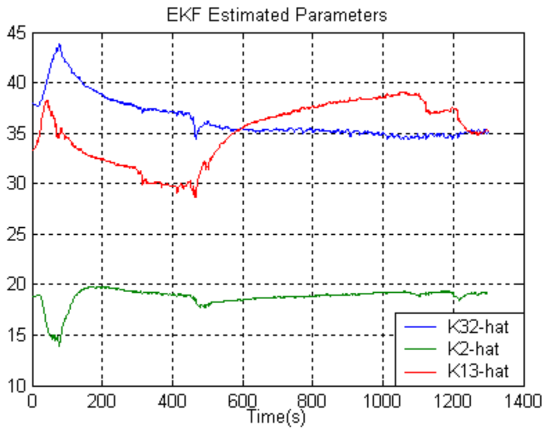

Fig. 11 - Stick-Slip influence on the estimated parameters

The prevention of this effect can be achieved by using non-mechanic sensors, such as ultrasound and capacitive sensors. However, other effects can appear with these sensors as consequence of environmental phenomena.

VI. CONCLUSIONS AND FUTURE WORK

This paper investigated the use of state space control and stochastic estimation in a third order liquid level experimental apparatus. Experimental evaluations indicated the importance of accurate model estimation in this process. This has been achieved, at least for steady state.

It has been verified that non-modelled effects trouble the stochastic estimator. In this case, sensor *stik-slip*, can lead to diverging estimation. Such a non-linear affect is difficult to be modeled by a stochastic distribution. This opens to two perspectives of investigation. One very interesting possibility is the use of EKF in the implementation of adaptive control for the liquid level process, with estimated parameters being used for real-time adaptive control. This, however, can only be achieved by an in dept study on how to detect and take into account non-modeled effects during operation.

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