CODIFICAÇÃO DE FONTE E CANAL COM INFORMAÇÃO LATERAL

UMA FORMA DE CODIFICAÇÃO CONJUNTA

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FEEC - Unicamp

Summary

- **INTRODUCTION**

- **DIRTY PAPER CODING**
  - CODING FOR MEMORIES WITH DEFECTS
  - PARTITIONED LINEAR BLOCK CODES
  - COSET CODES

- **DISTRIBUTED SOURCE CODING**
  - BINNING VS QUANTIZATION
  - COSET CODES

- **DISCUSSION**

Information Theory
Some seminal papers by Shannon

- **Channel Coding, 1948**
- **Source Coding, 1948, 1958**
- **Cryptography, 1949**
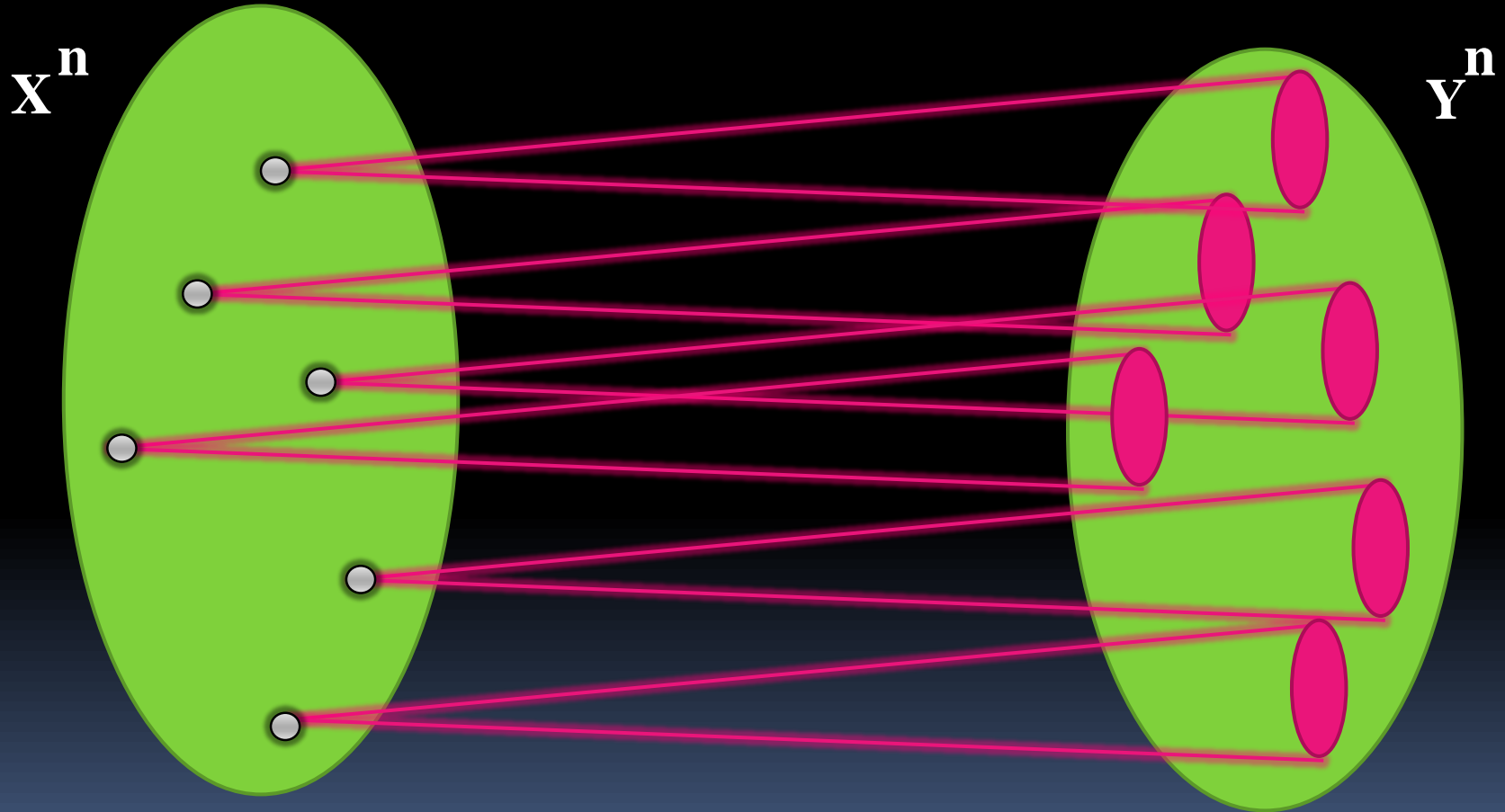
Channel coding - example
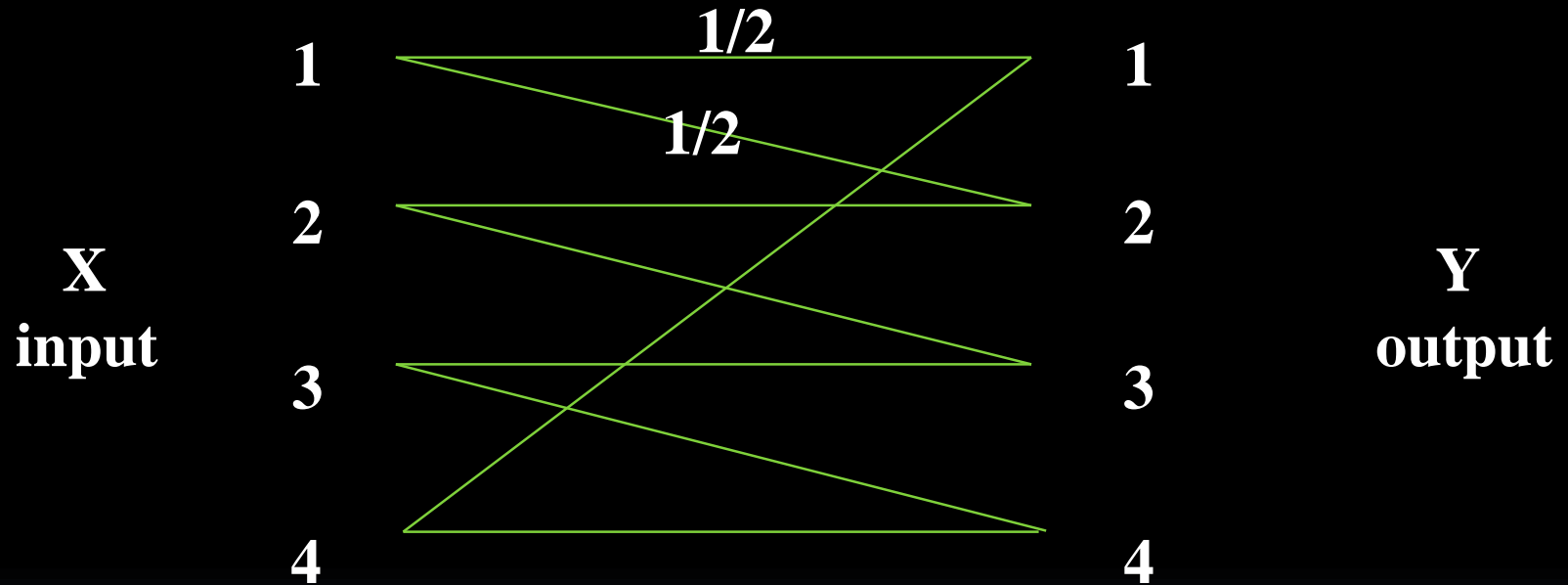
Capacity: 1 bit/transmission
Best code: Use only inputs {1 , 3}
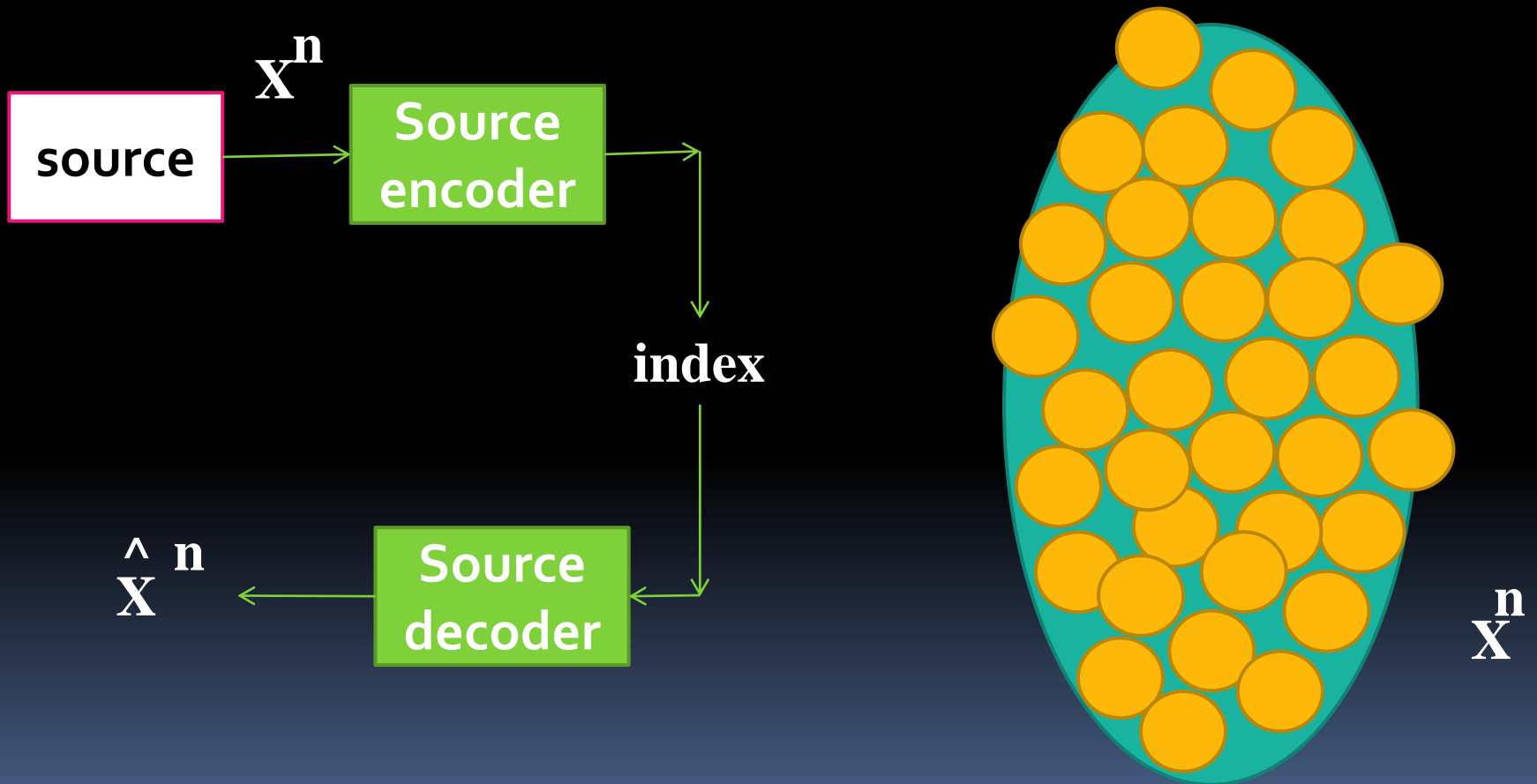
Exercise moderation!!

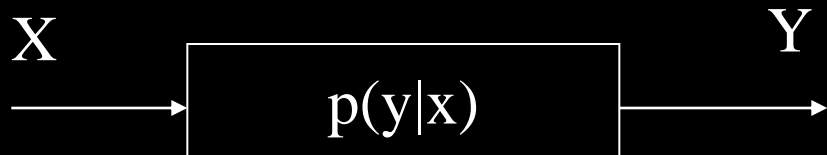# Channel coding

Typically need larger codes, n >>1

X^n

Y^n

Similar to sphere paching

Source coding:
Get good representation of source with few bits

X^n

source → **Source encoder** →

index

$\hat{X}^n$ ← **Source decoder** ←

X^n

Similar to sphere covering

Introduction

CHANNEL CODING

X ⟶ [p(y|x)] ⟶ Y

$$C = \max_{p(x)} I(X;Y)$$

SOURCE CODING

[p(x)] ⟶ X
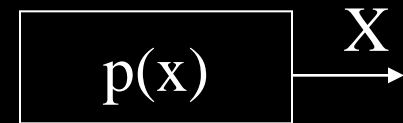
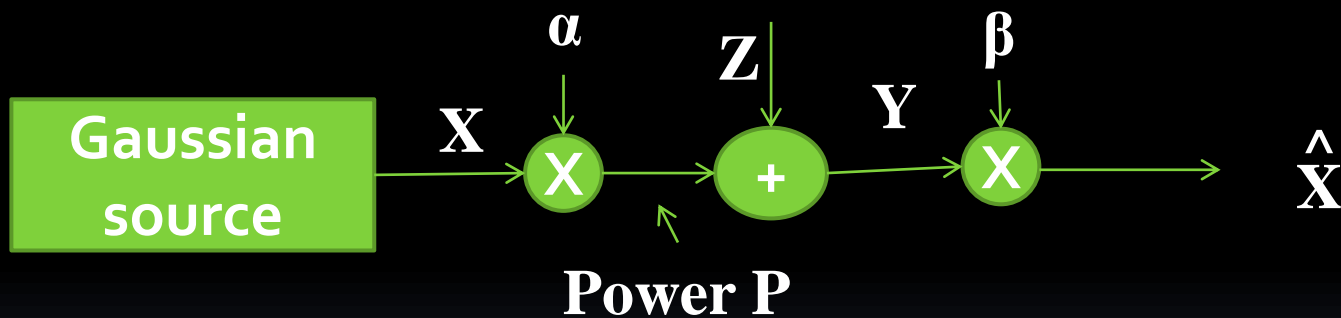$$R(D) = \min_{p(\hat{x}|x) \,:\, E\,d(x,\hat{x})<D} I(X;\hat{X})$$

Source and channel coding in communication system
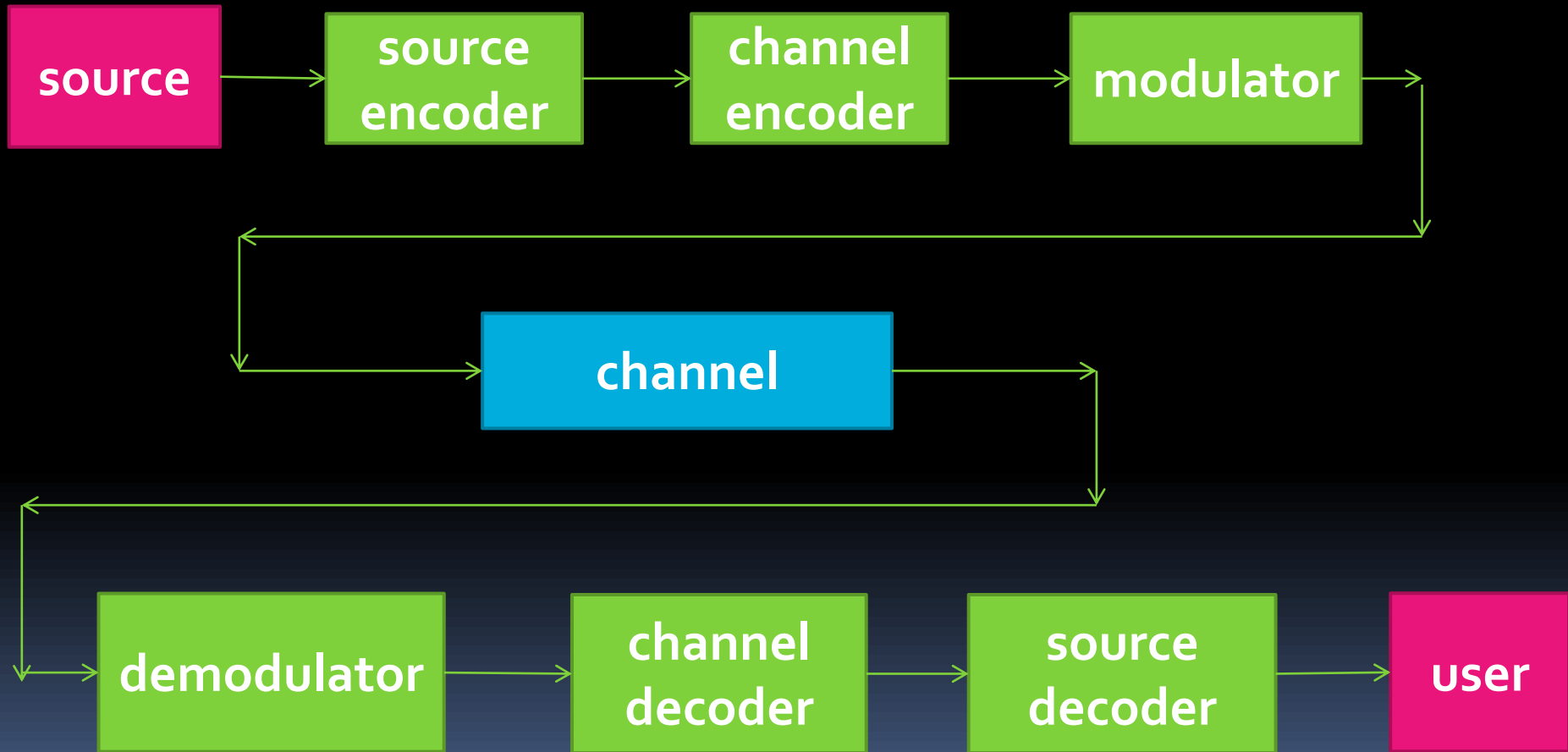
Joint source and channel coding

Can be simple if source and channel are matched

Gaussian noise

$$\alpha \qquad Z \qquad \beta$$

| Gaussian source | $\xrightarrow{X}$ | $\times$ | $\xrightarrow{}$ | $+$ | $\xrightarrow{Y}$ | $\times$ | $\xrightarrow{}$ | $\hat{X}$ |

Power P

Rate distortion theory
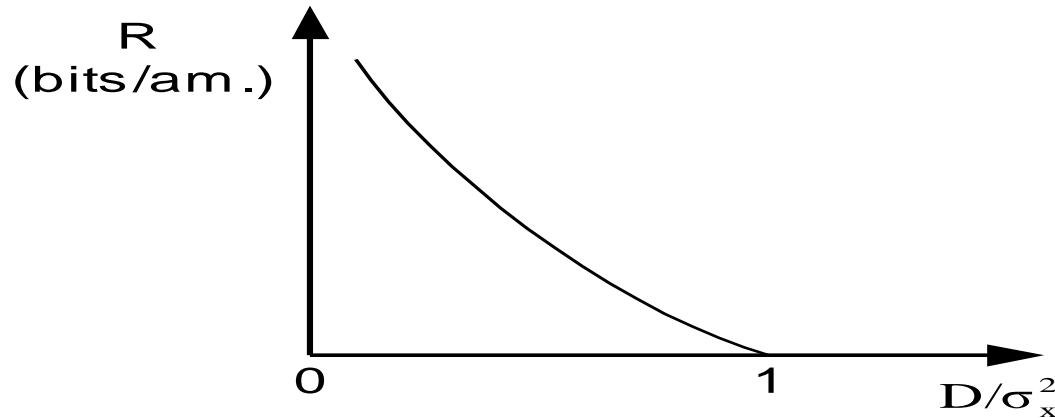
Example: Gaussian source with memory

$$D(R) = 2^{-2R} \sigma_x^2$$

$$or$$

$$R(D) = \frac{1}{2} \log_2 \left(\frac{\sigma_x^2}{D} \right)$$

$$\therefore \quad Max\ SNR(dB) = 10 \log_{10} \left(\frac{\sigma_x^2}{D(R)} \right) = 20R \log_{10} 2 \cong 6R$$
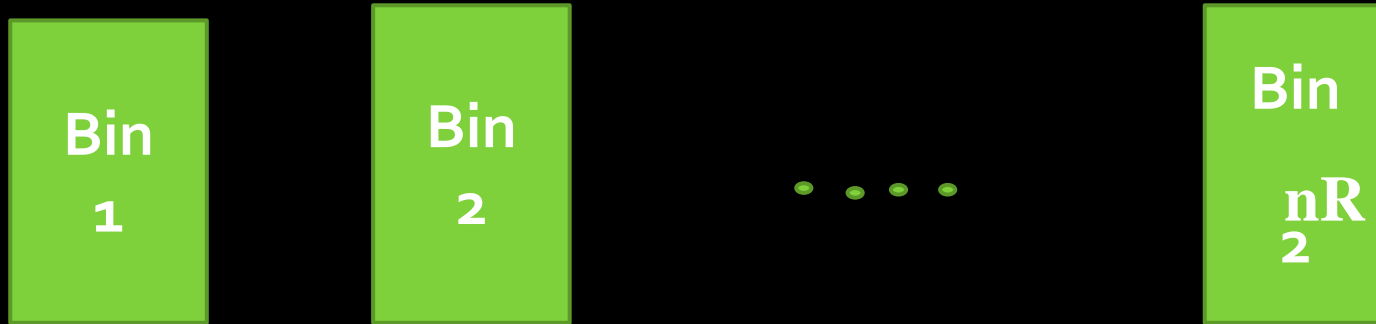
RMS distortion

Dirty paper coding

CODING FOR MEMORIES WITH DEFECTS

1	0		0	1	1	0

"stuck-at" defects – probability α

<u>Binning:</u> Randomly distribute all 2^n sequences into 2^{nR} "bins"

Bin 1   Bin 2   . . . .   Bin $\frac{nR}{2}$

$$\text{\# of sequences in each bin} = \frac{2^n}{2^{nR}} = 2^{n(1-R)}$$

$$E \text{ (\# of matching sequences in a bin)} = 2^{n(1-R)} \cdot 2^{-n\alpha}$$
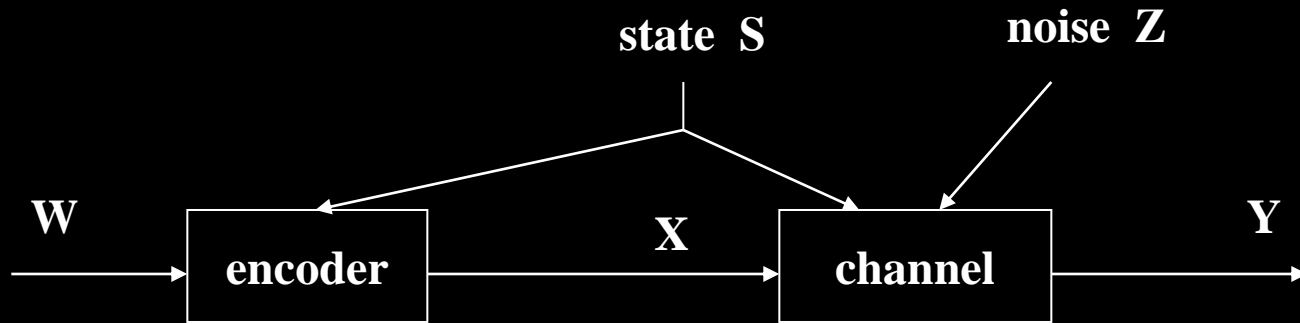
$$= 2^{n(1-\alpha-R)}$$

Note:

If R < 1- α → guaranteed to have a match

Thus Capacity = 1-α bits per memory cell

(same as if receiver knew defect positions)

Model

General solution (Gelfand and Pinsker):

C = max (I(U;Y) - I(U;S))
    $p(u,x|s)$

In the example: U = Y

Analog version

S ~ N(0,Q)   Z ~ N(0,N)

W → [Encoder] —X→ ⊕ —→ ⊕ —Y→ [Decoder] → Ŵ

Power P

channel
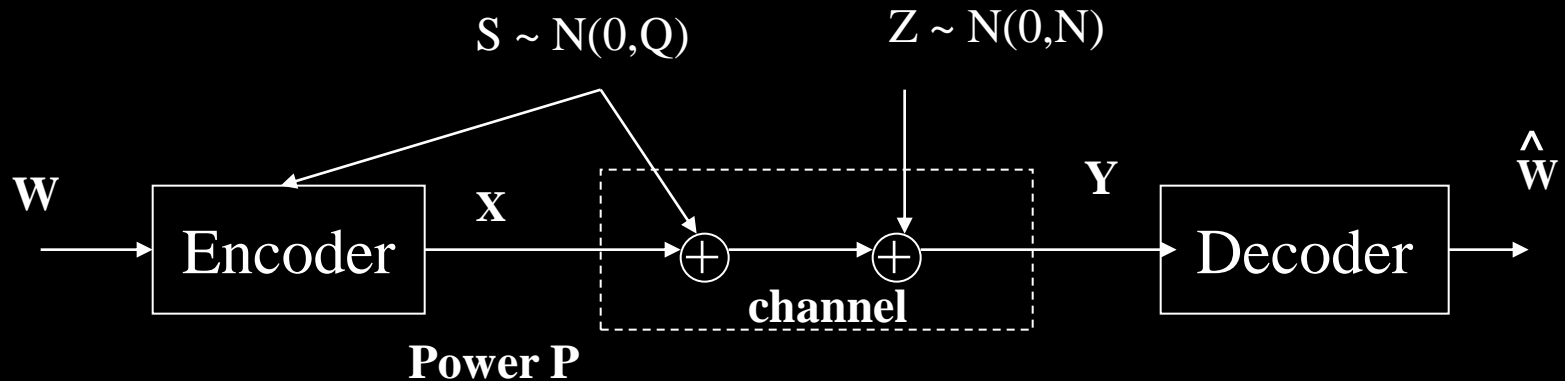
$$C = \max_{p(u,x|s)} (I(U;Y) - I(U;S))$$
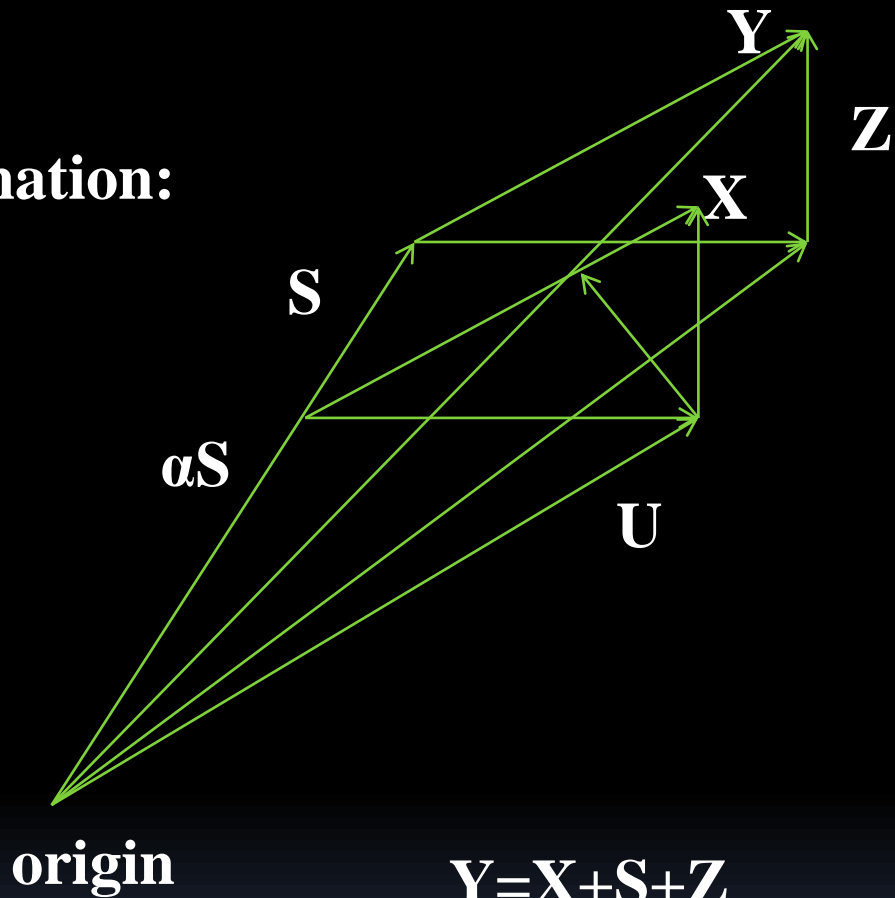
Adopt $U = X + \alpha S$, maximize over α

Result:
  $C = \tfrac{1}{2} \log (1 + P / N)$, independently of Q

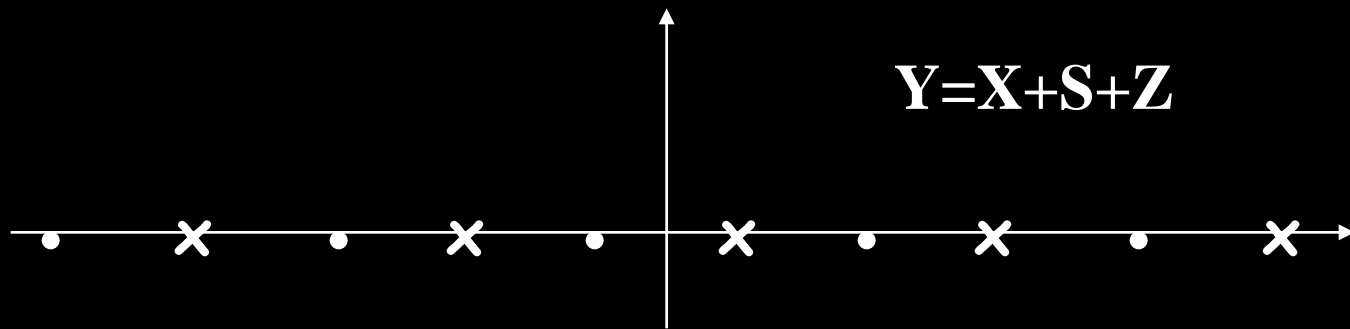 Obtained with $\alpha = P / (P+N)$

Geometrical explanation:

S

αS

Y

Z

X

U

origin

Y=X+S+Z

U=X+αS

Approximate methods

QIM (QUANTIZATION INDEX MODULATION)

$$Y = X + S + Z$$

$$f_\Delta (y) = \mathrm{mod}\,(y + \Delta/2\,,\,\Delta) - \Delta/2$$

Encoding: $X = f_\Delta (U\text{-}S) = U - S - k\,\Delta,$ k integer

Decoding: $\hat{W} = f_\Delta (Y) = f_\Delta(U\text{-}S\text{-}k\,\Delta + S + Z) = f_\Delta (U + Z)$

Watermark example
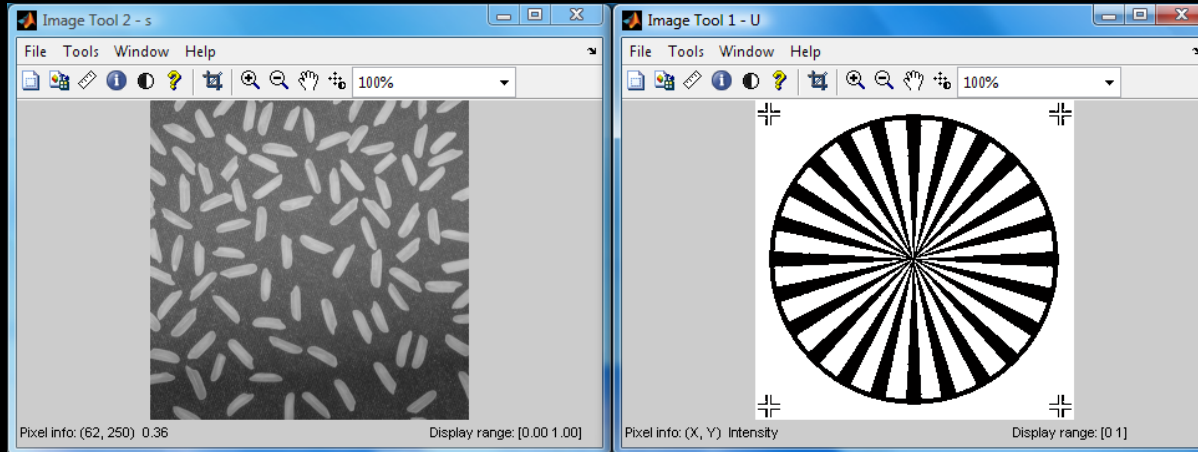
Images for host signal and watermark
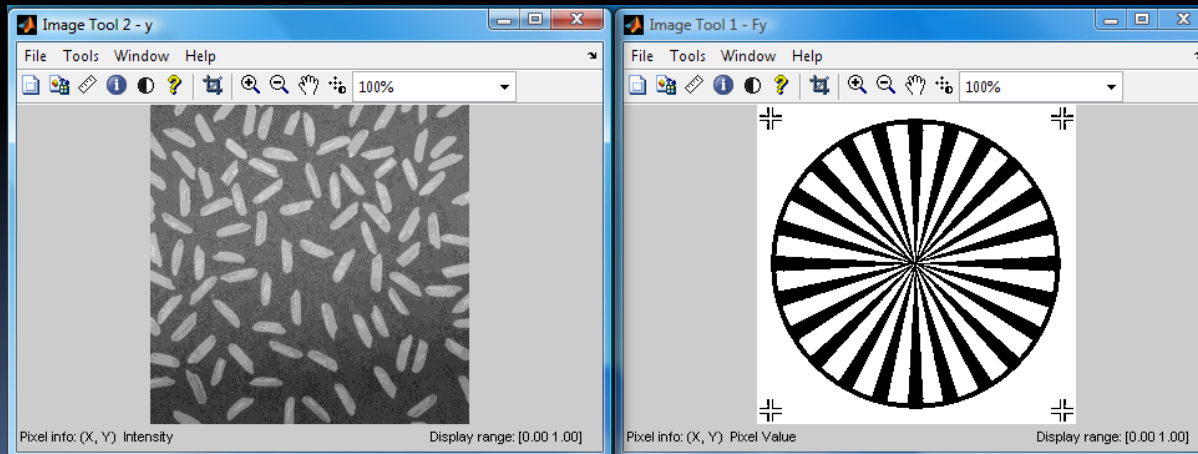
Images for received host signal and received watermark

Variations

- PARTITIONED LINEAR BLOCK CODES (HEEGARD, 1983)

- COSET CODES (FORNEY, RAMCHANDRAN)

- APPLICATIONS WITH BCH CODES, REED-SOLOMON CODES

- APPLICATIONS WITH LATTICES

- APPLICATIONS WITH LDPC, LDGM

Distributed source coding

$$R(D) = \min_{p(w|x)\,:\,Ed(X,\hat{X}) < D} (\; I\,(X\,;W) - I\,(Y\,;W)\;)$$

Simple example

- X and Y vectors of size 3

- Hamming distance ≤ 1

- **Case 1:** Y known by all (i.e., encoder and decoder)

     R = H(X|Y) = 2 bits   (just send X+Y)

- **Case 2:** Y known only by decoder – use coset codes

     Here too R = 2 bits

     Send index of coset of X (use a repetition code)

     Decoding: Using coset of X and Y, recover X exactly

Repetition code – standard array

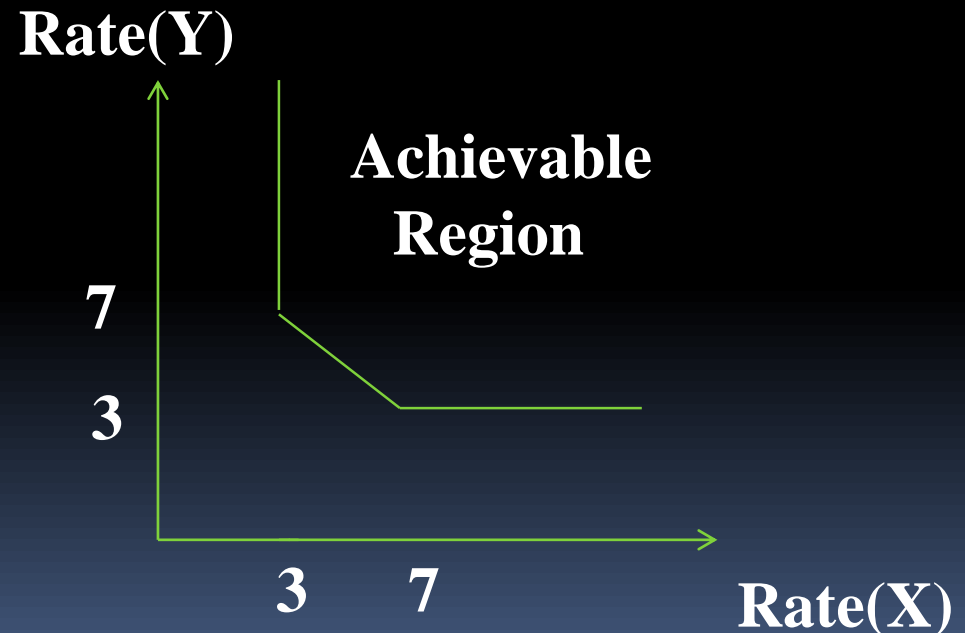|		
Code (coset 0)	**000**	**111**
Coset 1	001	110
Coset 2	010	101
Coset 3	100	011

Can get X from Y and coset number

Another simple example:

Let X and Y be unif. distributed length 7 binary sequences
Hamming distance (X,Y) ≤ 1

H(X) = H(Y) = 7 bits
H(X|Y) = H(Y|X) = 3 bits
H(X,Y) = 10 bits

Rate(Y)

Achievable Region

7

3

3   7

Rate(X)

Encoding: use 3 bits (8 possible cosets)
  To encode X use coset of a Hamming (7,4) code

Decoding:
  Based on coset number and on Y, find X

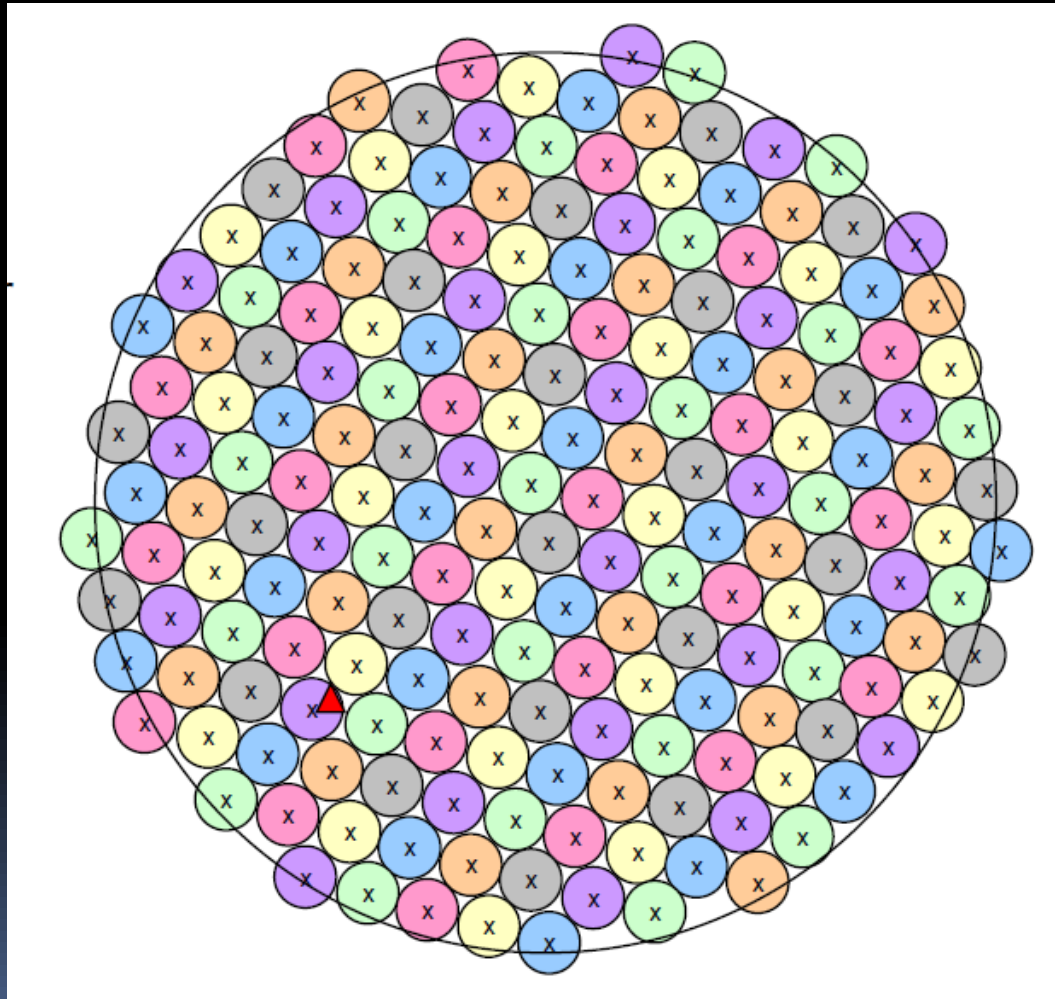# Binning operation

Dual operations

- **QUANTIZATION**
  integer division resulting in quotient

- **BINNING**
  integer division resulting in remainder

Applications

- DIGITAL WATERMARKING

- STEGANOGRAPHY

- CELLULAR TELEPHONY (DOWNLINK)

- COGNITIVE RADIO

- RADIO BROADCASTING
     DIGITAL-TV OVER ANALOG-TV (CHINOOK COMM., BOSTON AREA)
     DIGITAL RADIO OVER FM RADIO (ALTERNATIVE TO IBOC AND DRM)

- VIDEO COMPRESSION (DISTRIBUTED SOURCE CODING)

- VIDEO SYNCHRONIZATION

Information theory is alive and well !